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ABSTRACT

This is a project of the National Council of Teachers of Mathematics (NCTM) designed to assist leaders in mathematics education who conduct in-service programs for teachers. Six conferences were held at sites across the country during the summers of 1986 and 1987. Approximately 25 different 3-member teams consisting of school and college leaders attended each 4-day conference. The teams were selected on the basis of their qualifications and experience in the use of computers to enhance mathematics instruction and on their experience and expectations to conduct in-service programs. The materials in this packet were submitted to NCTM staff members to be used throughout the conferences and to be used as a resource by conference participants when they design in-service programs following the conference. Included are: (1) reference papers for conference presentations; (2) resource papers; (3) sample materials to be used in working with groups of teachers; and (4) sample student materials. (TW)

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Computers in Mathematics Classrooms

1986
1987

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NCTM

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Supported by a Grant from The National Science Foundation

A Project of: National Council of Teachers of Mathematics

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Printed in Athens, Ohio
July, 1987

Introduction

Computers in Mathematics Classrooms is an NCTM project designed to assist leaders in mathematics education to conduct in-service programs for teachers. Six conferences will be held at sites across the country during the summers of 1986 and 1987. Approximately twenty-five different three member teams consisting of school and college leaders will attend each four day conference. The teams were selected on the basis of their qualifications and experience in the use of computers to enhance mathematics instruction and on their experience and expectations to conduct in-service programs.

The materials in this packet were submitted by staff members to be used throughout the conference and to be used as a resource by conference participants when they design in-service programs in the coming months. Included are reference papers for conference presentations, resource papers, sample materials to be used in working with groups of teachers, and sample student materials. We are grateful, also for the help of Professors Bert Waits and Frank Demana, who provided a function grapher for 1987 participants and related materials in section 10S.

The project is supported by a grant from The National Science Foundation (NSF). Also, Apple Computer, Inc. provided approximately $80,000 worth of equipment and software for use in the project. Several other companies provided software and hardware for the project. They are mentioned in the list of display materials.

This material is based upon work supported by The National Science Foundation under Grant OSA 8470369 to The National Council of Teachers of Mathematics. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of The National Science Foundation or of The National Council of Teachers of Mathematics.

Special thanks are due to individuals who have contributed to its success. Joe Crosswhite was president of NCTM when the idea of the project was born. He encouraged the development of the proposal and actively participated in the project planning meetings. John Dossey. the current president of NCTM, continues that level of support and encouragement for the project. In the operational stage, Jim Gates, NCTM Executive Director of NCTM, has taken a genuine interest in the project and helped it develop. Betty Richardson, NCTM Director of Convention Services, completed the arrangements for each conference site. She is ably assisted by Betty Rollins. In Athens, Ohio I have had the assistance of Wanda Sheridan who managed a thousand details from operating the word processor and answering participant questions to preparing menus and housing lists. Peggy Sattler and Mark Penman developed the visual designs and Jane Dial was responsible for publishing the materials.

Our hope is that these conferences will stimulate an increase in the number of in-service offerings and improve the quality of programs to help teachers use computers effectively in mathematics classrooms.

Len Pikaart, Director
Athens, Ohio
July 1987

Session 2
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Computers in Mathematics Classrooms

Conference Agenda

Thursday

Session #	Time	Topic	Room*
0	9:30-10:00 a.m.	Registration	Near 1
1	10:00-10:30 a.m.	Welcome, Review of Materials Conference Goals	1
2	10:30-11:00 a.m.	Meet the Staff	1
3	11:15-Noon	Grade Level Meetings:	
3E		Elementary	2
3M		Middle	3
3S		Secondary	1
	Noon-1:00	Lunch	
4	1:00-1:45 p.m.	Grade Level Meetings:	
4E		Elementary	3
4M		Middle	2
4S		Secondary	1
5	1:55-2:40 p.m.	Grade Level Meetings:	
5E		Elementary	3
5M		Middle	1
5S		Secondary	2
6	2:50-3:35 p.m.	Ethics in Computer Uses	1
7	3:35-4:15 p.m.	Open Forum	1
	4:15-4:30 p.m.	Evaluation	1
	7:00-9:00 p.m.	Computer Laboratory (Optional)	2

Friday

Session #	Time	Topic	Room*
	8:15-8:25 a.m.	Evaluation Review	1
8	8:25-9:10 a.m.	Software Reviews and Evaluation	1
	9:10-9:25 a.m.	Coffee/Tea Break	
9	9:25-10:10 a.m.	Grade Level Meetings:	
9E		Elementary	1
9M		Middle	2
9S		Secondary	3
10	10:20-11:05 a.m.	Grade Level Meetings:	
10E		Elementary	1
10M		Middle	3
10S		Secondary	2
11	11:15-Noon	Grade Level Meetings:	
11E		Elementary	2
11M		Middle	3
11S		Secondary	1
12		Research Summary	
	Noon-1:10 p.m.	Lunch	

Room* 1: Large Conference Room
2: Computer Laboratory
3: Small Conference Room

13	1:10-1:55 p.m.	Grade Level Meetings:	
13E		Elementary	3
13M		Middle	1
13S		Secondary	2
14	2:05-2:50 p.m.	Grade Level Meetings:	
14E		Elementary	2
14M		Middle	1
14S		Secondary	3
15	3:00-3:45 p.m.	Grade Level Meetings:	
15E		Elementary	1
15M		Middle	2
15S		Secondary	3
16	3:55-4:40 p.m.	In-service Preparation	1
	4:40-4:50 p.m.	Evaluation	
	7:00-9:00 p.m.	Computer Laboratory (Optional)	2

Saturday

Session #	Time	Topic	Room*
17	8:15-8:25 a.m.	Evaluation Review	1
18	8:25-9:10 a.m.	Classroom Management	1
	9:10-9:25 a.m.	Coffee/Tea Break	
19	9:25-10:10 a.m.	Authoring Systems	1
20	10:20-11:05 a.m.	Grade Level Meetings:	
20E		Elementary	2
20M		Middle	3
20S		Secondary	1
21	11:15-Noon	Grade Level Meetings:	
21E		Elementary	3
21M		Middle	2
21S		Secondary	1
	Noon-1:10	Lunch	
22	1:10-1:55 p.m.	Grade Level Meetings:	
22E		Elementary	3
22M		Middle	1
22S		Secondary	2
23	2:05-2:50 p.m.	Equity and Careers	1
24	2:05-2:50 p.m.	Symbol Manipulation Systems	3
25	3:00-3:45 p.m.	Curriculum Implications	1
26	3:45-4:00 p.m.	Preparation for Team Meetings	1
27	4:00-4:45 p.m.	Team Meetings (No Evening Laboratory)	Choice

Sunday

Session #	Time	Topic	Room*
28	8:30-9:15 a.m.	Team Seminars Geographical Groups	To Be Assigned
29	9:20-10:15 a.m.	10 Minute Seminar Reports	1
	10:15-10:30 a.m.	Coffee/Tea Break	
30	10:30-11:15 a.m.	A Look at the Future	1
31	11:15-11:45 a.m.	Using Evaluation during In-service	1
32	11:45-Noon	Conference Charge Conference Evaluation	1

Session 2.1 Display Software

Addison Wesley
Reading, MA 01867

- Introduction to Program Language
- Strategy
- Calculus Tool Kit
- Statistics & Intuitions
- Computer Literacy

C & C Software
5713 Kantford Circle
Wichita, KS 67220

- Learning About Numbers

CBS
Interactive Learning
One Fawcett Place
Greenwich, CT 06836

- Success with Math
- Success with Algebra

Conduit Educational Software
University of Iowa
Oakdale Campus
Iowa City, IA 52242

- Algebra Drill & Practice I
- Algebra Drill & Practice II
- ARBPLOT
- Disc Learning in Trig.
- Graphing Equations
- Interpreting Graphs
- Math Program
- Surface
- Surface for Multivariate Calculus
- Exploratory Data Analysis
- Drill Shell

D. C. Heath & Company
125 Spring St.
Lexington, MS 02173

- Math Worlds: Sampling

DLM
One DLM Park
Allen, TX 75002

- Alien Addition
- Alligator Mixer
- Meteor Mission
- Meteor Multiplication
- Wiz Works

Educational Activities, Inc.
P. O. Box 392
Freeport, NY 11520

- Ratio & Proportion
- Salina Math Games
- South Dakota: An Economics and Mathematical Simulation
- EA Math. Worksheet Generator
- Basic Math. Competency Builder
- Read & Solve Math. Problems #1
- Read & Solve Math. Problems #2
- Algebra Coach
- Geometry Alive
- Math. for Everyday Living
- Arithmetic Doctor
- Comp-U-Solve
- Mindscape School Software

Gamco Industries, Inc.
Box 1911
Big Spring, TX 79721

- Number Sea Hunt
- Money

Grolier Electronic Publishing Co.
95 Madison Ave.
New York, NY 10016

- Easy Graph
- EduCalc
- EduCalc Templates
- Friendly Filer
- The Works (withguide)
- Using Grolier's Software

Harcourt Brace Jovanovich, Inc.
5 Sampson St.
Saddle Brook, NJ 07662

- Mathematics Today (K-8)
- Problem Solving (1-8)
- Microcomputer Mgnt. System for Tests (1-8)

Holt, Rinehart and Winston
383 Madison Avenue
New York, NY 10017

- Mimi "Maps and Navigation"

Houghton Mifflin Company
P. O. Box 683
Hanover, NH 03755

- Basic Math Facts
- Game Frame One (2)
- Game Frame Two (2)
- Math. Activities Courseware:1
- Math. Activities Courseware:2
- Math. Activities Courseware:3
- Math. Activities Courseware:4
- Math. Activities Courseware:5
- Math. Activities Courseware:6
- Math. Activities Courseware:7
- Math. Activities Courseware:8
- Math. Solving Story Problems (4)
- Basic Math Facts

HRM Software
175 Tompkins Ave.
Pleasantville, NY 10570

- Balance

Learning Well
200 South Service Road
Roslyn Heights, NY 11577

- Knowing Numbers
- Algebra Tutor
- Space Math

McGraw-Hill
Princeton Road
Highstown, NJ 08520

- Mathematics Software Problem Solving, Level K-8

Micro Computer Workshop
225 Westchester Ave.
Port Clinton, NY 10573

- Algebra Word Problem IV
- Equations
- Dividing Decimals
- 1-2-3 Digit Mulpl.
- Addition with Carry
- Rates & Proportion
- Subtracting Mixed Fractions

Quality Education Design
P. O. Box 12486
Portland, OR 97212

- Decimals
- Factoring and Whole Numbers
- Proportions and Percents

Random House
400 Hahn Road
Westminster, MD 21157

- Galaxy Math Facts Game
- Inside Outside Shapes
- Expanding Math Skills

Scholastic Software
730 Broadway
New York, NY 10003

- Math Assistant I
- Math Assistant II
- Math Shop
- Quations
- Arith-Magic II
- Arith-Magic

Scott, Foresman
1900 E. Lake Ave.
Glenview, IL 60025

- Addition and Subtraction 1-4
- Decimals, 1-3
- Division, 1-3
- Fractions, 1-5
- Geometry
  Angles of Triangles & Polygons
  Congruent Triangles
  More on Congruent Triangles
  Quadrilaterals
- Multiplication, 1-3
- Numeration, 1-2
- Percent, 1-2
- Picture Parts
- Pyramid Puzzler
- Number Bowling
- Space Journey
- Star Maze
- Frog Jump
- Dinosaurs and Dquids
- Spinners and Slugs

Silver Burdett Company
250 James Street
Morristown, NJ 07960-1918

- Addition and Subtraction (Grades 1-5)
- Multiplication and Division (Grades 3-6)
- Fractions (Grades 4-8)

Sunburst Communications
39 Washington Ave.
Pleasantville, NY 10570

- The Geometric Supposer: Triangles
- The Geometric Supposer: Quadrilaterals
- The Geometric Presupposer
- Green Globs & Graphing Equations
- Interpreting Graphs
- The Factory
- The Super Factory

University of Evansville Press
1800 Lincoln Ave.
Evansville, IN 47714

- Math Disc One
- Math Disc Two
- Math Disc Three
- Math Disc Four
- Math Disc Five
- 100 Math Programs for IBM
- Euclid

Weekly Reader Family Software
245 Long Hill Road
Middletown, CT 06457

- Sticky Bear Math 1
- Sticky Bear Math 2
- Sticky Bear Numbers
- Exploring Tables and Graphs 1
- Exploring Tables and Graphs 2

Addresses for Software Publishers

Commercial

Adventure International
P. O. Box 3435
Longwood, FL 32750

Apple Computer, Inc.
20525 Mariani Ave.
Cupertino, CA 95014

Baudville
1001 Medical Park Dr.S.E.
Grand Rapids, MI 49506

Carousel Software, Inc.
877 Beacon St.
Boston, MA 02215

CBS Software
1 Fawcett Pl.
Greenwich, CT 06836

City Software, Inc.
735 West Wisconsin Ave.
Milwaukee, WI 53233

Commodore Business Machines, Inc.
950 Airport Rd.
West Chester, PA 19380

COMPress
P.O. Box 102
Wentworth, NH 03282

Conduit
M310 Oakdale Hall
University of Iowa
P.O. Box C
Oakdale, IA 52319

Cross Cultural Software
5385 Elrose Ave.
San Jose, CA 95124

Cursor 64
P.O.Box 550
Santa Barbara, CA 93110

Cygnus Software
8002 E. Culver
Mesa, AZ 85207

Design Ware, Inc.
185 Berry St.
San Francisco, CA 94107

Educational Activities, Inc.
1937 Grand Ave.
Baldwin, NY 11510

Electronic Arts
2755 Campus Dr.
San Mateo, CA 94403

EQUALS
Lawrence Hall of Science
University of California
Berkeley, CA 94720

Hayden Software Co.
600 Suffolk St.
Lowell, MA 01853

Infocom, Inc.
55 Wheeler St.
Cambridge, MA 02138

Innovative Design Software, Inc.
P.O. Box 1658
Las Cruces, NM 88004

Koala Technologies Corp.
Suite 125
44962 El Camino Real
Los Altos, CA 94022

Krell Software Corp.
1320 Stony Brook Rd.
Stony Brook, NY 11790

L & S Computerware
1008 Stewart Dr.
Sunnyvale, CA 94086

Learning Company
545 Middlefield Road
Suite 170
Menlo Park, CA 94025

Lightning Software, Inc.
P.O. Box 11725
Palo Alto, CA 94306

Logo Computer Systems, Inc.
222 Brunswick Blvd.
Pointe Claire, Quebec
Canada H9R1A6

Math and Computer Education Project
Lawrence Hall of Science
University of California
Berkeley, CA 94720

McGraw-Hill
Princeton Road
Highstown, NJ 08520

Microcomputer Workshops
225 Westchester Ave.
Port Chester, NY 10573

Minnesota Education Computing
Consortium Publications
2520 Broadway Dr.
St. Paul, MN 55113

Odesta
3186 Doolittle Drive
Northbrook, IL 60062

Optimum Resource, Inc.
P. O. Box 100
Greenwoods Rd. East
Norfolk, CT 06058

Penquin Software
P.O. Box 136-M
Geneva, IL 60134

Personal Software
P.O. Box 136-M
Cambridge, MA 02138

Quality Educational Designs
P.O. Box 12486
Portland, OR 97212

Scarborough Systems, Inc.
25 N. Broadway
Tarrytown, NY 10591

Scholastic, Inc.
730 Broadway
New York, NY 10003

Sierra, Inc.
Sierra On-Line Building
Coersegold, CA 93614

Software Publishing Corp.
1901 Landings Drive
Mountain View, CA 94043

Spinnaker Software Corp.
215 First St.
Cambridge, MA 02142

Tom Smith
P.O. Box 345
Dedham, MA 02026

Sterling Swift
7901 South IH-35
Austin, TX 78744

Sunburst Communications
39 Washington Ave.
Pleasantville, NY 10570

Terrapin, Inc.
380 Greene St.
Cambridge, MA 02139

22nd Avenue Workshop
P.O. Box 3425
Eugene, OR 97403

Public Domain

San Francisco State University
Pet Shop - Thorton Hall
1600 Holloway Ave.
San Francisco, CA 94132

Softswap-Microcomputer Center
333 Main St.
Redwood City, CA 94063

Toronto Board of Education
65 Grace St.
Toronto, Ontario
Canada M6J2S4

Session 3E
Elementary

MICROS IN MATHEMATICS

TESTING/MANAGEMENT

DRILL AND PRACTICE

TUTORIAL

SIMULATIONS

PROGRAMMING/LOGO

PROBLEM SOLVING

Computer Use Questionnaire

We are asking educators to complete this questionnaire so that we might discover the extent of computer use at various levels and in various settings.

GRADE LEVEL TAUGHT __________ SCHOOL ____________________
# STUDENTS ______________ # TEACHERS ____________________
# OF COMPUTERS: IN SCHOOL ______ IN YOUR CLRM ____________________
AVG PER PUPIL MIN/WEEK ______________________________________

HRS COMPUTER TRAINING YOU'VE COMPLETED ____________________

MOST EFFECTIVE TRAINING YOU'VE COMPLETED
____

HOW DOES YOUR DISTRICT SELECT SOFTWARE FOR PURCHASE?
____

Please circle the number which best indicates the extent to which you personally utilize computers in your classroom/school:

	Extensive	Some	Little	None
Computer Awareness	4	3	2	1
Keyboarding	4	3	2	1
Grade introduced at your school	____________			
Grade stressed at your school	____________			
Computer Assisted Instruction:				
Drill and Practice	4	3	2	1
Tutorials	4	3	2	1
Simulations	4	3	2	1
For remedial use	4	3	2	1
For accelerated use	4	3	2	1
For grade level use	4	3	2	1
In Language Arts	4	3	2	1
In Social Studies	4	3	2	1
In Arithmetic/Math	4	3	2	1
In Science	4	3	2	1
Other ____________	4	3	2	1
Applications Software:				
Word Processing	4	3	2	1
Spreadsheets	4	3	2	1
Databases	4	3	2	1
Graphics ____________	4	3	2	1
Other ____________	4	3	2	1
Computer Managed Instruction:				
Record Keeping	4	3	2	1
Grade programs	4	3	2	1
IEP generation	4	3	2	1
Classroom management	4	3	2	1
Other ____________	4	3	2	1
Teacher Utilities:				
Test generation	4	3	2	1
Graphics programs	4	3	2	1
Newspaper generation	4	3	2	1
Other ____________	4	3	2	1
Programming [Language/s ________ ____________]	4	3	2	1
Networking	4	3	2	1
District In-Service for teachers	4	3	2	1

MICROCOMPUTER SOFTWARE EVALUATION

Evaluation Intended as First Screening

Program Name: ____________________ Reviewer: ____________________

Subject Area: ____________________ School: ____________________

Specific Topic: ____________________ District: ____________________

Area of Specialization: ____________________ Date: ____________________

Comments: ____

____

Overall Evaluation - Circle One:

5 Excellent program. Recommend for review without hesitation.
4 Very good program. Recommend for review.
3 Good program. Consider review.
2 Fair. Might want to wait for something better.
1 Not useful for this application/grade/etc. Do not recommend review.

PROGRAM

low			high		
1	2	3	4	5	Content is accurate.
1	2	3	4	5	Content has educational value.
1	2	3	4	5	Appropriate use of computer capabilities.
1	2	3	4	5	Content is user friendly.
1	2	3	4	5	Content is clear and logical.
1	2	3	4	5	Instructions well-organized, useful and easy to understand.
1	2	3	4	5	Flexible application.
1	2	3	4	5	Exhibits freedom from need for teacher intervention or assist.
1	2	3	4	5	Free of bias: Racial, sexual, or political.
1	2	3	4	5	Graphics and color.
1	2	3	4	5	Sound.
1	2	3	4	5	Grade level appropriate.
1	2	3	4	5	Quality of screen formats.
1	2	3	4	5	No need for external information.
1	2	3	4	5	Freedom from program errors.
1	2	3	4	5	Simplicity of user response.
1	2	3	4	5	Provides for self-pacing.
1	2	3	4	5	Appropriate and immediate feedback.
1	2	3	4	5	Branching occurs through student control.
1	2	3	4	5	Summary of student performance.
1	2	3	4	5	Degree of student involvement.

Session 3M
Pictorial Fractions

Graphical Representation of Fractions

Objective

The student will see how to make equivalent fractions.

Description

The purpose of this demonstration is to show pictorially equivalent fractions. The program asks the student to enter a fraction by typing in the numerator first, followed by a comma, followed by the denominator. Then the student is asked to place in a different denominator. The program then displays a unit whole divided into pieces corresponding to the first denominator. The second bar is the length of the fraction the student put in. The third bar is the unit whole divided into the number of pieces equal to the denominator the student put in last. The student can then count the number of pieces on the third bar to determine a fraction equivalent to his. The student will observe that if one of the division lines does not end exactly at the end of his bar then the equivalent fraction for that denominator does not exist.

Procedure

Run the program "Fraction Concept". When it asks for a fraction (a,b) put in 1/4 by typing 1,4. When it asks for a denominator type in 8. How many little 1/8's are the same length as 1/4?

Therefore 2/8 = 1/4

When it asks for the denominator type in 16. How many little 1/16's = 1/4?

Therefore ___/16 = 1/4.

When it asks for the denominator type in 12. Count how many 1/12's are the same as 1/4.

Therefore ___/12 = 1/4.

Predict how many 1/16's = 1/4? ____/16 = 1/4 Check your answer using the program. Were you right?

When it asks for the denominator type in 7. Can any number of 1/7's exactly equal 1/4? If so what is it? If not why not?

Notice the 1/4 is between 1/7 and 2/7. To which is it closest?

So we can say, "1/4 is close to 2/7 but not exactly equal to 2/7."

Predict what all of the possible denominators of fractions equivalent to 1/4 have in common. Use your rule to determine whether you can have fractions equal to 1/4 with each of the following denominators:

___/5 Yes or No

___/6 Yes or No

___/20 Yes or No

___/12 Yes or No

Check to determine of you were right.

Finding Common Denominators Graphically

Objective

Given any two fractions <1 the student will be able to determine a common denominator for the two and see that there are "many" common denominators.

Description

The student will first place two fractions into the program and will then predict a possible denominator which is common to each (i.e. denominator for which an equivalent fraction can be found for each fraction). The student will see the following on the screen:

> The first bar will be the unit bar. The second bar will be equivalent to the first fraction the student input. The third bar will be a copy of the unit bar divided into equal parts corresponding to the denominator requested. The fourth bar will be equivalent to the second fraction. The goal is to find a denominator which has parts which end exactly at the end of the first requested fraction AND the end of the second requested fraction. Any denominator which results in parts successfully meeting this requirement is a common denominator.

Procedure

Load the program "Common Denominators". Enter 1/3 into the program by typing 1,3 when requested. Place 1/4 into the program by typing the ordered pair 1,4 when requested. Now predict what would be a common denominator. Let's guess 7. Put 7 into the program where denominator is requested.

Does 1/3 = any ___/7 exactly?

Does 1/4 = any ___/7 exactly?

Since you cannot write either 1/3 or 1/4 in terms of ___/7 then 7 cannot be a common denominator.

Let's try 6. Place 6 into the program when denominator is requested.

Does 1/3 = any ___/6 exactly?

Does 1/4 = any ___/6 exactly?

Since 1/3 does, 6 is a possible candidate for a common denominator. But since 1/4 does not, 6 cannot be a common denominator.

Let's try 12.

Does 1/3 = any ____/12 exactly?

Does 1/4 = any ____/12 exactly?

Since it does for both then 12 is a common denominator. Are there any other common denominators?

Find two additional common denominators. What are they:

Are there any others?   How many others are there?

Are they all bigger than 12?   Can you find a common denominator less than 12?

We then say that 12 is the "lowest" common denominator. What do you know about the relationship between the two denominators in the problem and the denominator in the answer?   Will that always work?

When it asks for denominator type E. This clears the screen and allows you to put in two new fractions. Put in 1/4 and 1/6. Find the lowest common denominator. What is it?

Is it the product of 4 and 6?   Is the product of 4 and 6 a common denominator?

Type E. Put in the following two fractions: 1/2 and 5/6. Find the lowest common denominator. What is it?   Is it the product of 2 and 6?   Is the product of 2 and 6 a common denominator?

Type E. Try any two fractions. Find its common denominator. Repeat for 5 pairs of fractions.

Complete the following table. Sometimes the lowest common denominator is the product of the two denominators, sometimes it is smaller than the product of the two denominators, and sometimes it is one of the two original denominators in the problem. Use the program to collect as much data as you need to formulate hypotheses and check them.

First Fraction	Second Fraction	Common Denominator

Adding Fractions Graphically

Objective

To practice finding common denominators and the answer to fraction problems where the answer is less than 1 and to find common denominators pictorially.

Description

The program expects the first fraction to be entered as an ordered pair consisting of a numerator and denominator. After entering the first fraction, the program requests the second fraction to be entered in the same manner. The student will then observe three fraction bars as output on the screen. The first fraction bar is the unit bar which corresponds to a size of 1. The second bar represents the sum of the two fractions with the second fraction placed next to the first fraction. The third bar is the unit bar divided into parts corresponding to the requested common denominator.

Procedure

Load the program "Adding Fractions". We will illustrate with the problem:

1/4 + 1/3 =

Type in the fraction 1/4 by typing 1,4

Type in the fraction 1/3 by typing 1,3

Predict a common denominator (Let's try 7). Does the fraction sum end exactly at the end of one of the 7 parts?

Try another denominator. (Let's try 12). Since 12 results in parts which end exactly at the end of the fraction sum bar it is a common denominator. Count how many 1/12's equal the length of the fraction sum bar...

____/12 = 1/4 + 1/3

Repeat with 24 as the denominator. How many 1/24's equal the length of the fraction sum bar?

____/24 = 1/4 + 1/3

Count them to check.

Type E. Try any two fractions. Make sure the sum is not longer than 1.

SESSION 3S
STATISTICS
(45 MINUTE CLASS)

OBJECTIVE: To see how the computer can assist in the teaching of basic statistical topics, including mean, STD, frequency classes, histograms, and ogives. Emphasis is on (1) classroom use of programs that support textbook and chalkboard development of standard topics, (2) LISTing of short programs that are essentially step-by-step mathematical operations, and (3) homework problems that require the use or writing of programs.

PRELIMINARIES: RUN, LIST, CATALOG, Control-Reset and Control-S (15 minutes)

Transparency 1, top half. These standard DOS commands provide fast and direct control while running programs. Many teachers already use these commands as readily as they drive a car, but some do not.

Recommended sequence: (1) Put the Short Programs Disk into a drive; (2) turn computer on; (3) view the initial program; (4) LIST it; (5) use Control-S and S to stop-and-start scrolling; (6) LIST again and use Control-Reset to break; (7) CATALOG; and (8) announce that LISTings of all these programs are printed in the conference notebook.

MEAN and STANDARD DEVIATION (15 minutes)

Transparency 1, bottom half. With Program P47 showing on Transparency 1, comment:

An introduction to statistics programs on the Short Programs Disk. (See the pages beginning on 3S.1-1)

Line 30: Students can later type in their own test scores or other data

Line 40: Emphasize that S = S + X means

(New S) = (Old S) plus X

Tell exactly what Line 40 does, beginning with I=1 and ending with I=6. Then LIST the program, noting that it is exactly as shown on Transparency 1. Then RUN it.

Transparency 2, top half. Explain what RND(1) means. Show that Program P48 does the same thing that P47 does, but to 100 random numbers instead of 6 test scores.

While P48 runs, show bottom half of Transparency 2. That's Program P49. STD is computed by the formula at Line 40. Students should compare it with the same formula in their textbook or notes. Emphasize that the program is literally "doing" this formula.

Break the run of P48, and RUN P49. Comment that this little program can be easily adapted to find the mean and STD of almost any collection of data.

If time permits, examine P50. It computes percentages of data lying within 1 and 2 STDs of their mean.

FREQUENCY CLASSES (15 minutes)

Transparency 3, top half. Explain that Line 50 reads the data, Line 60 remembers the least, and Line 70 the greatest.

Bottom half. Call students' attention to range in their textbook, and identify it as B-A at Line 120.

There, K class intervals are computed. Each has length (B-A)/K. Class 1 extends from A=C(0) up to but not including C(1). Class 2 is from C(1) up to but not including C(2). ..., and Class K is from C(K-1) up to and including (because of Lines 140 and 190) B=C(K). Just like the construction of frequency classes in many textbooks.

At Lines 150-170, each datum (or "piece of data") X(I) is checked. If it lies in the Jth class, then the frequency counter F(J) is incremented. Thus, the program carries out the tallying process just as we ourselves do it.

Finally, Lines 210-240 print the K classes and their frequencies.

Note that the above program analysis is not intended as an introduction to frequency classes. Students should have already formed some frequency-class problems "the hard way." Program analysis then strengthens and extends students' grasps of the concept of frequency classes.

Run Program P44. When prompted, input K=2. Rerun using K=5, and then K=10.

Transparency 4. The step-by-step textbook development from data collecting to frequency classes and then to histograms and ogives is easily illustrated by programs on MATHDISK FIVE. Spend a several moments running Program 166 (type RUN HISTOGRAM and tap RETURN) and Program 167 (Break, and then type RUN OGIVE and tap RETURN).

If time permits, install Datadisk 5A in Drive 2, and use Program 166 to compute, for example, the percentage of 1983 American League batting averages between .22 and .27.

Emphasize that the chain of ideas on the transparency really is a chain: Program 166 starts with data and groups them into frequency classes. It then uses these classes to form a histogram, uses the histogram to form an ogive, and finally, uses the ogive to compute percentages.

Mention that percentages like this can also be found as the end-link of quite another chain of basic statistical ideas, shown on Transparency 5.

CONCLUDING REMARKS:

(1) The collection "Short Programs for Teaching and Learning High School Mathematics" is available not only for Apple, but also for IBM-PC. Ask a conference staff member or write to the University of Evansville Press, 1800 Lincoln Avenue, Evansville, IN 47722 (812-479-2488).

(2) Thirty-six fully developed programs comprise MATHDISK FIVE: PROBABILITY AND STATISTICS (University of Evansville Press, 1986). Its workbook contains two-hundred exercises, about 10% of which are marked "For programmers."

(3) On nearby pages are printed a selection of statistics problems based on "Short Programs." Included are solutions, objectives and suggested follow-ups.

STATISTICS PROBLEMS

Problems 1-7 require the use of "Short Programs" P47-P50 and P44. Problems 8-14 are for programmers.

1. Use Program P47 to find the mean of the following test scores: 95, 82, 90, 96, 72, 81.

   Mean = ______________

2. Continuing Problem 1, suppose the final test score is 87 instead of 81, and the other scores remain unchanged. Use Program P47 to find the mean score.

   Mean = ______________

3. The scores on a certain test were 82, 84, 67, 91, 75, 69, 72, 94, 81, 68, 78, 71, 80, 95, 82, 73, 72, 83, 70, and 93. Use Program P50 to find the mean and standard deviation (STD) of these scores.

   Mean = ______________
   STD = ______________

4. Continuing Problem 3, use the output of Program P50 to estimate the number of scores that are between the numbers Mean - STD and Mean + STD.

   Estimate = ______________

5. Run Program P44 (Frequency Classes). Input K=2. What number X is the "divider" between Class 1 and Class 2?

   X = ______________

   (Class 1 is the interval up to but not including X, whereas Class 2 does include X.)

6. Run Program P44 again, using K=3.

   45.8 lies in Class _______
   49.2 lies in Class _______
   55.0 lies in Class _______
   75.6 lies in Class _______
   Total number of data = _______

About what percentage of the data lie in the first third of the interval from the least data value to the greatest data value?

Percentage = _______

7. Run Program P44 again. This time demand to see 12 classes.

How many data lie in classes 1 to 3? ________
How many data lie in classes 4 to 6? ________
How many data lie in classes 7 to 9? ________
How many data lie in classes 10 to 12? ________

If you were to run the program once again, this time grouping the data into 4 classes, what would be the class frequencies?

Answer: ___, ___, ___, and ___

8. (This is the first of the problems for programmers, to be attempted only after solving Problems 1-7.)

Modify Program P49 so that you can input data from the keyboard. After each input, your program should print the "updated" mean and STD.

9. Continuing Problem 8, include in your program a report of a continuously updated percentage of data that lie within one STD of their mean, as in Program P50.

10. Write a program that computes the mean M of N user-chosen data X(I), then repeatedly lets the user input values of X which are used to form and print the sum

(X(1)-X)^2 + (X(2)-X)^2 + ... + (X(N)-X)^2.

Use your program to demonstrate that the value of X that causes this sum to be as small as possible is M.

Thus, in this sense, the mean is a very special number because it is the number that is "closest to" the collection of data.

11. Replace each term (X(I)-X)^2 in Problem 10 by ABS(X(I)-X). Demonstrate that the value of X that minimizes this new measure of spread of data about X is the median of the data.

Comparing this result with that of Problem 10 leads to the statement that "mean is to least-squares what median is to least absolute values."

12. Into Program P44 (Frequency Classes) insert the following line: 95 A=INT(A): B=1+INT(B)

    Explain the effect of this insertion. Then modify the program so that you can input data from the keyboard.

13. Write a program that groups data into frequency classes whose boundaries are input directly by the user.

14. Augment Program P44 so that the user can repeatedly input any two numbers D and E, and the computer will output an estimate for the percentage of the data that lie between D and E.

SOLUTIONS AND NOTES

for use with STATISTICS PROBLEMS

Almost all mathematics students who use computers should examine and run short programs that illustrate main topics of their coursework. To do this, they need not be programmers. However, able students who are programmers should certainly write some original mathematical programs.

Thousands of students are adept programmers who have yet to be exposed to the enormous value of program-writing as a way to really learn mathematics - not to mention the value of program-writing for developing general problem-solving skills.

Accordingly, there is a need for homework problems of two kinds: those that do not require programming, and those that do. Both kinds appear in this collection. Problems 1-7 do not require programming, and problems 8-14 do.

Just prior to assigning the problems, the teacher should discuss five DOS commands (RUN, LIST, CATALOG, Control-Reset, and Control-S) and ascertain that all the students are able to replace data that are given in data statements within a program.

1. Mean = 86

Objective: To be sure that the student's first venture is successful.

Follow-up: Have students LIST Program P47 and trace through each increment of the loop. They should see that the computer goes through the same steps that they themselves would go through.

2. Mean = 87

Objective: To indicate how a short program can handle many different data collections, especially if the user can modify the program.

Follow-up: Discuss other simple program modifications, such as changes in the number of data, printing the data, computing their range, etc.

3. Mean = 79 and STD = 9.02

Objective: To illustrate STD as a measure of spread

Follow-up: Use other data to illustrate further what it is that STD measures. In particular, show that the STD of a list of constant data equals zero.

4. Number of scores lying within one STD of mean: 13

Objective: To introduce the concepts of centrality and distribution, such as are later manifest in statements like "68% of normally distributed data lie within one STD of their mean."

Follow-up: Explain that "within 1 STD of" means "having distance less than one from";unfortunately, the phrase "within plus or minus 1 STD" seems to be gaining currency but suggests that some data lie within -1 STD of their mean; i.e., that they have negative distance from the mean. Rather than confuse students, let's say "within 1 STD," as when in calculus we say "within epsilon."

5. X = 56.45

Objective: To illustrate the grouping of data into frequency classes in the very simple case of two frequency classes.

Follow-up: Note that the sum of frequencies equals the total number of data. Thus each datum (or "piece of data") lies in one and only one class.

6. 45.8 lies in Class 1, 49.2 in Class 1, 55.0 in Class 2, and 75.6 lies in Class 3.

There are 24 data, found by summing the frequencies; 37.5% of these lie in the first of the three classes.

Objective: To indicate that frequency classes reveal clustering of data, as contrasted to uniform distribution of data.

Follow-up: Ask students to suggest other data collections that could be grouped into frequency classes. For each collection, ask whether the data would tend to be clustered, or to the contrary, uniformly distributed.

7. 5 data lie in Classes 1 to 3; 8 lie in Classes 4 to 7, 4 in Classes 7 to 9, and 7 in Classes 10 to 12. The same distribution of frequencies holds for K=4.

Objective: To observe the effect of grouping the data into a larger number of classes: smaller class frequencies, and further revelation of clusters.

Follow-up: Change the data in Program P44 and run the program using K=2, K=3, K=4, etc., thus reinforcing the objectives of Problems 5-7.

One good way to collect such data is as follows: Put two marks on the chalkboard. Have each student estimate the distance between the marks, to the nearest inch. The estimates are written on scraps of paper and collected. A student who types well enters the estimates as data into Program P44, and the program is run.

(These data will probably be rather nicely normally distributed.)

When students reach the topics of histograms and ogives, and when they run programs on these topics, be sure they understand how these graphs are being formed from frequency classes.

8. Problems 8-14 are intended for students (and teachers who are programmers. Tell them that most of the work should be done away from the computer.

Perhaps each programmer could give a five-minute presentation of finished work to his or her classmates.

LIST OF TRANSPARENCIES FOR SESSION 3S (STATISTICS)

1. DOS COMMANDS AND LISTing OF PROGRAM P47
2. LISTing OF PROGRAMS P48 AND P49
3. LISTing OF PROGRAM P44 (FREQUENCY CLASSES)
4. CHAIN OF BASIC STATISTICAL IDEAS (AND PROGRAMS)
5. ANOTHER CHAIN OF BASIC STATISTICAL IDEAS

DOS COMMANDS

RUN

LIST

CATALOG

Control-reset

Control-S and S

LISTing of Program P47

```
10 HOME
20 N = 6
30 DATA 95, 82, 90, 96, 72, 81
40 FOR I = 1 TO N : READ X : S = S + X : NEXT I
50 PRINT " MEAN = " ; S / N
```

Transparency 1.

LISTing of Program P48

```
10  HOME
20  N = 100
30  FOR  I = 1  TO  N :  S = S + RND(1) :  NEXT I
40  PRINT  " MEAN  =  " ;  S / N
50  S = 0 :  GOTO  30
```

LISTing of Program P49

```
10  HOME
20  N = 100
30  FOR I = 1 TO N:  X = RND(1): S = S + X:  T = T + X*X:  NEXT  I
40  M = S/N:  PRINT "MEAN = "; M
50  D = SQR(T/(N-1) - N*M*M/(N-1)):  PRINT "  STD = "; D
60  S = 0:  T = 0:  PRINT:  GOTO 30
```

Transparency 2.

PROGRAM P44 (Frequency Classes)

```
8  HOME
10  DIM X(100), C(21), F(100):  A = 10^10:  B = -A
11  DATA 56.2, 62.4, 45.8, 75.6, 45.6, 50.1, 38.9, 65.4, 70.3,
        74.2, 55.0, 64.5
12  DATA 46.5, 67.8, 37.6, 60.4, 78.1, 45.7, 75.4, 69.8, 49.2,
        52.6, 43.6, 34.8
49  DATA -1.572101
50  I = I+1: READ X(I): IF X(I) = -1.572101 THEN N = I-1: GOTO 90
60  IF A>X(I) THEN A = X(I)
70  IF X(I) >B THEN B = X(I)
80  GOTO 50
90  PRINT "LEAST:  "A"      GREATEST:  "B:  PRINT



100  PRINT "INPUT # CLASSES (2 TO 20) FOR GROUPING"
110  INPUT "DATA:  K = "; K
120  FOR I = 0 TO K:  C(I) = A + (B-A)*I/K:  NEXT I
130  FOR I = 1 TO N
140  IF X(I) = B THEN M = M+1
150  FOR J = 0 TO K-1
160  IF C(J) <= X(I) AND X(I) < C(J+1) THEN F(J) = F(J) + 1:  GOTO
     180
170  NEXT J
180  NEXT I
190  F(K-1) = F(K-1) + M
200  PRINT: PRINT "CLASS      INTERVAL           FREQUENCY":
     PRINT
210  FOR I = 0 TO K-1
220  PRINT  I+1"    "C(I)" TO "C(I+1);
230  PRINT TAB(35) F(I)
240  NEXT I
250  PRINT: PRINT "YOU MAY NOW TYPE YOUR OWN DATA INTO"
260  PRINT "LINES  11-48."
```

Transparency 3.

A CHAIN OF BASIC STATISTICAL IDEAS (AND PROGRAMS)

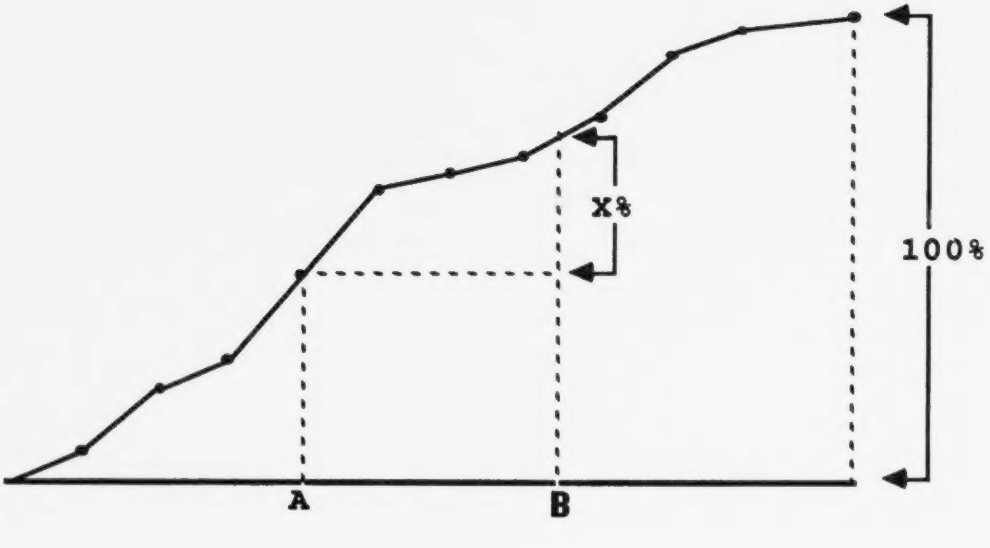
DATA ----> *FREQUENCY CLASSES -->*

----> *HISTOGRAMS -->*

----> *OGIVES -->*

----> *X% OF THE DATA LIE BETWEEN A AND B*

USING AN OGIVE TO ESTIMATE PERCENTAGES OF DATA -



Transparency 4.

ANOTHER CHAIN OF BASIC STATISTICAL IDEAS

(AND PROGRAMS)

DATA ---> SORTED DATA (PROGRAM P64) -->

---> PERCENTILES (PROGRAM P51) -->

---> X% OF THE DATA LIE BETWEEN A AND B

Transparency 5.

SHORT PROGRAMS FOR TEACHING AND LEARNING HIGH SCHOOL MATHEMATICS

DISK AND MANUAL WRITTEN FOR THE NSF/NCTM PROJECT

COMPUTERS IN MATHEMATICS CLASSROOMS

EQUIPMENT NEEDED:

The Short Programs disk runs on Apple II computers having the following specifications -

- DOS 3.3
- 32K or more memory
- One disk drive
- Printer (optional)
- Color or monochrome monitor

A similar disk is available for the IBM Personal Computer. To obtain the IBM-PC (or Apple) disk and manual, write to

UNIVERSITY OF EVANSVILLE PRESS
MATHDISK DIVISION
1800 LINCOLN AVENUE
EVANSVILLE, IN 47714

SHORT PROGRAMS FOR TEACHING AND LEARNING HIGH SCHOOL MATHEMATICS

Suppose, for example, you are teaching algebra. Step 1 toward using Short Programs is to determine which ones to use. Jot down the numbers of the ones you choose in your textbook, in a margin right next to the relevant material.

For algebra, you would want to include Program P2 (Quadratic Formula), so let's take it as an example. The main question is when and how to use the program.

Here is one approach to answering that question: first, introduce the quadratic formula just as you would if the nearest computer were miles away.

When you have reached the point of having a solved quadratic equation on the chalkboard, turn the computer(s) on, and use Program P2 to solve exactly the same equation. Then do another solution on the chalkboard, and again solve it with Program P2.

As each new aspect of the topic of quadratic equations unfolds from the textbook (e.g., repeated root, complex conjugate roots), exemplify it with Program P2.

The program works as fast as you can type A,B,C, so that the time required to run the program is minimal.

Eventually LIST Program P2, and discuss each of its lines with the class. It is easy, even for students who have never written a program, to see that the program *is* essentially the quadratic formula. In fact, the three cases, $D < 0$, $D = 0$, $D > 0$, stand out more clearly in the program than they do in many textbooks.

Many students who have not adequately grasped the quadratic formula previously will do so as a result of thinking through the five steps that the formula assumes in Program P2.

Each program on the Short Programs disk is comparable to Program P2 in its potential to reinforce a textbook topic or an enrichment topic. The main things to keep in mind while planning each program-assisted topic are these:

1. Introduce the topic *without* using the program, and leave a simple example on the chalkboard.
2. Reinforce that example by running the program.
3. Match selected portions of the textbook development of the topic with runs of the program.
4. LIST the program if appropriate, so that students can see that it is a step-by-step version of the formula or algorithm that they are studying as a mathematical topic.

Much more could be written about *teacher-directed* use of short programs, but let's consider the other kind of use: *student-directed.* We must make available programs that allow experimentation and creative modification.

Suppose one of your students asks for some modifiable algebra programs, because she likes algebra and programming. Right now, is there a disk of programs like that in her school library or computer lab? What about her math classroom?

The Short Programs are designed for teachers and students to use as tools and as reference materials. They are written to be run with as little fuss as possible.

Such flexibility calls for the user to control the Disk Operating System (DOS) directly, using commands like RUN, LIST, SAVE, CATALOG, Control-S, and Control-Reset. These are all explained in the Applesoft BASIC Reference Manual.

The mathematical programs on the Short Programs disk are numbered like this: P1, P2, P3, and so on. To run the program "Quadratic Formula," for example, type RUN P2 and then tap RETURN.

The complete list of numbered programs can be viewed by running the program INTRODUCTION, or by turning to the next two pages.

PROGRAMS FOR TEACHING AND LEARNING ALGEBRA

P1	EVALUATE Y(X)
P2	QUADRATIC FORMULA
P3	COEFFICIENTS (QUADRATIC)
P4	COEFFICIENTS (CUBIC)
P5	COEFFICIENTS (QUARTIC)
P6	HOT OR COLD (SEARCH)
P7	HALF-INTERVAL SEARCH
P8	HORNER'S METHOD (QUADRATIC)
P9	HORNER'S METHOD (CUBIC)
P10	HORNER'S METHOD (QUARTIC)
P11	DETERMINANT 2x2
P12	DETERMINANT 3x3
P13	DETERMINANT 4x4
P14	CRAMER'S RULE (2 EQ. IN 2 UNKN.)
P15	CRAMER'S RULE (3 EQ. IN 3 UNKN.)
P16	INVERT 2x2 MATRIX
P17	INVERT 3x3 MATRIX
P18	INVERT 4x4 MATRIX
P19	MULTIPLY 2x2 MATRICES
P20	MULTIPLY 3x3 MATRICES
P21	MULTIPLY 4x4 MATRICES

PROGRAMS FOR TEACHING AND LEARNING GEOMETRY

P22	DISTANCE BETWEEN POINTS
P23	GRAPH X=X(T) & Y=Y(T) (PARA. EQ.)
P24	INTERSECT LINES
P25	PROJECTION OF POINT ONTO LINE
P26	DISTANCE FROM POINT TO LINE
P27	REFLECTION OF POINT ABOUT LINE

PROGRAMS FOR TEACHING AND LEARNING TRIGONOMETRY

P28	EVALUATE TRIG FUNCTIONS
P29	EVALUATE INVERSE SINE
P30	EVALUATE INVERSE COSINE
P31	LAW OF COSINES
P32	CONVERT A+BI TO POLAR FORM
P33	DE MOIVRE'S FORMULA
P34	INVERSE OF DE MOIVRE'S FORMULA

PROGRAMS FOR TEACHING AND LEARNING PROBABILITY AND STATISTICS

P35	PERMUTATIONS (1 2 3)
P36	PERMUTATIONS (4 WORDS 3 AT A TIME)
P37	COMBINATIONS (4 NOS. 2 AT A TIME)
P38	COMBINATIONS (6 LET. 3 AT A TIME)
P39	FACTORIALS
P40	NUMBER OF PERMUTATIONS
P41	NUMBER OF COMBINATIONS
P42	TOSS COINS
P43	ROLL DICE
P44	FREQUENCY CLASSES
P45	INDIV. BINOMIAL PROBABILITIES
P46	CUM. BINOMIAL PROBABILITIES
P47	MEAN (OF 6 TEST SCORES)
P48	MEAN (OF 100 RANDOM NUMBERS)
P49	MEAN AND STD (100 RND NOS.)
P50	DISTR. OF DATA ABOUT THEIR MEAN
P51	PERCENTILES
P52	RANDOM NUMBER GENERATOR
P53	BALLOONS
P54	CRYSTALS
P55	FACES

PROGRAMS FOR TEACHING AND LEARNING ABOUT SEQUENCES AND LIMITS

P56	MULTIPLY INTEGERS
P57	FACTOR INTEGERS
P58	CONSECUTIVE PRIMES (TRY ALL N)
P59	CONSECUTIVE PRIMES (FASTER)
P60	ARITH. PROGRESSION (WITH LOOP)
P61	ARITH. PROGRESSION (WITHOUT LOOP)
P62	GEOMETRIC PROGRESSION
P63	GCD
P64	BUBBLESORT (10 RANDOM NUMBERS)
P65	ALPHABETIZE (VIA BUBBLESORT)
P66	ADD 1 + 2 + 3 + ... + N
P67	ADD 1^2 + 2^2 + ... + N^2
P68	ADD 1^3 + 2^3 + ... + N^3
P69	ADD 1/1 + 1/2 + ... + 1/N
P70	THE NUMBER e
P71	INTEREST COMP. N TIMES PER YEAR
P72	SQUARE ROOT

PROGRAMS FOR TEACHING AND LEARNING ALGEBRA

P1 EVALUATE Y(X)

```
10 HOME
20 DEF FN Y(X) = 2*X + 1
30 A = - 6: B = 6: S = 2
40 FOR X = A TO B STEP S
50 PRINT "Y("X") = " FN Y(X)
60 NEXT X
```

The enormous value of Program P1 is its openness to teachers' and students' choices of Y(X), A, B, and S. Students can strengthen their grasp of these fundamental concepts: (1) a function as a "rule," (2) variables (X and Y) and constants (A and B); and (3) arithmetic progression (first term A and common difference S).

P2 QUADRATIC FORMULA

```
10 HOME
20 INPUT "INPUT A,B,C = ";A,B,C
30 D = B*B - 4*A*C: V = B/(2*A)
40 IF D < 0 THEN W =  SQR (- D)/(2*A)
50 IF D = 0 THEN  PRINT "ONE REAL ROOT:  "V
60 IF D > 0 THEN  PRINT "TWO REAL ROOTS:  ";
           ( - B -  SQR (D))/(2*A): PRINT"   AND  "
           ( - B +  SQR (D))/(2*A)
70 IF D < 0 THEN  PRINT "COMPLEX CONJUGATE ROOTS:  "
           V" + i("W")  and  "V" - i("W")"
80 PRINT : GOTO 20
```

Program P2 purposefully allows the user to input A = 0. What happens is one more way the computer can be useful in teaching and learning mathematics. Line 80 enables the user to solve many quadratic equations rapidly. This is helpful when comparing several equation-and-solution pairs. It is worthwhile, for example, to keep A = 1 and B = 1 while varying C from -12 up to 4, stepping 2 at a time. Use CONTROL-RESET to halt the run.

P3 COEFFICIENTS (QUADRATIC)

```
10 HOME
20 INPUT "INPUT R,S = ";R,S
30 A1 = R + S
40 B = R*S
50 A = - A1
60 PRINT "(X-R)(X-S) HAS COEFFICIENTS  1, "A", "B
70 PRINT : GOTO 20
```

Program P3 is an "inverse" of Program P2. The two programs check each other. For a program that allows R and S to be any complex numbers, see MATHDISK THREE, Program 93.

P4 COEFFICIENTS (CUBIC)

```
10 HOME
20 INPUT "INPUT R,S,T = ";R,S,T
30 A1 = R + S + T
40 B = R*S + R*T + S*T
50 C1 = R*S*T
60 A = - A1:C = - C1
70 PRINT "(X-R)(X-S)(X-T) HAS COEFFICIENTS "
80 PRINT "1, "A", "B", "C
90 PRINT : GOTO 20
```

P5 COEFFICIENTS (QUARTIC)

```
10 HOME
20 INPUT "INPUT R,S,T,U = ";R,S,T,U
30 A1 = R + S + T + U
40 B = R*S + R*T + S*T + R*U + S*U + T*U
50 C1 = R*S*T + R*S*U + R*T*U + S*T*U
60 D = R*S*T*U
70 A = - A1: C = - C1
80 PRINT "(X-R)(X-S)(X-T)(X-U)  HAS COEFFICIENTS"
90 PRINT 1", "A", "B", "C", "D
100 PRINT : GOTO 20
```

P6 HOT OR COLD (SEARCH)

```
10 HOME
20 DEF FN Y(X) = 4*X^3 - 3*X^2 - 3*X - 7
30 INPUT "INPUT X = ";X
40 GOSUB 80
50 PRINT "Y("X") = " FN Y(X)
60 GOTO 30
70 END
80 VTAB ( PEEK (37)): PRINT "                    ": VTAB ( PEEK (37)): RETURN
```

As students use Program P6 to narrow down on a root (or max or min) of Y(X), they often gain a better understanding of a function as a "rule" that assigns to each numbers X a number Y(X).

Also, many a student discovers on her or his own the optimal way to search for a root: always take X to be half way between the previous X that gave the least positive Y(X) and the previous X that gave the greatest negative Y(X). This is a wonderful thing to happen, because this student has discovered the Half-Interval Search (Program P7).

P7 HALF-INTERVAL SEARCH

```
10 HOME
20 DEF FN Y(X) = 4*X^3 - 3*X^2 - 3*X - 7
30 INPUT "INPUT A,B = ";A,B
40 X = (A + B) / 2: Y =  FN Y(X)
50 C =  FN Y(A): D =  FN Y(B)
60 PRINT "Y("X") = ";Y
70 IF Y*C > 0 THEN A = X: GOTO 40
80 B = X: GOTO 40
```

P8 HORNER'S METHOD (QUADRATIC)

```
10 HOME
20 INPUT "INPUT A,B,C = ";A,B,C
30 INPUT "X = ";X
40 Y = A*X + B
50 Y = Y*X + C
60 PRINT "Y("X") = ";Y
70 PRINT : GOTO 30
```

P9 HORNER'S METHOD (CUBIC)

```
10 HOME
20 INPUT "INPUT A,B,C,D = ";A,B,C,D
30 INPUT "X = ";X
40 Y = A*X + B
50 Y = Y*X + C
60 Y = Y*X + D
70 PRINT "Y("X") = ";Y
80 PRINT : GOTO 30
```

P10 HORNER'S METHOD (QUARTIC)

```
10 HOME
20 INPUT "INPUT A,B,C,D,E = ";A,B,C,D,E
30 INPUT "X = ";X
40 Y = A*X + B
50 Y = Y*X + C
60 Y = Y*X + D
70 Y = Y*X + E
80 PRINT "Y("X") = ";Y
90 PRINT : GOTO 30
```

Program P9, for example, evaluates a cubic polynomial **A*X^3 + B*X^2 + C*X + D** faster and more accurately than the method of Program P6. The nickname "Rifle Test" corresponds to the fact that if Y(X) is very near zero for a particular value of X, then X must be a close approximation to a root of the polynomial P(X).

Unlike the Rifle Test, the "Shotgun Test" (Programs P3-P5) checks all the roots at once: if any one of them is "off" then this will be detected because the resulting coefficients will be "off."

P11 DETERMINANT 2x2

```
10 HOME
20 INPUT "INPUT A,B = ";A,B
30 INPUT "INPUT C,D = ";C,D
40 PRINT "DETERMINANT = ";A*D - B*C
```

P12 DETERMINANT 3x3

```
10 HOME
20 INPUT "INPUT A,B,C = ";A,B,C
30 INPUT "INPUT D,E,F = ";D,E,F
40 INPUT "INPUT G,H,I = ";G,H,I
50 P = A*E*I + B*F*G + C*D*H
60 N = A*F*H + B*D*I + C*E*G
70 PRINT "DETERMINANT = ";P - N
```

P13 DETERMINANT 4x4

```
10 HOME
20 INPUT "INPUT A,B,C,D = ";A,B,C,D
30 INPUT "INPUT E,F,G,H = ";E,F,G,H
40 INPUT "INPUT I,J,K,L = ";I,J,K,L
50 INPUT "INPUT M,N,O,P = ";M,N,O,P
60 S = A*F*K*P + A*G*L*N + A*H*J*O + B*E*L*O + B*G*I*P + B*H*K*M
70 S = S + C*E*J*P + C*F*L*M + C*H*I*N + D*E*K*N + D*F*I*O +D*G*J*M
80 T = A*F*L*O + A*G*J*P + A*H*K*N + B*E*K*P + B*G*L*M + B*H*I*O
90 T = T + C*E*L*N + C*F*I*P + C*H*J*M + D*E*J*O + D*F*K*M +D*G*I*N
100 PRINT "DETERMINANT = ";S - T
```

P14 CRAMER'S RULE (2 EQ. IN 2 UNKN.)

```
10  HOME
20  PRINT "THIS PROGRAM USES CRAMER'S RULE TO"
30  PRINT "SOLVE THE EQUATIONS"
40  PRINT
50  PRINT "AX + BY = R   AND   CX + DY = S"
60  PRINT
70  PRINT "FOR X AND Y."
80  PRINT : PRINT : PRINT
90  INPUT "INPUT A,B,R = ";A,B,R
100  INPUT "INPUT C,D,S = ";C,D,S
110  V = A*D - B*C: IF V < > 0 THEN 140
120  PRINT "NO SOLUTION.  (THE GRAPHS OF YOUR EQS."
130  PRINT "ARE PARALLEL LINES.)": END
140  PRINT : PRINT "X = "(D*R - B*S)/V", Y = "(A*S - C*R)/V
150  PRINT : PRINT "THE GRAPHS OF YOUR EQUATIONS ARE"
160  PRINT "LINES, AND (X,Y) IS THEIR POINT OF"
170  PRINT "INTERSECTION."
```

P15 CRAMER'S RULE (3 EQ. IN 3 UNKN.)

```
10  HOME
20  PRINT "THIS PROGRAM USES CRAMER'S RULE TO"
30  PRINT "SOLVE THE EQUATIONS"
40  PRINT
50  PRINT "    AX + BY + CZ = R"
60  PRINT "    DX + EY + FZ = S"
70  PRINT "    GX + HY + I Z = T"
80  PRINT
90  PRINT "FOR  X, Y, AND Z.": PRINT
100  INPUT "INPUT A,B,C,R = ";A,B,C,R
110  INPUT "INPUT D,E,F,S = ";D,E,F,S
120  INPUT "INPUT G,H, I,T = ";G,H,I,T
130  V = A*E*I + B*F*G + C*D*H - A*F*H - B*D*I - C*E*G:
     IF V < > 0 THEN 150
140  PRINT "NO UNIQUE SOLUTION.": END
150  N1 = R*E*I + B*F*T + C*S*H - R*F*H - B*S*I - C*E*T
160  N2 = A*S*I + R*F*G + C*D*T - A*F*T - R*D*I - C*S*G
170  N3 = A*E*T + B*S*G + R*D*H - A*S*H - B*D*T - R*E*G
180  PRINT : PRINT "X = "N1 / V
190  PRINT "Y = "N2 / V
200  PRINT "Z = "N3 / V
210  PRINT
220  PRINT "THE GRAPHS OF YOUR EQUATIONS IN 3-DIM."
230  PRINT "SPACE ARE PLANES, AND (X,Y,Z) IS THEIR"
240  PRINT "POINT OF INTERSECTION."
```

P16 INVERT 2x2 MATRIX

```
10 HOME
20 INPUT "INPUT A,B = ";A,B
30 INPUT "INPUT C,D = ";C,D
40 V = A*D - B*C: IF V < > 0 THEN 60
50 PRINT : PRINT "YOUR MATRIX HAS NO INVERSE.": END
60 A1 = D: B1 = - B: C1 = - C: D1 = A
70 PRINT : PRINT "THE INVERSE OF YOUR MATRIX RESULTS"
80 PRINT "FROM DIVIDING ALL FOUR ENTRIES OF THE"
90 PRINT "MATRIX"
100 PRINT "       "A1"  "B1
110 PRINT "       "C1"  "D1"  BY  "V
```

P17 INVERT 3x3 MATRIX

```
10 HOME
20 INPUT "INPUT A,B,C = ";A,B,C
30 INPUT "INPUT D,E,F = ";D,E,F
40 INPUT "INPUT G,H,I = ";G,H,I
50 V = A*E*I + B*F*G + C*D*H - A*F*H - B*D*I - C*E*G:
   IF V < > 0 THEN 70
60 PRINT : PRINT "YOUR MATRIX HAS NO INVERSE.": END
70 A1 = E*I - H*F: B1 = H*C - B*I: C1 = B*F - E*C
80 D1 = G*F - D*I: E1 = A*I - G*C: F1 = D*C - A*F
90 G1 = D*H - G*E: H1 = G*B - A*H: I1 = A*E - D*B
100 PRINT : PRINT "THE INVERSE OF YOUR MATRIX RESULTS"
110 PRINT "FROM DIVIDING ALL NINE ENTRIES OF THE"
120 PRINT "MATRIX"
130 PRINT
140 PRINT "  "A1"  "B1"  "C1
150 PRINT "  "D1"  "E1"  "F1
160 PRINT "  "G1"  "H1"  "I1"  BY  "V
```

P18 INVERT 4x4 MATRIX

```
10 HOME
20 INPUT "INPUT A,B,C,D = ";A,B,C,D
30 INPUT "INPUT E,F,G,H = ";E,F,G,H
40 INPUT "INPUT I,J,K,L = ";I,J,K,L
50 INPUT "INPUT M,N,O,P = ";M,N,O,P
60 S = A*F*K*P + A*G*L*N + A*H*J*O + B*E*L*O + B*G*I*P + B*H*K*M
70 S = S + C*E*J*P + C*F*L*M + C*H*I*N + D*E*K*N + D*F*I*O + D*G*J*M
80 T = A*F*L*O + A*G*J*P + A*H*K*N + B*E*K*P + B*G*L*M + B*H*I*O
90 T = T + C*E*L*N + C*F*I*P + C*H*J*M + D*E*J*O + D*F*K*M + D*G*I*N
100 V = S - T: IF V < > 0 THEN 120
110 PRINT : PRINT "  YOUR MATRIX HAS NO INVERSE.": END
120 A1 = F * K * P + J * O * H + N * G * L - F * O * L - J * G * P - N * K * H
130 B1 = B * O * L + J * C * P + N * K * D - B * K * P - J * O * D - N * C * L
140 C1 = D * G * P + F * O * D + N * C * H - B * O * N - F * C * P - N * G *D
150 D1 = B * K * H + F * C * L + J * G * D - B * G * L - F * K * D - J * C * H
160 E1 = E * O * L + I * G * P + M * K * H - E * K * P - I * O * H - M * G * L
170 F1 = A * K * P + I * O * D + M * C * L - A * O * L - I * C * P - M * K * D
180 G1 = A * O * N + E * C * P + M * G * D - A * G * P - E * O * D - M * C*H
190 H1 = A * G * L + E * K * D + I * C * H - A * K * H - E * C * L - I * G * D
200 I1  = E * J * P + I * N * H + M * F * L - E * N * L - I * F * P - M * J * H
210 J1 = A * N * L + I * B * P + M * J * D - A * J * P - I * N * D - M * B * L
220 K1 = A * F * P + E * N * D + M * B * H - A * N * H - E * B * P - M * F *D
230 L1 = A * J * H + E * B * L + I * F * D - A * F * L - E * J * D - I * B * H
240 M1 = E * N * K + I * F * O + M * J * G - E * J * O - I * N * G - M * F * K
250 N1 = A * J * O + I * N * C + M * B * K - A * N * K - I * B * O - M * J * C
260 O1 = A * N * G + E * B * O + M * F * C - A * F * O - E * N * C - M * B *G
270 P1 = A * F * K + E * J * C + I * B * G - A * J * G - E * B * K - I * G * C
280 PRINT : PRINT "THE INVERSE OF YOUR MATRIX RESULTS"
290 PRINT "FROM DIVIDING ALL 16 ENTRIES OF THE"
300 PRINT "MATRIX ": PRINT
310 PRINT : PRINT A1"   "B1"   "C1"   "D1
320 PRINT : PRINT E1"   "F1"   "G1"   "H1
330 PRINT : PRINT I1"    "J1"    "K1"   "L1
340 PRINT : PRINT M1"   "N1"   "O1"   "P1"   BY   "V
```

Program P18 illustrates the point that with a computer in the classroom, some conceptually valuable but computationally lengthy things can be done. Here are some notes about using this program:

1. The number V at Line 100 is the determinant of the coefficient matrix, computed exactly as in Program P13. The matrix has an inverse if and only if V is not equal to zero.

2. Students should use Programs P18 and P21 as a pair that check each other. The same holds for the program-pairs P16 and P19, and P17 and P20. This usage of these programs reinforces students' understanding of an inverse matrix as that matrix which, when multiplied by the original matrix, gives the identity matrix as the product.

P19 MULTIPLY 2x2 MATRICES

```
10  HOME
20  INPUT "INPUT A,B = ";A,B
30  INPUT "INPUT C,D = ";C,D
40  PRINT : INPUT "INPUT E,F = ";E,F
50  INPUT "INPUT G,H = ";G,H: PRINT
60  A1 = A*E + B*G: B1 = A*F + B*H: C1 = C*E + D*G: D1 = C*F + D*H
70  PRINT "A B        E F         "A1"  "B1
80  PRINT "     TIMES       EQUALS"
90  PRINT "C D        G H         "C1"  "D1
```

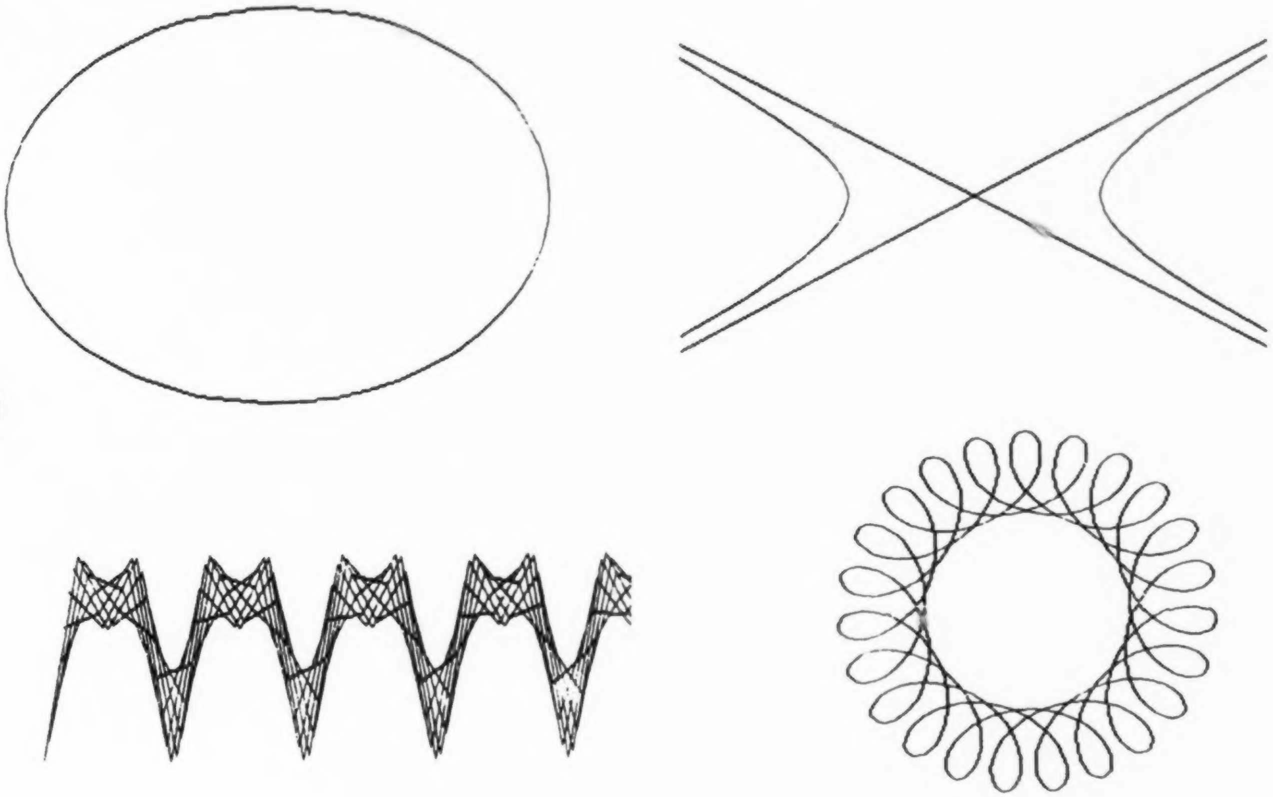
P20 MULTIPLY 3x3 MATRICES

```
8  HOME
11 A = 2: B = 3: C = 2
12 D = 4: E = 0: F =  - 1
13 G = 7: H = 1: I = 5
21 J = 1: K = 1: L =  - 2
22 M = 0: N = 2: O = 0
23 P = 2: Q = 5: R =  - 3
31 A1 = A*J + B*M + C*P: B1 = A*K + B*N + C*Q: C1 = A*L + B*O*C*R
32 D1 = D*J + E*M + F*P: E1 = D*K + E*N + F*Q:  F1 = D*L + E*O + F*R
33 G1 = G*J + H*M + I*P: H1 = G*K + H*N + I*Q:  I1 = G*L + H*O + I*R
41  PRINT "A B C              J K L"
42  PRINT "D E F  TIMES  M N O"
43  PRINT "G H I              P Q R"
45  PRINT
47  PRINT "              "A1"   "B1"   "C1
48  PRINT "EQUALS    "D1"   "E1"   "F1
49  PRINT "              "G1"   "H1"   "I1
50  PRINT
60  PRINT "THE TWO MATRICES ARE GIVEN BY THE FOL-"
70  PRINT "LOWING DATA, WHICH YOU MAY NOW REPLACE:"
80  PRINT : LIST 11 - 23
```

P21 MULTIPLY 4x4 MATRICES

```
10  HOME
20  PRINT "MATRICES WILL BE INPUT ONE ROW AT A"
30  PRINT "TIME.  FOR EXAMPLE, TO INPUT THE ROW"
40  PRINT "6  8  4  -3, YOU MUST TYPE  6,8,4,-3"
50  PRINT "AND THEN TAP THE ENTER KEY."
60  PRINT
70  FOR I = 1 TO 2: PRINT : FOR J = 1 TO 4
80  INPUT " #,#,#,# = ";M(I,J,1),M(I,J,2),M(I,J,3),M(I,J,4)
90  NEXT J,I: PRINT
100  FOR J = 1 TO 4: FOR K = 1 TO 4
110  FOR L = 1 TO 4: P(J,K) = P(J,K) + M(1,J,L)*M(2,L,K): NEXT L
120  NEXT K,J
130  PRINT : PRINT "PRODUCT OF YOUR TWO MATRICES:": PRINT
140  FOR J = 1 TO 4
150  FOR K = 1 TO 4: PRINT P(J,K)" ";: NEXT K
160  PRINT : NEXT J
```

Several curves produced by Program P23.



PROGRAMS FOR TEACHING
AND LEARNING GEOMETRY

P22 DISTANCE BETWEEN POINTS

```
 10  HOME
 20  INPUT "INPUT A,B = ";A,B
 30  INPUT "INPUT C,D = ";C,D
 40  PRINT
 50  PRINT "THE DISTANCE BETWEEN (A,B) AND (C,D) IS"
 60  PRINT  SQR ((A - C) ^ 2 + (B - D) ^ 2)
```

P23 GRAPH X=X(T) & Y=Y(T) (PARA. EQ.)

```
 20  DEF  FN X(T) =  SQR (T) *  COS (T)
 30  DEF  FN Y(T) =  SQR (T) *  SIN (T)
 40  P = 8*ATN (1): S = 10
 50  DEF FN U(X) = 140 + S*X
 60  DEF FN V(Y) = 96 - S*Y
 70  DEF FN A(T) =  FN U( FN X(T))
 80  DEF FN B(T) =  FN V( FN Y(T))
 90  HOME : PRINT "THIS PROGRAM GRAPHS CURVES GIVEN BY"
 100  PRINT "PARAMETRIC EQUATIONS.  AFTER THE FIRST"
 110  PRINT "RUN, TYPE YOUR COORDINATE FUNCTIONS"
 120  PRINT "X(T) AND Y(T) AS FNX(T) AND FNY(T) AT"
 130  PRINT "LINES 20 AND 30.": PRINT
 140  PRINT "PRESS 'RETURN' TO START GRAPHING.";
 150  GET A$: HOME : PRINT
 160  HGR : POKE 49234,0: HCOLOR= 3
 170  G = 0: H = 10000
 180  I = P/32
 190  FOR T = G TO H STEP I:W = T + I
 200  C =  FN A(T): IF C < 0 OR C > 279 THEN 250
 210  D =  FN A(W): IF D < 0 OR D > 279 THEN 250
 220  E =  FN B(T): IF E < 0 OR E > 191 THEN 250
 230  F =  FN B(W): IF F < 0 OR F > 191 THEN 250
 240  HPLOT C,E TO D,F
 250  NEXT T
```

To graph any curve Y = F(X), use X = T and Y = F(T). To graph any equation R = R(theta), in polar coordinates, use X = R(T)*COS(T) and Y = R(T)*SIN(T). These two comments and the graphics output on a nearby page indicate the wide range of capabilities of Program P23.

P24 INTERSECT LINES

```
 10 HOME
 20 INPUT "INPUT A,B = ";A,B
 30 INPUT "INPUT C,D = ";C,D
 40 INPUT "INPUT E,F = ";E,F
 50 INPUT "INPUT G,H = ";G,H
 60 Z = (C - A) * (F - H) - (E - G) * (D - B)
 70 PRINT : IF Z = 0 THEN PRINT "THESE LINES ARE IDENTICAL OR
     PARALLEL.": END
 80 T = ((E - A) * (F - H) - (E - G) * (F - B)) / Z
 90 X = A + (C - A) * T:Y = B + (D - B) * T
100  PRINT "THE LINE JOINING (A,B) AND (C,D) INTER-"
110  PRINT "SECTS THE LINE JOINING (E,F) AND (G,H)"
120  PRINT "IN THE POINT ("X", "Y")."
```

P25 PROJECTION OF POINT ONTO LINE

```
 10 HOME
 20 INPUT "INPUT A,B = ";A,B
 30 INPUT "INPUT C,D = ";C,D
 40 INPUT "INPUT E,F = ";E,F
 50 Z = (C - A) * (C - A) + (D - B) * (D - B)
 60 T = ((E - A) * (C - A) + (F - B) * (D - B)) / Z
 70 X = A + (C - A) * T:Y = B + (D - B) * T
 80 PRINT
 90 PRINT "THE PROJECTION OF POINT (E,F) ONTO THE"
 100  PRINT "LINE JOINING POINTS (A,B) AND (C,D) IS"
 110  PRINT "THE POINT ("X", "Y")."
```

P26 DISTANCE FROM POINT TO LINE

```
 10 HOME
 20 INPUT "INPUT A,B = ";A,B
 30 INPUT "INPUT C,D = ";C,D
 40 INPUT "INPUT E,F = ";E,F
 50 Z = (C - A) * (C - A) + (D - B) * (D - B)
 60 T = ((E - A) * (C - A) + (F - B) * (D - B)) / Z
 70 X = A + (C - A) * T:Y = B + (D - B) * T
 90 PRINT "THE DISTANCE FROM THE POINT (E,F) TO"
 100  PRINT "THE LINE JOINING POINTS (A,B) AND (C,D)"
 110  PRINT "IS "; SQR ((E - X) ^ 2 + (F - Y) ^ 2)"."
```

P27 REFLECTION OF POINT ABOUT LINE

```
 10 HOME
 20 INPUT "INPUT A,B = ";A,B
 30 INPUT "INPUT C,D = ";C,D
 40 INPUT "INPUT E,F = ";E,F
 50 Z = (C - A) * (C - A) + (D - B) * (D - B)
 60 T = ((E - A) * (C - A) + (F - B) * (D - B)) / Z
 70 X = A + (C - A) * T:Y = B + (D - B) * T
 80 PRINT
 90 PRINT "THE REFLECTION OF THE POINT (E,F) ABOUT"
 100 PRINT "THE LINE JOINING POINTS (A,B) AND (C,D)"
 110 PRINT "IS THE POINT ("2 * X - E", "2 * Y - F")."
```

Note that Program P25-P27 differ only in Lines 90-110. All three programs should be taught as corollaries of Program P24.

PROGRAMS FOR TEACHING AND LEARNING TRIGONOMETRY

P28 EVALUATE TRIG FUNCTIONS

```
10  HOME
20  INPUT "INPUT ANGLE (IN DEGREES):  A = ";A
30  R = A*ATN (1) / 45
40  PRINT
50  PRINT "SIN("A") = " SIN (R)
60  PRINT "COS("A") = " COS (R)
70  PRINT "TAN("A") = " TAN (R)
80  PRINT : GOTO 20
```

P29 EVALUATE INVERSE SINE

```
10  HOME
20  PRINT "THIS PROGRAM COMPUTES THE ANGLE BETWEEN"
30  PRINT "-90 DEGREES AND 90 DEGREES WHOSE SINE"
40  PRINT "IS THE NUMBER Y."
50  PRINT
60  INPUT "INPUT  Y = ";Y: IF  ABS (Y) <  = 1 THEN 80
70  PRINT "Y MUST SATISFY -1<=Y<=1.  TRY AGAIN.": GOTO 60
80  IF  ABS (Y) = 1 THEN A = 90*SGN (Y): GOTO 100
90  A = ATN (Y/SQR (1 - Y*Y)): A = 45*A/ATN (1)
100  PRINT "ANGLE:  "A" DEGREES"
110  PRINT : GOTO 60
```

P30 EVALUATE INVERSE COSINE

```
10  HOME
20  PRINT "THIS PROGRAM COMPUTES THE ANGLE BETWEEN"
30  PRINT "0 DEGREES AND 180 DEGREES WHOSE COSINE"
40  PRINT "IS THE NUMBER Y."
50  PRINT
60  INPUT "INPUT  Y = ";Y: IF  ABS (Y) < = 1 THEN 80
70  PRINT "Y MUST SATISFY -1<=Y<=1.  TRY AGAIN.": GOTO 60
80  IF  ABS (Y) = 1 THEN A = 90 - 90*SGN (Y): GOTO 100
90  A = ATN (Y/SQR (1 - Y*Y)): A = 90 - 45*A/ATN (1)
100  PRINT "ANGLE:  "A" DEGREES"
110  PRINT : GOTO 60
```

P31 LAW OF COSINES

```
10  HOME
20  PRINT "INPUT TWO SIDE-LENGTHS AND THEIR"
30  PRINT "INCLUDED ANGLE (IN DEGREES):"
40  PRINT
50  INPUT "L,M,A = ";L,M,A
60  N =  SQR(L*L + M*M - 2*L*M*COS (A*ATN (1)/45))
70  PRINT "LENGTH OF THIRD SIDE:  N = ";N
```

P32 CONVERT A+BI TO POLAR FORM

```
10  HOME
20  INPUT "INPUT  A,B = ";A,B
30  R = SQR (A*A + B*B)
40  IF A = 0 THEN T = 1.570797: GOTO 60
50  T = ATN (B/A): IF A < 0 THEN T = T + 3.141593
60  PRINT "A+BI HAS POLAR FORM R*COS(T)+I*R*SIN(T)"
70  PRINT "     WHERE  R = "R
80  PRINT "           AND  T = "T"  RADIANS"
90  PRINT "                      = "T*45/ATN (1)"  DEGREES"
100  PRINT : GOTO 20
```

P33 DE MOIVRE'S FORMULA

```
10  HOME
20  PRINT "(INPUT T IN DEGREES.)": PRINT
30  INPUT "INPUT R,T,N = ";R,T,N
40  T = T*ATN (1)/45
50  PRINT "[R*COS(T) + I*R*SIN(T)]^N = ";
60  Q = R ^ N: A = N*T: PRINT Q*COS (A)" + I("Q*SIN (A)")"
70  PRINT : GOTO 30
```

P34 INVERSE OF DE MOIVRE'S FORMULA

```
10  HOME
20  PRINT "(INPUT T IN DEGREES.)": PRINT
30  INPUT "INPUT R,T,N = ";R,T,N
40  T = T*ATN (1)/45
50  PRINT "THE N COMPLEX NUMBERS Z THAT SATISFY"
60  PRINT "THE EQUATION  Z^N = R*COS(T)+I*R*SIN(T)"
70  PRINT "ARE AS FOLLOWS:": PRINT
80  FOR K = 0 TO N - 1
90  U = R^(1/N): V = (T + 6.283195*K)/N
100  PRINT U*COS (V)" + I*("U*SIN (V)")"
110  NEXT K
```

PROGRAMS FOR TEACHING AND LEARNING PROBABILITY AND STATISTICS

P35 PERMUTATIONS (1 2 3)

```
10 HOME
20 FOR I = 1 TO 3: FOR J = 1 TO 3: FOR K = 1 TO 3
30 IF J = I OR K = I OR K = J THEN 50
40 PRINT I;J;K"  ";
50 NEXT K,J,I
```

With pencils, students should trace the action of Program P35: first, Line 30 rejects the triple 1,1,1. Next, I=1, J=1, K=2, but Line 30 rejects this triple also. Students find the first survivor to be 1,2,3 and by then, the program is clearly succeeding as a learning tool.

Program P35 extends easily to programs that permute N things taken R at a time. Here is a version for N=4 and R=3:

P36 PERMUTATIONS (4 WORDS 3 AT A TIME)

```
10 HOME
20 S$(1) = "HAPPINESS":S$(2) = "IS":S$(3) = "WARM":S$(4) = "PUPPY"
30 FOR I = 1 TO 4: FOR J = 1 TO 4: FOR K = 1 TO 4
40 IF J = I OR K = I OR K = J THEN 60
50 PRINT S$(I)" "S$(J)" "S$(K)
60 NEXT K,J,I
```

P37 COMBINATIONS (4 NOS. 2 AT A TIME)

```
10 HOME
20 FOR I = 1 TO 3: FOR J = 2 TO 4
30 IF I < J THEN  PRINT I;J
40 NEXT J,I
```

P38 COMBINATIONS (6 LET. 3 AT A TIME)

```
10 HOME
20 S$(1) = "A":S$(2) = "B":S$(3) = "C":S$(4) = "D":S$(5) = "E":S$(6) = "F"
30 FOR I = 1 TO 4: FOR J = I + 1 TO 5: FOR K = J + 1 TO 6
40 PRINT S$(I);S$(J);S$(K)" ";
50 NEXT K,J,I
```

P39 FACTORIALS

```
10 HOME
20 F = 1
30 FOR M = 1 TO 10
40 F = F*M: PRINT M"! = ";F
50 NEXT M
```

P40 NUMBER OF PERMUTATIONS

```
10 HOME
20 INPUT "INPUT N,R = ";N,R: IF R = 0 THEN 20
30 P = 1
40 FOR M = R + 1 TO N: P = P*M: NEXT M
50 PRINT "P(N,R) = ";P
60 PRINT : GOTO 20
```

P41 NUMBER OF COMBINATIONS

```
10 HOME
20 INPUT "INPUT N,R = ";N,R: IF R = 0 THEN 20
30 P = 1: D = 1
40 FOR M = 1 TO N - R: D = D*M: NEXT M
50 FOR M = R + 1 TO N: P = P*M: NEXT M
60 PRINT "C(N,R) = ";P/D
70 PRINT : GOTO 20
```

P42 TOSS COINS

```
10 HOME
20 PRINT "THIS PROGRAM SIMULATES THE TOSSING OF"
30 PRINT "N COINS, EACH HAVING PROBABILITY P OF"
40 PRINT "FALLING 'HEADS'."
50 PRINT : INPUT "INPUT  N,P = ";N,P
60 FOR I = 1 TO N: IF  RND (1) < P THEN H = H + 1: PRINT "H";: GOTO 80
70 PRINT "T";
80 NEXT I
90 PRINT : PRINT : PRINT "TOTAL OF "H" HEADS.": H = 0: GOTO 50
```

P43 ROLL DICE

```
10 HOME
20 X = 1 + INT(6*RND (1)): Y = 1 + INT(6*RND (1))
30 PRINT X" DOTS AND "Y" DOTS   SUM =";X + Y
40 GOTO 20
```

P44 FREQUENCY CLASSES

```
8  HOME
10  DIM X(100),C(21),F(100): A = 10^10: B = - A
11  DATA  56.2,62.4,45.8,75.6,45.6,50.1,38.9,65.4,70.3,74.2,55.0,64.5
12  DATA  46.5,67.8,37.6,60.4,78.1,45.7,75.4,69.8,49.2,52.6,43.6,34.8
49  DATA  -1.572101
50  I = I + 1: READ X(I): IF X(I) =  - 1.572101 THEN N = I - 1: GOTO 90
60  IF A > X(I) THEN A = X(I)
70  IF X(I) > B THEN B = X(I)
80  GOTO 50
90  PRINT "LEAST: "A"    GREATEST: "B: PRINT
100  PRINT "INPUT # CLASSES (2 TO 20) FOR GROUPING"
110  INPUT "DATA:  K = ";K
120  FOR I = 0 TO K: C(I) = A + (B - A) * I / K: NEXT I
130  FOR I = 1 TO N
140  IF X(I) = B THEN M = M + 1
150  FOR J = 0 TO K - 1
160  IF C(J) <  = X(I) AND X(I) < C(J + 1) THEN F(J) = F(J) + 1: GOTO 180
170  NEXT J
180  NEXT I
190  F(K - 1) = F(K - 1) + M
200  PRINT : PRINT "CLASS      INTERVAL         FREQUENCY": PRINT
210  FOR I = 0 TO K - 1
220  PRINT I + 1"    "C(I)" TO "C(I + 1);
230  PRINT  TAB( 35)F(I)
240  NEXT I
250  PRINT : PRINT "YOU MAY NOW TYPE YOUR OWN DATA INTO"
260  PRINT "LINES 11-48."
```

P45 INDIV. BINOMIAL PROBABILITIES

```
10 HOME : PRINT "JUST A MOMENT PLEASE ..."
20 DIM L(300): FOR I = 1 TO 300: L(I) = L(I - 1) +  LOG (I): NEXT I
30 INPUT "INPUT X,N,P = ";X,N,P
40 Y = X*LOG (P) + (N - X)*LOG (1 - P) + L(N) - L(X) - L(N - X)
50 PRINT "P[ X = "X" ]  =  "; EXP (Y)
60 PRINT : GOTO 30
```

P46 CUM. BINOMIAL PROBABILITIES

```
10 HOME : PRINT "JUST A MOMENT PLEASE ..."
20 DIM L(300): FOR I = 1 TO 300: L(I) = L(I - 1) +  LOG (I): NEXT I
25 DIM P(300)
30 INPUT "INPUT N,P,A,B = ";N,P,A,B
35 FOR X = 0 TO N
40 P(X) =  EXP(X*LOG (P) + (N - X)*LOG (1 - P) + L(N) - L(X) - L(N - X))
50 NEXT X
60 FOR X = A TO B: S = S + P(X): NEXT X
70 PRINT "P[ "A" <= X <= "B" ]  =  "S
80 S = 0: PRINT : INPUT "INPUT  A,B = ";A,B
90 PRINT : GOTO 60
```

Programs P45 and P46 serve the same purposes as binomial probability tables found in the backs of textbooks. However, the programs are more extensive (N up to 300), more accurate (7 decimal places), and faster.

Program P45 can be taught as a practical application of the LOG and EXP functions: the binomial probability formula is

P[X = x] = C(N,x)*(P^x)*(1-P)^(N-x)

Line 40 computes the LOG of this probability (that random variable X takes the value x), and Line 50 prints the antilog, or EXP, of the LOG. Without LOG, the computer would experience overflow at Line 20 long before I reaches 200.

P47 MEAN (OF 6 TEST SCORES)

```
10 HOME
20 N = 6
30 DATA 95,82,90,96,72,81
40 FOR I = 1 TO N: READ X: S = S + X: NEXT I
50 PRINT "MEAN = "; S / N
```

P48 MEAN (OF 100 RANDOM NUMBERS)

```
10 HOME
20 N = 100
30 FOR I = 1 TO N: S = S +  RND (1): NEXT I
40 PRINT "MEAN = "; S / N
50 S = 0: GOTO 30
```

Students should be asked what number they think will be the mean of random numbers from the computer. It is remarkable how fast the computer can then print sample means, which, as Program P48 shows, almost always lie between .46 and .54. It is a good idea to change 100 to 1000 and have students see that the sample mean for the larger samples tends to be considerably closer to .5.

P49 MEAN AND STD (100 RND NOS.)

```
10 HOME
20 N = 100
30 FOR I = 1 TO N: X = RND(1): S = S + X: T = T + X*X: NEXT I
40 M = S/N: PRINT "MEAN = ";M
50 D = SQR(T/(N - 1) - N*M*M/(N - 1)): PRINT " STD = "D
60 S = 0: T = 0: PRINT : GOTO 30
```

P50 DISTR. OF DATA ABOUT THEIR MEAN

```
2 HOME : DIM X(500)
3 DATA 1.7,2.3,2.4,1.3,3.4,2.0,2.6,3.1,3.4,2.7
4 DATA 3.5,1.4,5.6,1.4,2.6,3.0,2.5,3.4,2.3,1.6
20 DATA -.324354
30 I = I + 1: READ X(I): IF X(I) = - .324354 THEN N = I - 1: GOTO 50
40 GOTO 30
50 FOR I = 1 TO N:S = S + X(I):T = T + X(I) * X(I): NEXT I
60 M = S/N: PRINT "MEAN = ";M
70 D = SQR(T/(N - 1) - N*M*M/(N - 1)): PRINT "STD = ";D
80 FOR I = 1 TO N
90 IF ABS (X(I) - M) < D THEN A = A + 1
100 IF ABS (X(I) - M) < 2*D THEN B = B + 1
110 NEXT I
115 PRINT
120 PRINT 100*A/N"% OF THE DATA LIE WITHIN 1 STD OF"
125 PRINT "       THEIR MEAN.": PRINT
130 PRINT 100*B/N"% OF THE DATA LIE WITHIN 2 STDs OF"
135 PRINT "       THEIR MEAN.": PRINT
140 PRINT "YOU MAY NOW TYPE YOUR OWN DATA INTO"
145 PRINT "LINES 3-19."
150 LIST 3 - 19
```

P51 PERCENTILES

```
10 HOME : DIM X(500)
20 DATA 1,4,2,2,3,1,4,4,5,2,3,2,7,3,4,2,3,3,2,1
30 DATA 8,0,3,3,3,2,5,2,0,1,5,1,4,4,3,3,2,3,1,2
40 DATA -.324354
50 I = I + 1: READ X(I): IF X(I) = - .324354 THEN N = I - 1: GOTO 70
60 GOTO 50
70 PRINT "DATA BEFORE RANKING:"
80 FOR I = 1 TO N: PRINT X(I)" ";: NEXT I
90 FOR I = 1 TO N - 1: FOR J = I + 1 TO N: IF X(I) < X(J) THEN 110
100 X = X(I): X(I) = X(J): X(J) = X
110 NEXT J,I
120 PRINT : PRINT : PRINT "DATA AFTER RANKING:"
130 FOR I = 1 TO N: PRINT X(I)" ";: NEXT I: PRINT : PRINT
140 PRINT "INPUT DESIRED PERCENTILE (1 TO 99):"
150 INPUT " K = ";K
160 Q = N*K/100: IF Q =INT(Q) THEN P = .5*(X(Q) + X(Q + 1)): GOTO 180
170 I = 1 + INT(Q): P = X(I)
180 PRINT "THE "K" PERCENTILE EQUALS "P
190 PRINT : GOTO 140
```

P52 RANDOM NUMBER GENERATOR

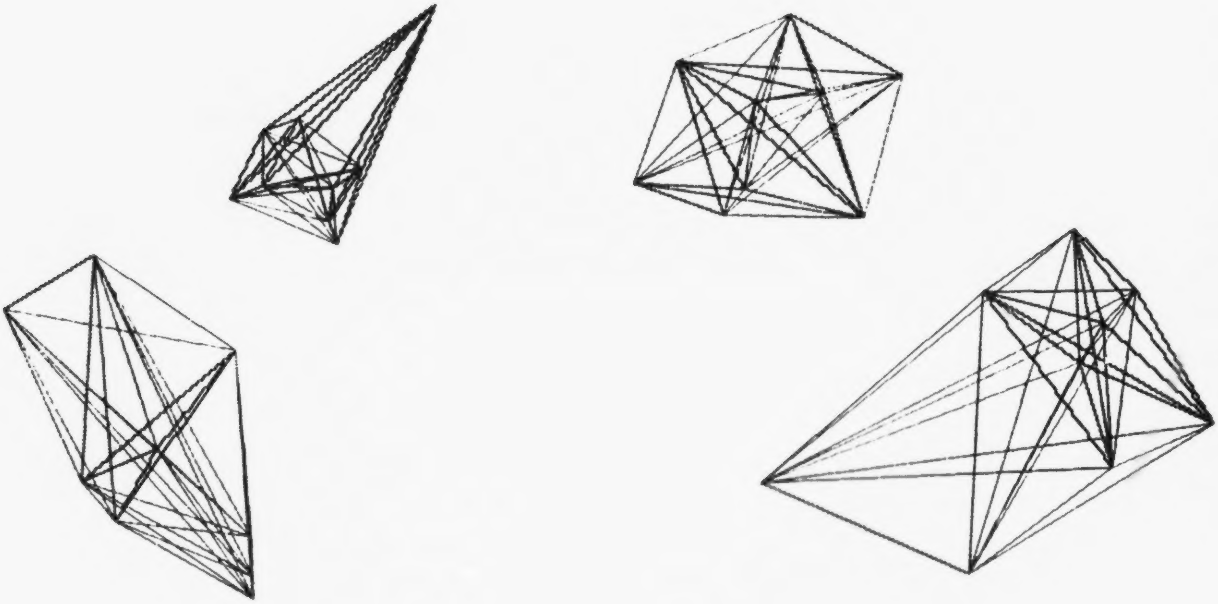
```
10 HOME
20 X = 47394118: A = 23: M = 10000001
30 Y = A*X - M*INT(A*X/M)
40 PRINT Y/M
50 X = Y
60 GOTO 30
```

The built-in random number generator on many computers is this Linear Congruence Method. (What an excellent example of a practical application of number theory!) Various L.C. generators use various seeds, or starting values of X, A, and M (at Line 20). The values given here are the original values used by D. H. Lehmer when he introduced this L.C. Method in 1948.

Students should write programs using the L.C. Method. They may turn random numbers into random motions, letters, colors, directions, words, shapes, faces, etc. Following are three examples. Program P55 uses the L.C. Method, and Programs P53 and P54 use the computer's built-in generator (which quite possibly also uses the L.C. Method).

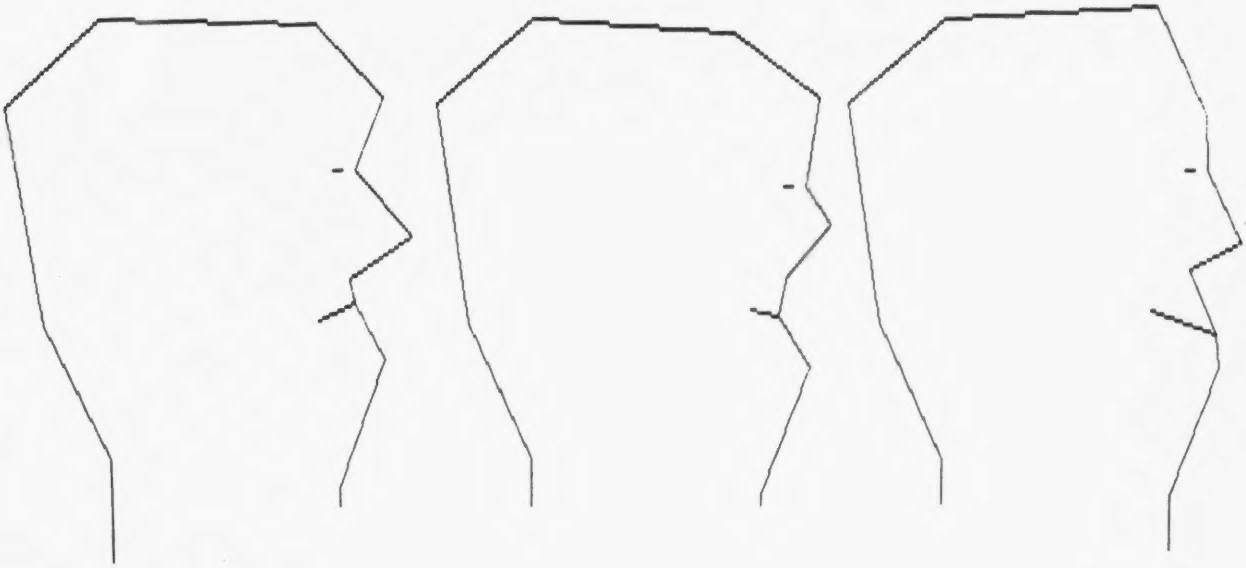
P53 BALLOONS

```
10 REM    BALLOONS
20 HOME : VTAB 7: PRINT "FORTY BALLOONS WILL RISE AT RANDOM."
30 PRINT "THE FIRST TO REACH THE TOP OF THE"
40 PRINT "SCREEN IS THE WINNER.": PRINT
50 PRINT : PRINT "PRESS 'RETURN' TO START THE RACE.";: GET A$
60 GOTO 80
70 PRINT "PRESS 'RETURN' TO START ANOTHER RACE.";: GET A$
80 DIM R(39)
90 GR : HOME
100 W =  INT (9 + 30*RND (1))
110 FOR C = 0 TO 39: R(C) = 0
120 COLOR= C + 1 - 15*INT (C / 15)
130 PLOT C,W + 1: NEXT C: PRINT :T = 0
140 PRINT "STARTING LEVEL FOR THIS RACE: "W
150 C =  INT(40*RND (1))
160 R(C) = R(C) + 1: T = + 1
170 COLOR= C + 1 - 15*INT (C / 15)
180 PLOT C,W - R(C): COLOR= 0
190 PLOT C,W - R(C) + 1
200 IF R(C) < W THEN 150
210 COLOR= C + 1 - 15*INT (C / 15)
220 FOR X = 1 TO W: PLOT C,X: NEXT X
230 PRINT "AVG HT ABOVE STARTING LEVEL: "T / 40
240 RUN 70
```



ABOVE: Random Crystals produced by Program P54.

BELOW: Random Faces produced by Program P55.



P54 CRYSTALS

```
10  REM     CRYSTALS
20  DIM X(100),Y(100)
30  HOME : VTAB 8: PRINT "THIS PROGRAM USES RANDOM NUMBERS TO"
40  PRINT "GENERATE RANDOM POLYGONS WITH DIAGONALS."
50  PRINT "PRESS 'RETURN' TO START THE ACTION.";: GET A$
60  HCOLOR= 3: FOR T = 3 TO 30
70  HOME : HGR : POKE 49234,0: FOR I = 1 TO T
80  X(I) = 280*RND (1): Y(I) = 192*RND (1)
90  HPLOT X(I),Y(I): NEXT I
100  FOR I = 1 TO T: FOR J = I TO T
110  HPLOT X(I),Y(I) TO X(J),Y(J)
120  NEXT J,I,T: GOTO 60
```

P55 FACES

```
10  S = 12: T = 11
20  DEF FN U(X) = 140 + S*X
30  DEF FN V(Y) = 80 - T*Y
40  DIM X(16),Y(16),A(16),B(16)
50  HOME : HGR : HCOLOR= 3
60  DATA  -4,-10,-4,-7,-6,-3,-7,3.5,-4,6,3,6.5,5,4,4.5,2,5.5,0
70  DATA  4.3,-1,4.5,-2.5,3.2,-2.5,4.5,-4.3,4.5,-4.3,3,-8,3,-10
80  FOR I = 1 TO 16: READ X(I),Y(I): NEXT I
90  FOR I = 6 TO 13: A(I) = X(I): B(I) = Y(I): NEXT I
100  X = 7: A = 23: M = 101: W = 23: B = 19: N = 179
110  FOR I = 6 TO 13: X(I) = A(I): Y(I) = B(I)
120  Y = A*X - M*INT(A*X/M): X(I) = X(I) - 1 + Y/M
130  Z = B*W - N*INT(B*W/N): Y(I) = Y(I) - 1 + W/N
140  X = Y: W = Z: X(13) = X(11): Y(13) = Y(11)
150  NEXT I
160  FOR I = 1 TO 15: U =  FN U(X(I)): V =  FN V(Y(I))
170  U1 =  FN U(X(I + 1)): V1 =  FN V(Y(I + 1))
180  HPLOT U,V TO U1,V1
190  NEXT I
200  U =  FN U(X(8)): V =  FN V(Y(8))
205  HPLOT U - 4,V TO U - 2,V
210  FOR P = 1 TO 700: NEXT P
220  HGR : GOTO 110
```

PROGRAMS FOR TEACHING AND LEARNING ABOUT SEQUENCES AND LIMITS

P56 MULTIPLY INTEGERS

```
 10  HOME : DIM K(100): P = 1
 20  DATA  2,3,5,7,11,13,17,19
 30  DATA  5.7
 40  I = I + 1: READ K(I): IF K(I) = 5.7 THEN N = I - 1: GOTO 60
 50  GOTO 40
 60  FOR I = 1 TO N: P = P*K(I): NEXT I
 70  FOR I = 1 TO N - 1: PRINT K(I)"*";: NEXT I: PRINT K(N)" = "P
 75  PRINT
 80  PRINT "TYPE THE NUMBERS YOU WISH TO MULTIPLY"
 90  PRINT "INTO LINE 20."
 100  LIST 20
```

P57 FACTOR INTEGERS

```
 8  HOME
 10  INPUT "INPUT A = ";A
 20  FOR I = 2 TO A
 30  C = 0
 40  IF A/I < > INT(A/I) THEN 80
 50  A = A/I
 60  C = C + 1
 70  GOTO 40
 80  IF C = 0 THEN 100
 90  PRINT "("I"^"C")";
 100  IF A = 1 THEN 120
 110  NEXT I
 120  PRINT : PRINT : RUN 10
```

P58 CONSECUTIVE PRIMES (TRY ALL N)

```
 8  HOME
 10  FOR N = 2 TO 1000
 20  FOR I = 2 TO SQR(N + 1)
 30  IF N/I = INT(N/I) THEN 60
 40  NEXT I
 50  PRINT N" ";
 60  NEXT N
```

P59 CONSECUTIVE PRIMES (FASTER)

```
  8  HOME
 10  FOR N = 2 TO 10000
 20  IF 2*INT(N/2) = N THEN 90
 30  IF 3*INT(N/3) = N THEN 90
 40  FOR  I = 6 TO 1 + SQR (N) STEP 6
 50  J = 1 - 1: IF J*INT(N/J) = N THEN 90
 60  J = I  + 1 : IF J*INT(N/J) = N THEN 90
 70  NEXT I
 80  PRINT N"";
 90  NEXT N
```

P60 ARITH. PROGRESSION (WITH LOOP)

```
8 HOME
10 INPUT "INPUT A,D = ";A,D
20 FOR X = A TO A + 10*D STEP D: PRINT X: NEXT X
30 PRINT : RUN 10
```

P61 ARITH. PROGRESSION (WITHOUT LOOP)

```
8 HOME
10 INPUT "INPUT A,D = ";A,D
15 T = A: PRINT T" ";
20 T = T + D: PRINT T" ";: GOTO 20
```

Students should experiment with reformatting the output of Programs P60-P62 and adding lines to print a running sum of terms.

P62 GEOMETRIC PROGRESSION

```
8 HOME
10 INPUT "INPUT A,R = ";A,R
15 T = A: PRINT T" ";
20 T = T*R: PRINT T" ";: GOTO 20
```

P63 GCD

```
8 HOME
10 PRINT : PRINT : INPUT "INPUT A,B = ";A,B
15 PRINT
20 C = A: D = B
30 R = A - B*INT (A / B)
40 PRINT A,B,R
50 IF R > 0 THEN 60
55 PRINT : PRINT "     GCD(A,B) = "B: PRINT : GOTO 10
60 A = B: B = R
70 GOTO 30
```

Program P63 is the Euclidean Algorithm. Students should pencil through it entirely, using, for example, A = 105 and B = 66. The GCD of A,B is the last nonzero remainder R (3rd column of output). The LCM of A,B equals the product AB divided by the GCD.

P64 BUBBLESORT (10 RANDOM NUMBERS)

```
8 HOME : PRINT "BEFORE:": PRINT
10 FOR I = 1 TO 10: X(I) =  RND (1): PRINT X(I)" ";: NEXT I
20 FOR I = 1 TO 9: FOR J = I + 1 TO 10: IF X(I) < X(J) THEN 40
30 X = X(I): X(I) = X(J): X(J) = X
40 NEXT J,I: PRINT : PRINT
45 PRINT "AFTER:": PRINT
50 FOR I = 1 TO 10: PRINT X(I)" ";: NEXT I
```

The bubblesort (already used to rank data in Program P51) is the easiest way to rank, or sort, a sequence to numbers from least to greatest. For more than 50 numbers, the bubblesort is much slower than more sophisticated sorts. Program P65 bubblesorts strings in order to alphabetize them.

P65 ALPHABETIZE (VIA BUBBLESORT)

```
4 HOME
5 DATA SMILE, B.B.KING,TRIG,ZILCH,75,NEW YORK,B.A.D.,SASKATOON,AMY,YES
10 FOR I = 1 TO 10: READ S$(I): NEXT I
20 FOR I = 1 TO 9: FOR J = I + 1 TO 10: IF S$(I) < S$(J) THEN 40
30 X$ = S$(I): S$(I) = S$(J): S$(J) = X$
40 NEXT J,I
50 FOR I = 1 TO 10: PRINT I,S$(I): NEXT I
```

P66 ADD 1 + 2 + 3 + ... + N

```
8  HOME
10 FOR N = 1 TO 20
20 FOR K = 1 TO N
30 S = S + K
40 NEXT K
50 PRINT N,S,N * (N + 1) / 2
60 S = 0
70 NEXT N
```

P67 ADD 1^2 + 2^2 + ... + N^2

```
8  HOME
10 FOR N = 1 TO 20
20 FOR K = 1 TO N
30 S = S + K * K
40 NEXT K
50 PRINT N,S,N * (N + 1) * (2 * N + 1) / 6
60 S = 0
70 NEXT N
```

P68 ADD 1^3 + 2^3 + ... + N^3

```
8  HOME
10 FOR N = 1 TO 20
20 FOR K = 1 TO N
30 S = S + K * K * K
40 NEXT K
50 PRINT N,S,(N * (N + 1) / 2) ^ 2
60 S = 0
70 NEXT N
```

P69 ADD 1/1 + 1/2 + ... + 1/N

```
8  HOME
10 FOR N = 1 TO 10000
20 S = S + 1 / N
30 PRINT N,S
40 NEXT N
```

P70 THE NUMBER e

```
8  HOME
10 FOR N = 1 TO 20
20 PRINT N"   "(1 + 1 / N) ^ N"    " EXP (1)
30 NEXT N
```

P71 INTEREST COMP. N TIMES PER YEAR

```
8  HOME
10  INPUT "INPUT P,R,T = ";P,R,T
20  PRINT : PRINT " N","  AMOUNT"
30  FOR N = 1 TO 12
40  A = P*(1 + R/N) ^ (N*T)
50  PRINT N,"$".01*INT(.5 + 100*A)
60  NEXT N
70  PRINT : PRINT "UNDER CONTINUOUS COMPOUNDING, THE"
80  PRINT "AMOUNT WOULD BE  $"P*EXP(R*T)
```

For example, the input P,R,T = 1000,.1,10 represents a principal of \$1000 invested at 10% annual interest for ten years. The output gives the total value of the deposit for N compoundings per year, for N = 1, 2, 3, ...

P72 SQUARE ROOT

```
8  HOME
10  PRINT "INPUT THE NUMBER  N  WHOSE SQUARE ROOT"
15  INPUT "IS TO BE COMPUTED:  N = ";N
18  PRINT
20  R = N
30  R = .5*(R + N/R)
40  PRINT R, R*R - N
50  IF ABS(N - R*R) > 10^ - 5 THEN 30
60  PRINT : GOTO 10
```

Session 4E
Elementary

Software Evaluation

Subject Area ____________ Microcomputer ____________
Topic(s) ____________
Requires ___ K of memory Grade Level (estimate) ____________
Program Name ____________ Author ____________
Publisher ____________ Copyright Date ____________
Publisher Address ____________
Storage Medium: Tape Cassette ___ Cartridge ___ Diskette ___
Type of Package: Single Program _____ Part of a Series ____ Other: ____________
Price: ____________ Reviewer's Name ____________
Application(s) under consideration ____________
Additional Hardware Required ____________
Possible Grouping Arrangements: Individual __ Small Group __ Large Group __ Other _____
Special Format: Game sequences _____ Drill and Practice _____
Sequences _____ Management System _____ Pretests _____
Posttests _____ Linear _____ Branching ____ Other ____

Brief Description of Program:

Characteristics: Program Operation and Documentation

		Rating	Weight	Total
1.	Allows user to correct typing errors	____	____	____
2.	Documentation available and clearly written	____	____	____
3.	Clear, nicely formatted screen displays	____	____	____
4.	Incorrect selection of commands or keys does not cause program to abort	____	____	____
5.	Menus and other features make the program "user friendly"	____	____	____
6.	Instructions can be skipped if already known	____	____	____
7.	Uses correct grammar, punctuation, and spelling	____	____	____
8.	Clear and useful summary of program operation provided	____	____	____
9.	Loading instructions clear; program easy to load	____	____	____
10.	Bug-free: program runs properly	____	____	____
11.	Uses computer capabilities well	____	____	____
12.	Accepts abbreviations for common responses (i.e., Y for YES)	____	____	____
13.	Operation of program does not require user to turn computer on and off	____	____	____

14. Readability of text appropriate for intended user	____	____	____
15. ____________________ ____________________	____	____	____
16. ____________________ ____________________	____	____	____

Total Program Operation/Documentation Score: _____

Characteristics: Student Use

	Rating	Weight	Total
1. Requires no computer knowledge	____	____	____
2. Does not require student reference to manuals	____	____	____
3. High student involvement	____	____	____
4. Provides student with summary of performance	____	____	____
5. Shows no racial, sexual discrimination, and so forth	____	____	____
6. Effective feedback for correct responses	____	____	____
7. Effective feedback for incorrect responses	____	____	____
8. Positive reinforcement more attractive than negative reinforcement	____	____	____
9. Encourages cooperation	____	____	____
10. Student control over rate of presentation	____	____	____
11. Student control over sequence of lesson	____	____	____
12. Student control over selection of lesson	____	____	____
13. Student can select to go back and review previous frames of information	____	____	____
14. Student can select various styles of presentation	____	____	____
15. Length (time) of lesson appropriate	____	____	____
16. ____________________ ____________________	____	____	____
17. ____________________ ____________________	____	____	____

Total Student Use Score: _____

Characteristics: Instructor Use

	Rating	Weight	Total
1. Instructional objectives clearly stated	____	____	____
2. Easily integrated into curriculum	____	____	____
3. Random generation of problems contributes to usefulness of program	____	____	____

4.	No need for instructor to assist users			
5.	Useful teacher manual and/or accompanying materials provided			
6.	Suggested lesson plans			
7.	Suggested grouping arrangements			
8.	Useful student workbook provided			
9.	Useful blackline/ditto masters provided			
10.	Interesting follow-up activities and/or projects suggested			
11.	Management system easy to use and flexible			
12.	Provides whole class summaries of performance			
13.	Other educational materials suggested or provided			
14.	When reentering program, student begins at appropriate spot			
15.	Software does not require intermittent operation of disk drive or cassette recorder			
16.				
17.				

Total Instructor Use Score: ______

Characteristics: Content

		Rating	Weight	Total
1.	Follows sound educational techniques			
2.	Follows sound educational theory			
3.	Accurate content			
4.	Amount of learning justifies time spent by users			
5.	Appropriate use of color			
6.	Appropriate use of graphics and/or animation			
7.	Appropriate use of sound			
8.	Accomplishes stated objectives			
9.	Sequence of lesson and instructions logical and clear			
10.				

11. ____________________
____________________ ____ ____ ____

Total Content Score: ____

Evaluation Summary: (Main strengths and weaknesses, and so forth)

Total points for all characteristics: ____

Total points possible: ____
(Add all weights and multiply by 5 to determine
total points possible)

Percent Rating: ____ percent

Describe Backup Policy:

Backup policy is: acceptable _____ not acceptable _____

Recommendation: Worth the price ____ Do not purchase ____
Can recommend only if certain changes are
made ______________________________

Integrating Software into Curriculum

Mathematical topic/level: Ordering sets of numbers up to 100 - Grade 1

Software title/source: Sequence - teacher-written (attached)

Type: BASIC program

Brief Description: Teacher selects how many numbers (up to 10) and largest number to be used. Program is continuous (recycles).

Other materials: Cuisenaire materials or Dienes blocks, hundreds board

Teaching plan

Apperceptive basis: comparing two numbers (less than 100) using Cuisenaire or Dienes materials.

1. Review apperceptive basis.

2. Introduce hundred board
   a. Compare two numbers - record results.
   b. Discover rule for order on board.
   c. Relate to apperceptive basis.

3. Use overhead (or chalkboard)
   a. Have class order - smallest to largest - sets of three or four numbers.
   b. Discuss strategy.

4. Use computer program (two students per computer)
   a. Computers preloaded to randomly generate sets of four numbers to order smallest to largest.
   b. Explain how to enter numbers.
   c. Let students work on computers. Have them write ordering on paper before entering on computer.
   d. For students doing well, reset computer for larger sets (6, 7, or 8).

5. Closure
   a. Review rule for ordering.
   b. Use terms "face value" and "place value".
   c. Tell students that I appreciate their good behavior while using the computer.
   d. Acknowledge students with at least 10 "good job" reports.

Source: Ken Stilwell, Kirksville, Missouri

```
PR#0
]LIST

10      REM  PROGRAM BY KEN STILWELL
40      HOME
50      PRINT "HOW MANY NUMBERS DO YOU WANT TO ORDER"
60      PRINT "THIS MUST BE A NUMBER BETWEEN 2 AND 9"
65      PRINT
70      INPUT A
75      PRINT: PRINT "ENTER THE LARGEST NUMBER TO BE USED"
78      PRINT: INPUT B
100     FOR I = 1 TO A
104     LET K = INT (B * RND (1)) + 1
110     LET X (I) = K
120     NEXT  I
130     FOR I = 1 TO A
140     FOR J = I + 1 TO A
150     IF X (I) = X (J) THEN 100
160     NEXT J
170     NEXT  I
175     HOME
180     FOR  I = 1 TO A
190     PRINT X (1); "  ";
200     NEXT  I
210     PRINT
220     PRINT
225     GOSUB 500
230     PRINT "ENTER THE NUMBER IN PROPER ORDER"
232     PRINT : PRINT "PRESS RETURN KEY"
233     PRINT "AFTER EACH ENTRY"
235     PRINT
238     Z = 0
240     FOR  I = 1 TO A
245     Z = Z + 4
250     INPUT Y (I): POKE 36,Z
270     IF Y (I) < > X (I) THEN 600
280     NEXT I
290     PRINT : FOR  I = 1 TO A
300     PRINT Y (I); "  ";
310     NEXT  I: PRINT
320     PRINT: PRINT
330     PRINT "GOOD JOB!!!"
340     FOR  I = 1 TO 2000
350     NEXT  I
360     GOTO 100
500     REM  SUBROUTINE TO SEQUENCE NUMERALS
510     LET K = A - 1
520     FOR  I - 1 TO K
530     IF X (I) <X (I + 1) THEN 570
540     LET C = X (I)
550     LET X (I ) = X (I + 1)
```

```
560    LET X (I + 1) = C
570    NEXT I
575    LET K = K - 1
580    IF K > 0 THEN 520
588    PRINT
600    REM ERROR NOTIFICATION
610    PRINT : PRINT : PRINT
620    PRINT "YOUR NUMBERS ARE OUT OF ORDER"
630    PRINT
632    PRINT : PRINT "THE CORRECT ORDER IS : ": PRINT
634    FOR I = 1 TO A
636    PRINT X (I); "  ";
638    NEXT I: PRINT
640    FOR T = 1 TO 2000
650    NEXT T
660    GOTO 100
670    END

]
```

Integrating Software into Curriculum

Mathematical topic/level: Word problems - Grade 3

Software title/source: Easy Graph (Grolier)

Type: Tutorial/application

Brief description: to help student define and make graphs

Other materials: worksheet plus concrete materials

Teaching plan

The software is to be used in the third-grade "Shoe Unit".

1. Come up with shoe categories - have children look around the room, then brainstorm.
2. List types of shoes on board (e.g., sneakers, tie shoes, loafers, docksiders, heels, Mary Janes).
3. Give blank table, send home -- ask them to check house for their shoe collections.
4. Use tutorial aspect of software program "Easy Graph" -- tell children that they will be asked to pick a type of graph to display the shoe data -- they should look at all types of graphs to make choices.
5. Have children create own graphs, using software.
6. Hang up different choices or put on overhead -- discuss options as well as summary of findings.

Source: Harriet Fayne, Columbus, Ohio

Mathematical topic/level: Problem solving - Grades 4-6

Software title/source: The Factory (Sunburst)

Type: Perceptive - Problem Solving

Brief description: Three machines can fabricate a product; can play against computer or other user.

Other materials: none

Teaching plan

1. Demonstrate the "test of machines" part.
2. Have a group of three students play as a team against the computer.
3. Set up tournament for teams of 3 to try to solve other teams' product.
4. Teacher creates product (8 steps); challenge teams to recreate in fewer than 8 steps (find a minimum).
5. Careful: plant a spy in one team who gives information to a member of another team. Talk about computer crime!
6. Students make up activities.

Source: Unknown

Mathematical topic/level: Problem solving - Grade 6

Software title/source: Problem Solving (McGraw-Hill)

Type: Problem solving

Brief description: mathematical decision-making

Other materials: related resource book, notebook

Teaching Plan

1. Introduce procedure for solving story problems.
2. Give guided practice solving word problems.
3. Students practice solving word problems from McGraw-Hill resource book.
4. Student groups of three or four design word problems to be solved by their classmates.
5. Student groups of three or four solve word problems from McGraw-Hill disk.
6. Individual student assignments on solving word problems.
7. Individual testing on problem solving.

Source: Joe Orf, St. Louis; Ramona Choos, Highland Park, Illinois; Rich Little, Berea, Ohio

Software Evaluation (NCTM)

- instructional range
- instructional grouping for use
- executive time
- program use(s)
- user orientation: instructor's point of view
  - student's point of view
- content
- motivation and instructional style
- social characteristics

Integrating Software into Curriculum

Mathematical topic/level ______________________________

Software title/source: ______________________________

Type: ______________________________

Brief description: ______________________________

____

Other materials (e.g., manipulatives): ______________________________

Teaching Plan (indicate at what point software should be used):

Problem Solving: Research Background

For years, research focused on --

- characteristics of problems
- characteristics of good or poor solvers
- teaching strategies to build success

Recently, focus has shifted --

- strategies children use: PROCESS

Data from Priorities in School Mathematics (PRISM) survey:

- consistently ranked high in priority for increased support
- at least lip-service to more than routine exercises
- support for teaching a range of problem-solving strategies and for teaching problem solving as a process
- support for providing in-service on problem-solving methods
- consistently strong support for increasing emphasis on applications throughout the curriculum

Problem solving, grades 3-6 (Stockdale, 1985):

"Stronger problem-solving programs in the 1980's"--

- more problems
- greater variety
- more multi-step
- fewer like prior problem
- fewer merely computation practice
- more strategies taught
- more clusters around theme

Generalizations from Research on Problem Solving

- Problem-solving strategies can be specifically taught; when they are, they are used more and students attain correct solutions more often.
- Learning strategies gives students a repertoire from which to draw.
- There is no one optimal strategy.
- Students need problems (in which the approach is not apparent) and need encouragement to test many alternative approaches.
- Some strategies are used more than others, with various strategies used at different stages of the problem-solving process.
- Developmental level is related to a student's problem-solving achievement.
- Problem-solving skills are improved by incorporating them throughout the curriculum.

Good Problem Solvers --

- understand concepts, terms
- note likenesses, differences -- analogies
- identify critical elements
- note irrelevant details
- evaluate and select alternative solution routes
- estimate, approximate, and check for reasonableness
- switch methods readily
- generalize from few examples
- learn from mistakes
- had less anxiety, more confidence
- transfer learning to similar problems
- remember mathematical structure of problem -- forget context, details

Children

- can solve many problems before they come to school
- use procedures which model the structure of a problem -- until after they have instruction
- use less efficient processes when materials are present, even when they don't need to
- find making a drawing and restating a problem particularly helpful
- have much more trouble with multistep problems than with single-step problems
- develop misconceptions when all problems in a lesson are solved by the same algorithm or procedure
- may be unable to solve problems even when they know all the words
- calculators and computers help children to concentrate on the problem, and not get bogged down in computation

Teaching Strategies for Problem Solving

- present many, varied problems
- teach variety of problem-solving strategies, plus overall plan
- give opportunities to analyze problem situations
- encourage using a strategy to solve many problems, and many strategies to solve one problem
- have student determine question asked, necessary and unnecessary information, process: discuss why appropriate
- have students seek similarities across problems
- provide time for discussion, practice, reflection
- have students put problems into own words, compose problems
- provide problems at appropriate levels of difficulty
- have students estimate, analyze estimates, test reasonableness of answers
- have students use dramatization, manipulatives, models, pictures, diagrams, charts, tables, graphs as aids to solving problems
- teach students to select main idea, make inferences, construct sequences
- help students to simplify a problem
- vary the wording in problems of the same type
- consistently ask questions encouraging involvement and thinking
- encourage students to look back and reflect on solved problems

Session 4S.1 Algebra and Geometry

ACTIVITY ONE

OBJECTIVE

To help participants become familiar with a graphics package.

DESCRIPTION

Three separate examples will be graphed to illustrate the operation of the graphics package we'll be using called "Chalkboard Graphics Tool Box I" by Scharf Systems, Inc.; P.O. Box 712; Lyndhurst, New Jersey 07071. Each example is intended to illustrate another feature of this graphics package.

PROCEDURE

- Boot the disk for "Chalkboard Graphics Tool Box I" by Scharf Systems, Inc.
- After the initial introductory screens, you'll be given four options.
- Choose option # 1, FUNCTION ANALYZER.
- After FUNCTION ANALYZER loads, enter the function F(X) = X*SIN(1/X).

This session is designed to help participants become familiar with some of the software they will be using during the upcoming conference.

-

DOMAIN	MIN: -4	RANGE	MIN: -2
	MAX: 4		MAX: 2
	INC: 1		INC: 1

- Press "P" to select "PLOT SPEED/RESOLUTION" and then select option #1, "Medium/High".
- Press <RETURN> and wait for the graph.
- A great deal of activity appears to be happening at the origin. To better see what is going on near the origin, let us choose option #2, "Change Domain and Range".
-

DOMAIN	MIN: -.4	RANGE	MIN: -.5
	MAX: .4		MAX: .5
	INC: .2		INC: .2

- Again, press <RETURN> to see the graph generated.
- It's certainly better, but we'd still like an even better picture.
- Again choose option #2, "Change Domain and Range".
- This time:

DOMAIN	MIN: -.2	RANGE	MIN: -.1
	MAX: .2		MAX: .1
	INC: .1		INC: .1

- By this time you should have an excellent picture of the activity near the origin on the function F(X) = X*SIN(1/X).

__________next__________

- Now select option #3 - "Select Another Function".
- Enter $F(X) = X^4 - X^3 - 5X^2 - 1$ in the form F(X) = X^4-X^3-5*X^2-1.
- Assuming that we have no idea as to what the function will do, we select a purely arbitrary domain and range.
- Let's use:

DOMAIN	MIN: -2	RANGE	MIN: -5
	MAX: 2		MAX: 5
	INC: 1		INC: 2

- Press "P" to change the plot speed to #3, "Very Fast/Sketch".

- This function is a fourth degree polynomial - therefore once the graph is completed, it should be obvious that much of the graph is still not visible.
- Let's change the domain and range (selecting option #2) by a factor of 3.
- DOMAIN MIN: -6 RANGE MIN: -15
  MAX: 6 MAX: 15
  INC: 3 INC: 6
- We now note that the function appears to have two real zeros and two imaginary zeros.
- Let's attempt to isolate the two real zeros to the nearest tenth using option #1, "Display Table of Values".
- By observing the graph, it is clear that the two real zeros occur between -3 and 3. Therefore - start the table at -3 and stop it at 3 using an increment of 1.
- From the table, it's can be seen that the real zeros occur between -2 and -1 --- also between 2 and 3.
- Now, continue to work with the table choosing option #1 to "Recalculate Table" until the real zeros can be determined to the nearest tenth.
- You should find that the real zeros, correct to the nearest tenth, are -1.9 and 2.8.

_________next_________

We will now illustrate how "Chalkboard Graphics Tool Box I" will overlay one graph upon another. To accomplish this, you must select option #5 "Return to the Main Menu".

- At this point, select option #2, RELATION GRAPHER.
- After RELATION GRAPHER loads, select option #6, "Any Function".
- Enter $F(X) = X^2$ in the form F(X) = X^2.
- Select :

  DOMAIN MIN: -2 RANGE MIN: -5
  MAX: 2 MAX: 5
  INC: 1 INC: 2
- Press "P" to select a plot speed, then option #2 for "Fast/Low".

- After the graph is drawn, select option #1 - "Add Another Relation" - then #6 for "Any Function".
- This time, use the function F(X) = X^3+2. Don't forget to enter it as F(X) = X^3 + 2.

We'll explore additional relations that can be graphed as well as features of this graphics package later.

ACTIVITY TWO

OBJECTIVE

To enable participants to become familiar with the operation of the "GEOMETRIC SUPPOSER: TRIANGLES" from Sunburst Communications.

DESCRIPTION

Specific directions will be supplied to enable the participants to move through some of the features of the "GEOMETRIC SUPPOSER: TRIANGLES" from Sunburst in order to utilize the program in the lab period.

PROCEDURE

- Boot the disk for the "GEOMETRIC SUPPOSER: TRIANGLES" from Sunburst. After the program automatically moves through its several introductory screens, the drawing screen appears.
- As directed, press "N" to begin.
- Select "6 YOUR OWN" for the triangle you wish to draw.
- Now choose "3 ANGLE-SIDE-ANGLE". Note the unit length in the upper right hand corner of the screen.
- Enter "40" (degrees) for the size of angle BAC, "6" (units "u" - see upper right hand corner of the screen) for the length of side AB and "110" (degrees) for the size of angle CBA.
- Now press "D" to complete the labeling, then <RETURN> to remove the construction lines.
- Next, choose the following in succession:

"1" for "DRAW"

"9" for "MIDSEGMENT", from midpoint of "BC", to midpoint of "AC"

"1" for "DRAW"

"5" for "PARALLEL", through point "C" parallel to "BA"

"1" for "LENGTH" definition, "u" for "unit" , "3" as constant, then <RETURN>

"1" for "DRAW"

"0" for "EXTENSION", "DE" for segment to be extended, from point "E"

"1" for "LENGTH" definition, "u" for "unit", and

"3"as constant, then <RETURN>

- Now to extend segment BA through point A, press :

"1" for "DRAW"

"0" for "EXTENSION", "BA" for segment to be extended, from point "A"

"1" for "LENGTH" definition, "u" for "unit", and

"3" as constant, then <RETURN>

- Look at the figure drawn. We will now attempt to "discover" a few relationships through experimentation. To measure the size of angle FCE, proceed as follows :

"M" for "MEASURE"

"4" for "ANGLE", "FCE" for the angle's name and <RETURN>.

Then the <SPACE BAR>.

- Now measure the size of angle HEA in the same manner.
- Since both angles measure 40 degrees, lines FC and HD must be parallel.
- Also since lines FC and BA are parallel by construction, lines HD and BA must be parallel giving rise to the conjecture that the line joining the midpoints of two sides of a triangle is parallel to the third side.

*** NOTE ***

Although we really did not need to construct line FC parallel to line BA to achieve this conjecture, it was done to familiarize the participant with that feature of this software package which produces such parallels.

- Again press the <SPACE BAR> in order to continue to measure, then <ESC> since we will now measure segments :

"1" for "LENGTH", "DE" for the segment followed by a <RETURN>

<space bar> to continue to measure, "BA" for the length of that segment (in case you've forgotten it was constructed to be 6 units) followed by a <RETURN>.

- You're now on your own to draw the segment from the midpoint of AC to the midpoint of BA and compare its length to the length of segment BC.
- You'll note that when you draw this segment, the program labels point E with a second letter J. You may use either the new or the old letter in naming the segment when requesting its length.
- Finally draw the segment between the remaining unconnected midpoints.
- Compare the length of this segment with side AC.
- Conjectures should now be made.
- Students should now be encouraged to make conjectures concerning how the perimeters of these figures compare. Reasons for their responses should be expected.
- At this point students should be asked to compare the areas of the four small triangles formed inside the large triangle ABC. Again, formulate conjectures.
- Attempts should be made to "discover" other relationships by utilizing the features of "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst.
- Finally, students should be challenged to try to prove their conjectures deductively.

Session 4S.2 Student Worksheets

Teacher Notes for Worksheets on Lines (1, 2, 3)

OBJECTIVE

To familiarize the student with the graphs of equations in the form of $y=mx+b$. Worksheet 1 establishes the relationship between the value of m and the resulting inclination of the line. In worksheet 2 the student varies the value of b to determine the relationship between that value and the y intercept. The student will also work with parallel lines and changes in the slopes and y intercepts of those lines. Worksheet 3 develops the concepts of negative reciprocals and perpendicular lines.

DESCRIPTION

Any graphics program for functions or relations will work with these three worksheets. Since no instructions for use with a particular software package are included in the worksheets, the teacher will have to provide general information in that area. Worksheets are written so that students may work alone, in pairs, or small groups at computers or they may take notes from a lesson presented on a computer at the front of the classroom.

SPECIAL NOTES

Each of these worksheets takes from 45 to 55 minutes for students to complete.

Many students may forget that the coefficient of x in an equation of the form $y = x + c$ is one. This is a good time to reinforce that idea.

The students may want to label each graph with the corresponding problem number rather than the equation due to space limitation.

These worksheets work especially well when proceding actual plotting of points by students. The shapes of the graphs seem to stay in their minds and future graphing seems to make more sense to them.

These worksheets should not be used to replace actual graph plotting by students, but to introduce or reinforce those ideas.

Student Worksheet
Graphing Lines (1)

You are familiar with the equation y=mx+b. It is called a linear equation because its graph is always a line. In that equation, the x and y are the coordinates of a particular point on that line. The m describes the slope of the line. The value of b, called the y intercept, determines where the line crosses the y axis.

I. In the first set of exercises you will examine lines whose y intercept (the value of b) is zero. These would be lines in the form y=mx+0, which simplifies to y=mx.

A. Type the following equations into the computer. All lines will be graphed on the same set of axes. Sketch and label the lines on the graph provided below.

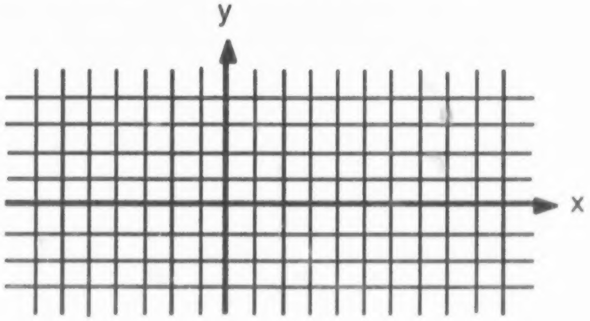
1) $y = x$

2) $y = 2x$

3) $y = 4x$

4) $y = 7x$

5) $y = 15x$



a) What happened to the numerical value of m in equations 1-5 above?

b) Describe what effect this change in m had on the inclination of the slope of the lines.

c) Notice that as the numerical value of m gets larger and larger the graph of y=mx+b gets closer and closer to the y axis.

B. Erase all of the lines from part A and type in this new set of equations. Sketch and label them below.

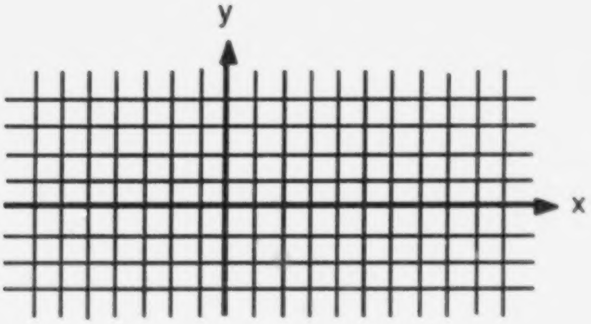
1) y = x

2) y = .8x

3) y = .5x

4) y = .3x

5) y = .12x



a) What happened to the numerical value of m in the equations above?

b) What effect did this change in m have on the inclination of the lines?

C. Erase the previous lines and type in the following. Sketch and label below.

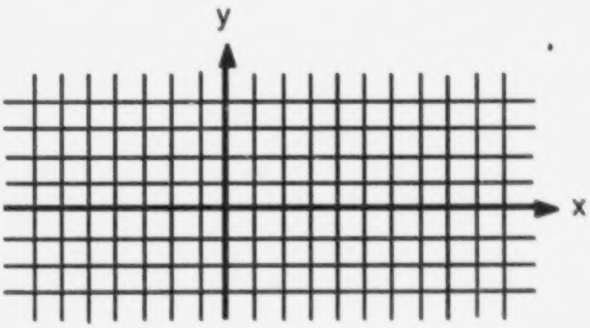
1) y = -.1x

2) y = -.32x

3) y = -.6x

4) y = -.75x

5) y = -x



a) What happened to the numerical value of the slope in each of the equations above?

b) What was the effect on the inclination of the lines?

D. Erase the lines from above and type in these equations. Sketch and label them below.

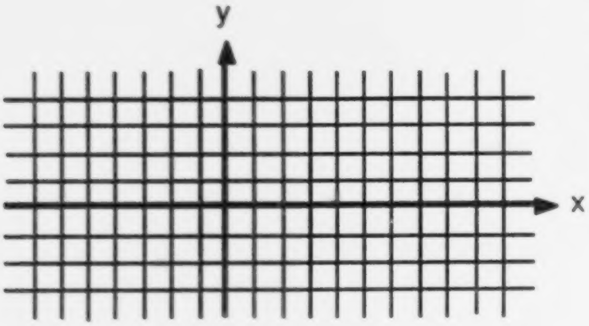
1) $y = -x$

2) $y = -2x$

3) $y = -4.5x$

4) $y = -7x$

5) $y = -12x$



a) What happened to the numerical value of the slope in the equations above?

b) What was the effect on the inclination of the lines?

E. In general, when the slope, m, of a line is greater than zero, describe the inclination of the line as you observe it from left to right on the graph.

Describe the inclination of a line whose slope is less than zero as you observe it from left to right on the graph.

If $y=x$ is used as a reference line, describe the effect on the graph of lines when the slope, m, is increased. That is, when $m>1$.

If $y=x$ is used as a reference line, describe the effect on the graph of lines when the slope $m<1$ and m gets closer to 0.

If $y=-x$ is used as a reference line, describe the effect on the graph of the lines when the absolute value of the slope $m>1$.

If $y=-x$ is used as a reference line, describe the effect on the graph of the lines when the absolute value of the slope $m<1$ and gets closer to 0.

Can you predict what the graph of a line with slope $m=0$ will look like? Sketch your guess on the graph below. Type the equation $y=0x$ into the computer to check your answer.

Student Worksheet
Graphing Lines (2)

This worksheet will allow you to work with linear equations and their graphs. Recall that a linear equation is in the form $y=mx+b$, where x and y are the coordinates of a point on that line. The coefficient of x is m, the slope of the line. The value of b, the y intercept, determines where the line crosses the y axis.

II. You will work with groups of equations in which m will remain the same and b, the y intercept, will change.

A. Type the following equations into the computer. Sketch and label the lines on the graph provided below.

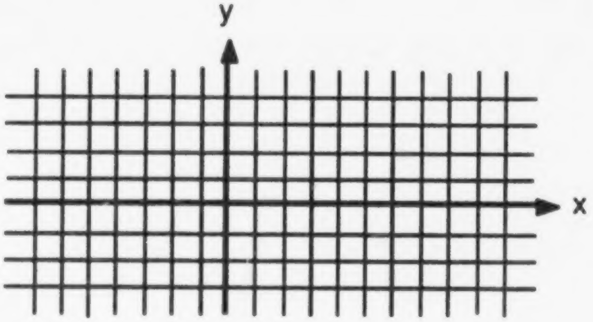
1) $y = .5x + 2$

2) $y = .5x$

3) $y = .5x - 4$

4) $y = -2x$

5) $y = -2x - 3$



a) What is the numerical value of the slope in equations 1-3 above?
In equations 4-5?

b) What is the graphical result when linear equations have the same slope?

B. Erase all the lines from part A and type in this new set of equations. Sketch and label below.

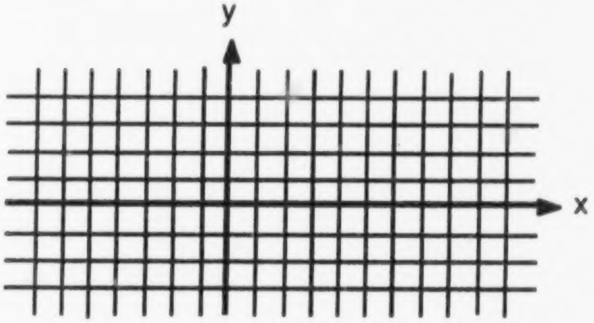
1) $y = x$

2) $y = x + .5$

3) $y = x + 3$

4) $y = x - 2$

5) $y = x - 4.6$



a) Did you notice that all of the lines had the same slope?

b) In general, what must be true about lines if they are parallel?

C. Erase the equations from above. Retype them one at a time and answer the appropriate question below. Erase each line before typing the next one.

1) On equation 1, where does the line cross the y axis?
What is the value of b in that equation?

2) On equation 2, where does the line cross the y axis?
What is the value of b in that equation?

3) Line 3 above crosses the y axis at _____ and b is equal to _____.

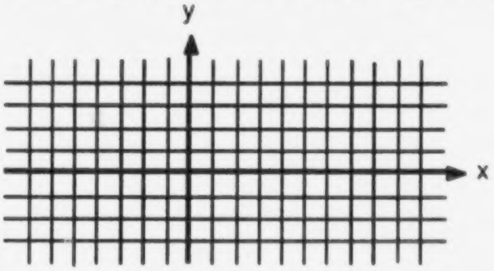
4) After answering the same questions about graphs 4 and 5, can you conclude that when graphing a line in the form y=mx+b, b describes where the line will cross the y axis?

D. Erase the lines from part B. Type the following equations in and observe their graphs.

1) y = 2

2) y = -3

3) y = 0



In the above equations, the slope, m, was equal to 0. The equation y=mx+b could be written as y=0x+b. The first equation would be y = 0x + 2. Rewrite equations 2 and 3.

Notice that all three lines are parallel to each other and to the x axis.

E. Test your understanding of these lessons by considering the following equations. Answer the questions without typing the equations into the computer.

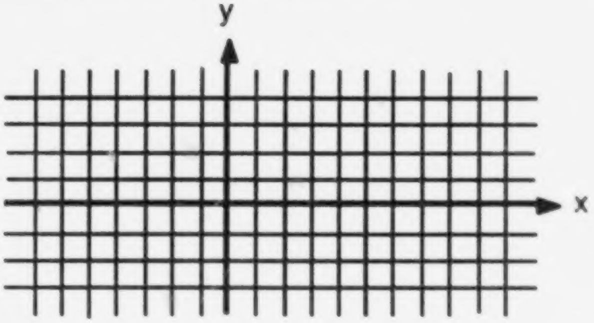
1) $y = 2x - 3$

2) $y = .7x + 2$

3) $y = .7x$

4) $y = .7x - 3$

5) $y = 7x - 1$



a) Which equations above have graphs that are parallel?

b) Which equations above have graphs that cross the y axis in the same place?

c) Now type the equations in to verify your answers.

Student Worksheet
Graphing Lines (3)

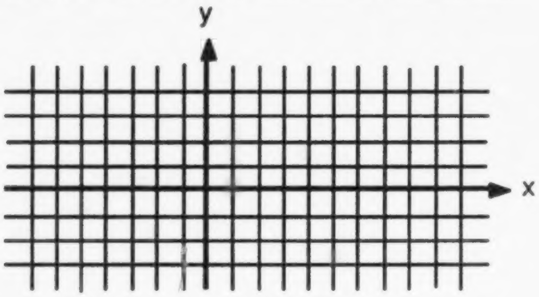
This worksheet will allow you to work with linear equations and their graphs. Recall that a linear equation is in the form y=mx+b, where x and y are the coordinates of a point on that line. The coefficient of x is m, the slope of the line. The value of b, the y intercept, determines where the line crosses the y axis.

III. In this lesson you will work with pairs of equations whose slopes have a special relationship--they are negative reciprocals. Remember that reciprocals are two numbers whose product is 1. Negative reciprocals are two numbers whose product is -1.

A. Type the following pairs equations into the computer. Sketch and label the lines on the graph provided below. Erase each pair of lines before continuing with the next pair.

1) y = 1/2x

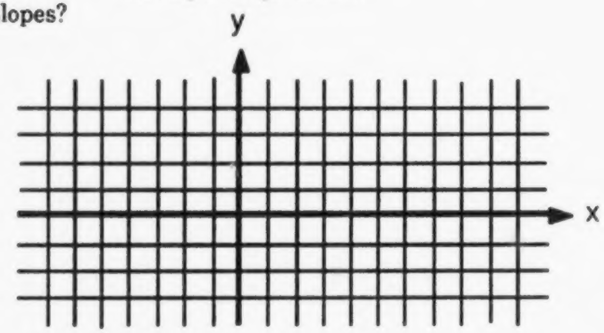
2) y = -2x



What is the slope of equation 1 above? The slope of equation 2?
What is the product of these two slopes?

3) y = x

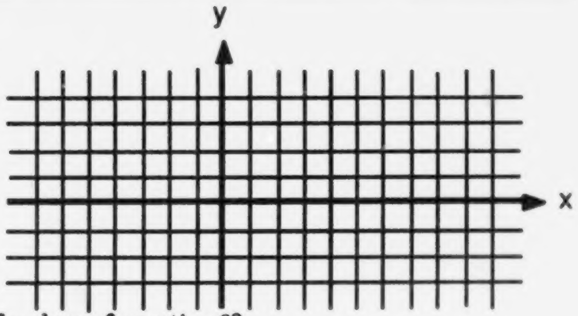
4) y = -x



What is the slope of equation 3? The slope of equation 4?
What is the product of these two slopes?

5) y = 4x

6) y = -1/4x



What is the slope of equation 5? The slope of equation 6?
What is the product of these two slopes?

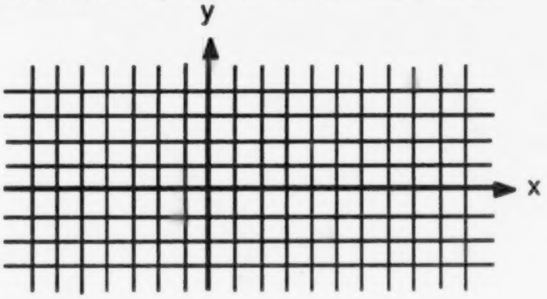
As you looked at these three pairs of lines, what seemed to be the relationship between the slopes of the lines and the resulting graphs?

If the slopes of two linear equations are negative reciprocals, the lines are perpendicular to each other.

B. Erase the graphs from above. Enter the following pairs of equations and answer the questions about their graphs. Erase each pair of graphs before continuing on to the next pair.

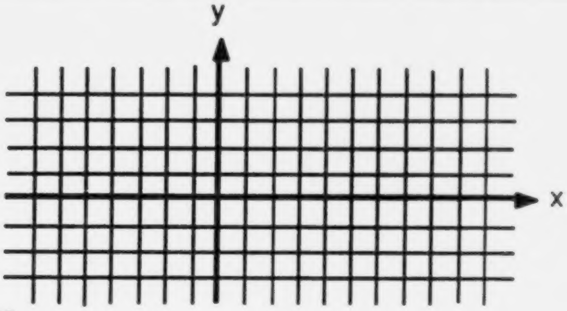
1) y = 2x + 3

2) y = -1/2x - 4



What are the slopes for equations 1 and 2?

Do their graphs appear to be perpendicular?



3) y = -x - 2

4) y = x - 3

What are the slopes for equations 3 and 4?

What is the relation between the graphs of these lines?

C. Without graphing the equations below, tell which pairs will have graphs that are perpendicular.

1) y = 3x + 2

2) y = -7x - 1

3) y = 1/5x - 3

4) y = -1/3x

5) y = 7x + 2

6) y = 1/3x - 4

7) y = -1/7x

8) y = -5x + 1

Teacher Notes for Worksheet on Domains and Ranges

OBJECTIVE

To introduce students to the concepts of the domain and range of a function from a purely visual standpoint.

DESCRIPTION

Any graphics package which graphs functions or relations including lines, parabolas, hyperbolas, square roots, and trigonometric functions will be suitable for use with this worksheet. Student worksheets contain no instructions for use of a particular graphics package, so directions for use must be provided by the teacher. This worksheet can be used for students with access at individual computers or as a worksheet to accompany a lesson given on a computer in front of the classroom.

SPECIAL NOTES

This worksheet takes from 45 to 55 minutes for the student to complete.

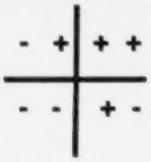
Many students have trouble when first encountering the definitions for domain and range of a function. This worksheet presents a visual description of those concepts on a more intuitive level which may help pave the way for a more formal discussion afterwards.

Students need to be familiar with the ideas of integers and real numbers before using this worksheet.

Although the word "asymptote" is not used in this worksheet, this would be an appropriate time to introduce the concept.

The teacher may want to show the student how to write the domain and range values for this worksheet in set notation as a follow-up for this activity.

Some students may not readily recall the signs of the x and y coordinates in each quadrant. It may be helpful to draw a diagram as shown below for them to refer to.

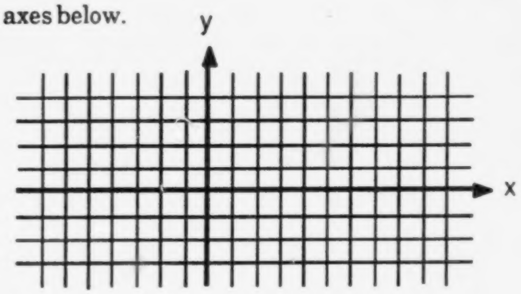


Student Worksheet
Domains and Ranges

This worksheet will introduce you to the ideas of the domain and range of a function. You will start by examining the graphs of a number of different functions in order to find a definition for domain and range.

Type in this equation. Sketch and label its graph on the axes below.

1) $y = 2x + 3$



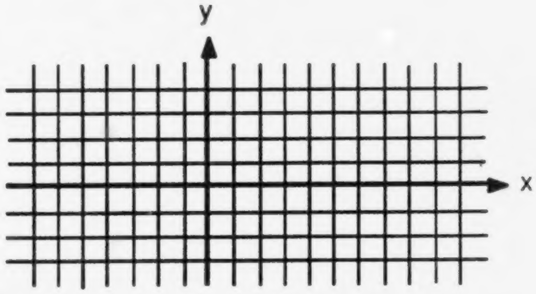
You know that equations of the form y=mx+b are called linear equations and that their graphs are lines.

Notice that the graph of y=2x+3 is a smooth graph with no holes or gaps. It seems to extend from left to right and from top to bottom on the graph without any interruption. Any values substituted for x in the equation will result in a y value.

We say that the domain of the function (all of the values for x that are allowable in the equation) is the set of real numbers. We say that the range (the y values that result after substituting the allowable x values) is the set of real numbers.

Erase the above graph. Type in the following equation. Sketch and label below.

2) $y = x^2$

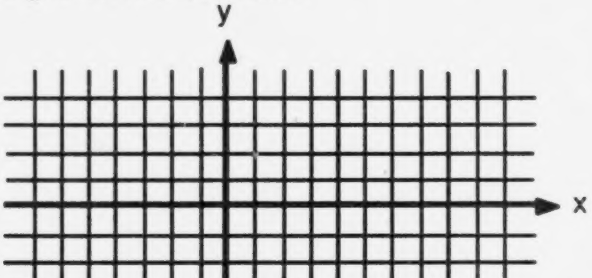


The graph of $y=x^2$ is called a parabola. Notice that the parabola is found in the first and second quadrants of the coordinate plane. Although any value for x can be substituted into the equation, only positive values of y result since any squaring any real number yields a positive answer. The value of y is positive in the first and second quadrants.

The domain (allowable x values) is the set of real numbers. The range (resulting y values) is the set of real numbers greater than or equal to 0.

Erase the screen and type in the following. Sketch and label below.

3) y = 1 / x



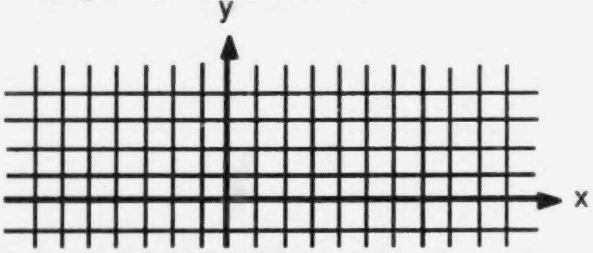
The figure sketched above is known as a hyperbola. As you examine its graph from left, notice that the curve gets closer and closer to the y axis without ever touching it. As you examine the graph from the right, you notice the same thing happening from the opposite side. It seems that the hyperbola has no value when x=0. Now look at the equation y=1/x. What happens if you substitute 0 into that equation?

As you follow the graph downward from the top of the screen, notice that the hyperbola approaches the x axis, but never touches it. As the hyperbola approaches the x axis from quadrant three, the same thing is true. In either case, there is no value of x which will make y=0. Observe the equation y=1/x. Can you think of any value which will divide into one and give 0 for an answer?

The domain of this function is the set of all real numbers except 0. The range of this function is the set of all real numbers except 0.

Erase the above graph. Type in the following. Sketch and label below.

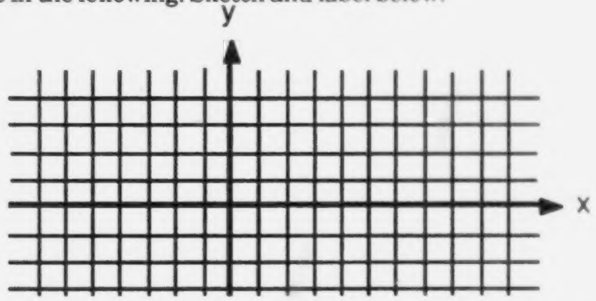
4) y = √x



Notice that the above graph is located entirely in the first quadrant. Taking the square root of a negative number does not give a real number answer. The domain of this function is the set of all real numbers greater than or equal to 0. The range of this function is the set of all real numbers greater than or equal to 0.

Erase the graph from above. Type in the following. Sketch and label below.

5) $y = (x + 2)^2 - 3$



The graph of the function above is a parabola. Notice that this parabola does not have its lowest point at the origin. Where is the lowest point on this parabola located?

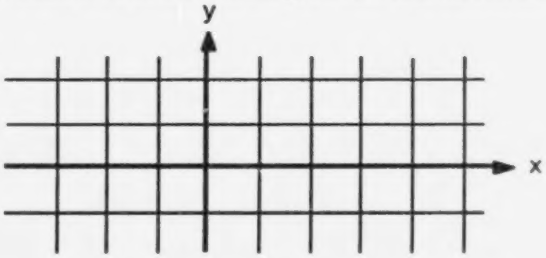
Are there any values of x that this function would not be defined for?

What is the domain of this function?

What are the resulting y values (look at the graph.)

Erase the graph from above and type in the following. Sketch and label it below.

6) $y = \sin(x)$



Even though you may not be familiar with the trigonometric function above, you can still describe its graph. Notice that the graph is very regular and repeats itself.

Do there seem to be any holes or gaps in the graph?

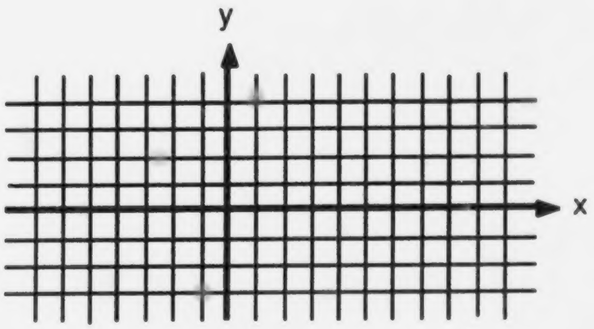
Are there any x values that are not allowable?

What is the domain of this function?

Notice that the y values seem to stay between two lines --- y=1 and y=-1. What would the range of this function be?

Erase the graph from above and type in this final equation. Sketch and label it below.

7) $y = \dfrac{x+2}{x-1}$



Are their any gaps or holes in this graph?

Where do they occur?

Looking at the equation of this graph, which values for x are not allowable?
How does that compare to the graph?

What is the domain of this function?

What is the range of this function?

For the following equations, give their general shape, domain, and range. Check your answers afterwards by graphing them with the computer.

1) y = 3x - 7
2) y = x2 - 3
3) y = /x + 2
4) Y = -2x + 1
5) y = - x2
6) y = /x - 3

Teacher Notes for Worksheet on Polynomial Graphs

OBJECTIVES

To present general shapes of polynomial equations of the form $y = x^n$ where n is an even or odd integer greater than one and of the form $y = x^{(a/b)}$ where a and b are even or odd integers greater than one.

DESCRIPTION

A graphics package such as "Chalkboard Graphics Tool Box I" by Scharf Systems Inc. works well for this activity since it has special odd number roots built in. This is a feature not available in most other graphics packages. Students can work alone, in small groups, or together with the class in a lecture format with a single computer to discover relationships between the values of the exponents in polynomial equations and the resulting graphs.

SPECIAL NOTES

This worksheet takes approximately 45 minutes to complete.

Most graphics packages only plot the positive portion of an odd number root graph. This type of software would be very misleading for use in this activity.

You may want to further the discussion of polynomial equations by showing inverse functions on the same axes. For instance, you could put both $y = x^3$ and $y = x^{(1/3)}$ on the screen at the same time and observe their reflection about the line $y = x$. This line could be graphed, too, to make the reflection clearer.

A natural extension of this lesson would be to consider polynomial equations of the form $y = x^n$ where n is a negative number.

Student Worksheet
Polynomial Graphs

This worksheet will allow you to explore the graphs of polynomial equations in the form of $y=x^n$.

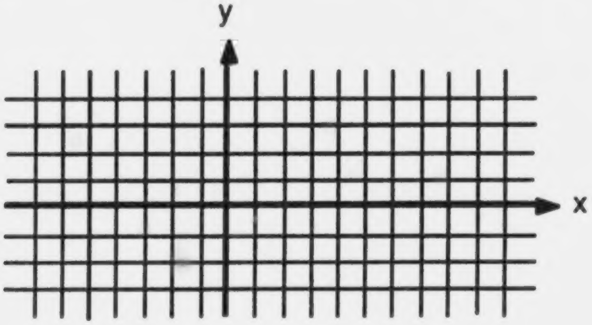
A. First you will examine graphs of $y=x^n$ where n is an integer greater than one. Type the following equations into the computer. Sketch and label their graphs on the axes below.

1) $y = x^2$

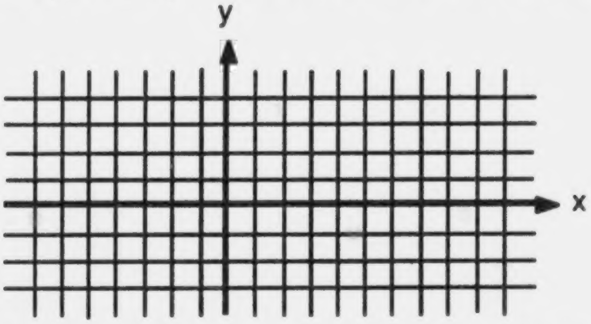
2) $y = x^3$

3) $y = x^4$

4) $y = x^5$



Can you predict what the graphs of $y = x^6$ and $y = x^7$ should look like? Sketch them on the axes below.



Now erase the previous graphs and type in the pair from above to test your prediction.

Notice that when n is an even integer exponent, the graph of $y = x^n$ is in quadrants one and two. Why?

When n is an odd integer exponent, the graph of $y = x^n$ is in quadrants one and three. Why?

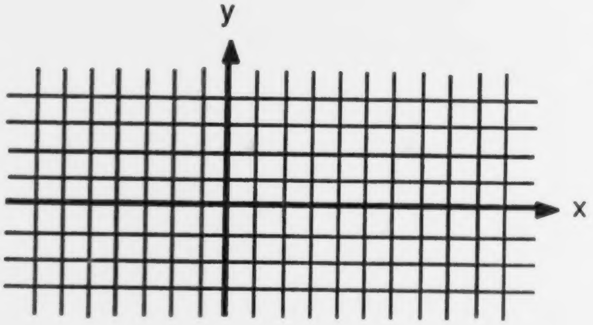
B. Now consider equations of the form $y = x^{1/n}$ where n is an integer greater than one. Erase all graphs and type in the following. Sketch and label on the axes provided below.

1) $y = x^{(1/2)}$

2) $y = x^{(1/3)}$

3) $y = x^{(1/4)}$

4) $y = x^{(1/5)}$



Did you recognize $y = x^{(1/2)}$ as another way of writing $y = \sqrt{x}$? Rewrite equations 2-4 above in a similar form.

Notice that all of these equations have part or all of their graphs in the first quadrant. The equations that also have part of their graphs in the third quadrant have odd integer denominators.

Even number roots of positive numbers are positive. Even number roots of negative numbers are imaginary and cannot be graphed on the real coordinate plane.

Odd number roots of positive numbers are positive. Odd number roots of negative numbers are negative and will be graphed in the third quadrant of the coordinate plane.

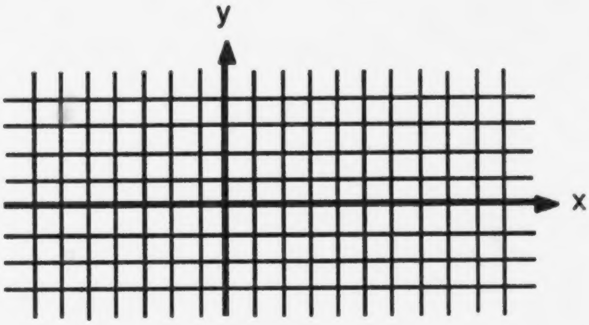
What quadrants would the graphs of the following equations be found in?

1) $y = x^{(1/11)}$ 2) $y = x^{(1/12)}$

3) $y = x^{(1/7)}$ 4) $y = x^{(1/70)}$

C. Can you predict what equations of the form $y = x^{(a/b)}$ will look like where a and b are integers greater than one and a/b is less than one? Erase all graphs from above and type in the following. Sketch and label their graphs on the axes below.

1) $y = x^{(2/3)}$
2) $y = x^{(4/5)}$
3) $y = x^{(3/5)}$
4) $y = x^{(5/11)}$
5) $y = x^{(3/7)}$



Did you notice that graphs 1 and 2 and graphs 3, 4, and 5 had the same general shape?

Why were graphs 1 and 2 located in the first and second quadrants?

Why were graphs 3, 4, and 5 located in the first and third quadrants?

What general statements can you form about the values of a and b in an equation of the form $y = x^{(a/b)}$ and the resulting graphs?

Session 5E
Elementary

SOFTWARE BY OBJECTIVE - ADDITION

Grade Objective	Software
Grade K Given a set of objects, the learner will construct a set that has one more object than the given set.	Early Childhood Learning Program *Educational Activities
Grade 1 Given an addition fact in horizontal or vertical format, sums through 12 the learner will identify the sum.	Alien Addition/Alligator Mix Developmental Learning Materials (DLM) Spaceship Minnesota Education Computing Consortium (MECC)
Grade 2 Given an addition fact in horizontal or vertical format, sums through 18, the learner will identify the sum.	Math Blaster *Davidson and Associates Mathsheet (teacher tool) *Houghton Mifflin
Grade 3 Given (two) two and three digit addends with no regrouping, the learner will identify the sum.	The EA Mathematics Worksheet Generator (teacher tool) *Educational Activities Monster Math IBM *IBM Software
Grade 4 Given an addition problem with two or more regroupings, the learner will identify the sum.	Mathsheet (teacher tool) *Houghton Mifflin The Game Show (teacher modifiable) *Advanced ideas, Inc. Create-Lessons (teacher modifiable) *Hartley Courseware, Inc.
Grade 5 Given an addition problem with three or more four-digit addends, with and without regrouping, the learner will identify the sum.	Mathsheet (teacher tool) *Houghton Mifflin The Game Show (teacher modifiable) *Advanced Ideas, Inc. Create-Lessons (teacher modifiable) *Hartley Courseware, Inc. The EA Mathematics Worksheet Generator (teacher tool) *Educational Activities
Grade 6 Given two addends 0-1000, the learner will estimate the sum by rounding.	Create-Lessons (teacher modifiable) *Hartley Courseware, Inc. The EA Mathematics Worksheet Generator (teacher tool) *Educational Activities

Sample Lesson Plan

Lesson Plan: Math Detectives

Grade Level: 4-6

Objectives: Students will enhance their problem-solving skills by solving a mystery simulation. (Identify specific textbook exercises to math software objectives.)

Subject: Math--problem solving

Materials Required: The Mystery of the Hotel Victoria (McGraw-Hill), Apple IIe or IIc, copies of worksheets.

Preparation: Before working with the program, students can practice problem-solving skills by completing preliminary worksheets provided with the courseware. These worksheets help students solve problems presented by Blabbers (those who provide too much information) and by Quiet Types (those who provide too little information). Students learn to identify useful data and record it on their Notepads just as they must do while working through the simulation. Explain to students that they have been hired as detectives to help solve the mystery of the Hotel Victoria.

Activity: After students have practiced problem-solving skills, they can begin the simulation. They can move through the hotel where they will meet a number of characters who require that students solve a series of problems. There are six minor mysteries that students must solve before they can resolve the Big Mystery. The lesson can be repeated until students have met all the characters in the hotel and solved all the problems they present. Students can either share information or work independently to solve the mysteries.

Evaluation: Students who successfully solve the six minor mysteries and the Big Mystery have completed the assignment. Refer to the program's individual student records for specific results.

Follow-up: The courseware includes a number of supplementary worksheets to reinforce a variety of problem-solving strategies practiced during the simulation.

Session 5M
Numbers

Prime Factorization

Objective

To determine relationships between numbers and their factors. To determine what numbers have what factors.

Description

The student types in any number after the question mark prompt. The computer then writes the number as a product of its prime factors. The student will examine which numbers always have 2 as a factor, which numbers always have three as a factor, which numbers always have both 2 and 3 as factors. They will also examine which numbers have only the number itself and 1 as factors and those numbers which have other prime factors. Finally, this program can be used to help students discover the various divisibility rules (for 2,3,4,5,6,8,and 9)

Procedure

Advanced Organizer:

The purpose of this session is to demonstrate number concepts on the computer. The calculating power of the computer will be used to demonstrate prime factorization, equivalent fractions and decimals, and percent.

Load the program "Computing Factors". From the question mark prompt type in any whole number bigger than 1. In this case we will type 6. What do you get:

6=

Type 42. What do you get:

Type 14. What do you get:

Type 10. What do you get?

Type 30. What do you get?

What factor do they all have in common?

Do all numbers have 2 in common?

Try 15. What do you get?

Try 17. What do you get?

Predict what type of numbers always have 2 as a factor. What do they have in common in terms of appearance (hint: look at the values in the ones place)?

Check your hypothesis by checking several with the program.

Find some numbers which have 3 as a factor. Write them all down. Sum the digits of each number. What do you get? (if you get more than a 1 digit number, sum the digits in the answer. Keep doing this until you get a 1 digit answer). What do all of the answers have in common?

Select any number where the sum of the digits is 3,6,or 9. Put it into the program. Does it have 3 as a factor?

Hypothesize what type of number always has 3 as a factor?

Find a number (using a calculator or by hand) which has 6 as a factor. (For example, multiply 6 by some random whole number on a calculator or by hand.) Put the number into the program. Does it also have 3 as a factor? Does it also have 2 as a factor?

If a number always has 6 as a factor does it always have 2 as a factor? Does it always have 3 as a factor? Why?

If a number has 2 and 3 as factors, is 6 always a factor too? Check by using the program to find a number which has both 2 and 3 as factors. Using a calculator (or by hand) determine by dividing if 6 is also a factor.

If a number has 2 and 4 as factors, does it necessarily have 8 as a factor too?

Notice that some numbers have many factors and others have only the number itself and 1. (1 is not printed. It is understood to be a factor of every number) Any number which has only itself and 1 as factors is called a PRIME number. Numbers having additional factors are called COMPOSITE numbers. Use the program to determine all of the prime numbers between 2 and 50. Write them down.

Are any of them divisible by 2? If so which ones?

Are most prime numbers odd or even?

If John tells you that 76 is a prime number, can you believe him? If so why? If not, why not?

Are any prime numbers divisible by 3? If so which ones?

If John tells you that 87 is prime, can you believe him? If so why? If not, why not?

If a number has 9 as a factor does it automatically have 3 as a factor? How many 3's does it have as factors?

Find 5 numbers less than 100 which have 9 as a factor (Use the program to check to see if it has two 3's as factors). Sum the digits of each number. What do you notice?

Does this work for numbers bigger than 100 also?

Use the program to check whether a number bigger than 100 is where the sum of the digits is 9 has two 3's as factors.

Equivalent Fraction Table

Objective

To determine the common denominator of a pair of fractions using "families" of equivalent fractions.

Description

This spreadsheet template allows the user to input a first fraction. The program then displays 50 fractions equivalent to this fraction. The user then inputs a second fraction below the first. Fifty equivalent fractions are displayed. The user can then examine the two lists of 50 fractions each to find a pair with the same denominator.

Procedure

Load the template "Fractions1". Put the fraction 1/5 into the template. Put the fraction 2/3 below. Find the first pair of fractions with the same denominator.

1/5 =

2/3 =

Find another "common" denominator.

1/5 = ?/30

2/3 = ?/30

Predict what the next denominator is?

Check. What is it?

The set of common denominators is 15, 30, 45,... What do you think the next denominator is? What do all of the denominators have in common?

What is the lowest common denominator? Put the information into the table.

Now enter 1/7 and 3/4 into the spreadsheet. What is the first (lowest) common denominator? Find two more common denominators.

Is there a greatest common denominator?

Put the information into the table.

Using the data in the table, predict what the lowest common denominator of 2/5 and 3/8 is.

Were you right?

Using the data in the table, predict what the lowest common denominator of 1/6 and 2/9 is.

Were you right? Is 54 a common denominator?

Find the GCF of 6 and 9. Find the GCF of all of the denominator pairs. What is the relationship between the product, the GCF, and the lowest common denominator?

Check out your idea by trying any two fractions with denominators less than 50.

What is the rule for finding the lowest common denominator?

Fractions and Their Decimal Equivalents

Objective

To observe that every fraction can be written as a repeating decimal and to observe various patterns of repetition.

Description

The student will use the program to put in any fraction and see what its decimal equivalent is. The sound feature will allow the student to hear the repetition pattern as well as see it.

Procedure

Load the program "Fractions to Decimals". Using a calculator (or by hand if one is unavailable) convert each fraction to a decimal:

1/4=

3/5=

2/7=

3/8=

5/9=

How are they the same?

How are they different?

Use the computer program and put each fraction in one at a time. Did you get the same result as before?

Place any fraction into the program. Write the equivalent decimal.

/ =

Predict what fractions have decimals which terminate in 0's and which do not. Check your prediction by using the program.

Now place the following fractions into the computer one at a time and write down the decimal equivalent.

1/9=

2/9=

3/9=

4/9=

5/9=

6/9=

Predict what 7/9 = Check your prediction on the computer. Were you right?

Predict what 8/9 = Check your prediction on the computer. Were you right?

If the pattern holds what would 9/9 = Check your prediction. Were your right? But since 9/9 = 1 then what do you know about:

.999999999999999999.... ? 1

Put in the following:

45/99 =

38/99 =

What do your predict 56/99 = Check your answer. Were you right?

Predict what 5/99 = Check your answer. Were you right?

Predict what 354/999 = Check your answer. Were your right?

State in your own words how to determine the equivalent decimal of any fraction with only 9's in the denominator. Check your answer by testing it out with any fraction with only 9's in the denominator.

Use your definition to predict what 123/99 = (Notice that the program only prints the decimal part, not the whole number part and ignores negative signs in the numerator) Check your answer. Were you right? If not, can you modify your definition? Do so here:

Be sure to check your definition with all possible cases.

Put in the following into the program:

`1/11 =

2/11 =

3/11 =

Predict what 4/11 = Check your answer. Were you right?

Predict what 5/11 = Check your answer. Were you right?

Predict what 45/121 = Check your answer. Were you right?

Predict what 45/111 = Check your answer. Were you right?

Predict what 46/111 = Check your answer. Were you right?

Determine a pattern by placing in fractions where the denominator is 111 and the numerator is < 111.

Put your predictions and actual results in the table. When you can predict what will happen, state the rule in your own words.

Predict what will happen with 465/1111 =   Check your answer. Were you right?

Use the program to find any other patterns which may exist.

Put in any 10 fractions one at a time. Count the number of digits in the period of repetition (number of digits in a repeating block). Insert the information into the table. What is the pattern? Will the period ever exceed the denominator?

Will it ever equal the denominator?

Can you predict when the period of repetition will exactly equal 1 less than the denominator?

Demonstrating Percents

Objective

To see the relationship between part, whole, percent and a pictorial representation of percent.

Description

The student inputs the part corresponding to the amount s/he wishes to find the percent of. S/he is then prompted for the amount corresponding to the whole or entire amount (i.e. the base). The program draws the unit whole on the top, subdivides it into 10 equal pieces (corresponding to 10% each) and then proceeds to color in the amount equal to the percent. The actual number percent is given on the bottom of the screen. Using this program the student can visualize the various percents and see, for example that 50% is twice as large as 25%. Also the student can see what it means to have more than 100% (Up to 700% is allowed in the program) and that more than 100% corresponds to a result larger than the unit whole.

Procedure

Load the program "Visualizing Percents". When it prompts for the part put in 6. When it asks for the whole put in 10. The problem this could correspond to is a John takes a 10 question quiz and gets 6 right. What is is percent correct?

Draw the bars the way you see them on the top of the screen.

If Susan takes a 20 question quiz and gets 12 right is her percent the same? Draw the bars the way you see them on the top of the screen.

If Alan takes a quiz and gets 3 right out of 4 who got the better score, Alan or John?

John's percent (from above) :

Alan's percent:

Which is a higher score : a) 5 right out of 7 or

b) 6 right out of 9

To solve put 5 in for the part and 7 for the whole. Copy the results.

Then put 6 in for the part and 9 for the whole. Copy results.

Which resulted in a longer bar at the top (higher percent?)

A local community center is offering an aerobics class. The class fits 70 students comfortably. 60 students sign-up. How full is the class in terms of percent?

The same class the following month has 90 students sign-up. How full is the class now in terms of percent?

The next month 300 students sign-up. How many sections of the class will they have to offer and how full will each one be if every class is full but the last one?

Least Common Multiples and Greatest Common Factors

Objective

To determine what pairs of numbers have 1 as their greatest common factor (GCF), what pairs of numbers have a number larger than 1 as their GCF, what pairs of numbers as their GCF one of the numbers, what pairs of numbers have as their least common multiple (LCM) the product of the pair, what pairs of numbers have as their LCM a number smaller than their product, and what pairs of numbers have one of the numbers as their LCM. Also, to determine what the relations hip between the GCF, LCM and the product of the two numbers is.

Description

The student places two numbers separated by a comma into the program. The program then prints the prime factorization of each number, the GCF of the pair, the LCM of the pair, and the product of the pair.

Procedure

Load the program "LCM". Put in the numbers 6 and 8 as 6,8. What is the prime factorization of 6

What is the prime factorization of 8

What is GCF(6,8)=

What is LCM(6,8)=

What is 6 * 8 =

Use the data sheet. Select any two numbers collect the data and enter into the sheet. Repeat for at least 5 pairs of numbers. What do the pairs which have 1 as the GCF have in common?

What do the pairs which have something other than 1 as the GCF have in common?

What pairs of numbers have one of the numbers as its GCF?

What pairs of numbers have the product as its LCM?

What pairs of numbers have something less than the product as its LCM (Hint: look at the GCF)

If the GCF = 1 what do you know about the LCM?

Is the converse true: If the LCM = the product of the two numbers what is the GCF?

Is there a relationship between the GCF, LCM and the product? What is this relationship?

Does the relationship always work?

Session 5S Algebra and Geometry

ACTIVITY ONE
Lab

OBJECTIVE

To illustrate to participants a method of solving maximum-minimum problems with arithmetic and algebraic methods, using the computer, BEFORE students have a course in the Calculus.

DESCRIPTION

A maximum-minimum problem will be presented and analyzed. An algebraic function representing the quantity to be maximized or minimized will be determined. Using a graphics program such as "Chalkboard Graphics Tool Box I" by Scharf Systems, the function will be graphed and the maximum or minimum point being sought will be approximated. Then, through the use of the TABLE OF VALUES option, the solution will be found to any degree of accuracy desired.

PROCEDURE

In this session, participants will explore a problem usually reserved for the Calculus - yet will only use Algebra and the computer. Experimentation with the "Geometric Supposer: Triangles" will also be accomplished.

1. The problem -

A shop teacher was going to have his students make open boxes in which to store small nails, screws, nuts, bolts and washers. He gave each student a 9" by 12" sheet of heavy paper. He instructed them to cut small squares from each corner and fold up the sides to form open boxes. Further, he told the students, extra credit would be given to that student who constructs the box of largest volume and is able to compute the volume.

2. The solution :

   a. First draw a diagram to represent the sheet of heavy paper with equal squares cut from each corner.

   b. Label the diagram, calling a side of each of the equal squares to be cut from the corners, "X". Now form the function which represents the volume of the open box - the quantity you wish to maximize.

   c. Consider any restrictions on the value of the independent variable, "X", in the function.

   d. Boot the disk for "Chalkboard Graphics Tool Box I" and graph the function using FUNCTION ANALYZER. Set the bounds on "X" (the domain) between 0 and 4.5 with an increment of .5 and the bounds on "Y" (the range) between 0 and 50 with an increment of 10. Select plot speed option #3, "Very Fast/Sketch".

   e. After the graph is complete, you'll find that the maximum value cannot be read since it lies above the screen's "window". Select option # 2 to "Change Domain and Range". Now set the domain between 0 and 4.5 with increment of .5 and the range between 50 and 100 with increment of 10.

   f. You will now observe that the maximum volume occurs when a side of one of the equal squares to be cut from the corner of the rectangular sheet of heavy paper is between 1.5 and 2 inches and the maximum volume is a bit over 80 cubic inches.

   g. Use the "table of values" option to determine the size of side of each of the equal squares to the nearest hundredth of an inch and then read the corresponding maximum volume.

3. More problems for you to do on your own -

   a. An open box is to have a capacity of 36,000 cubic inches. If the box must be twice as long as it is wide, find the dimensions that would require the least amount of building material.

   b. Find two numbers whose sum is 36, and further, the product of the first and the cube of the second is a maxim.

   c. At 7:00 a.m., a ship was 60 miles due east of a second ship. If the first ship sailed west at 20 miles per hour and the second ship sailed south-east at 30 miles per hour, when were the two ships closest together?

ACTIVITY TWO

Lab

OBJECTIVE

To practice with the "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst to gain insight into the many features it contains.

DESCRIPTION

The participant will be given many tasks from geometry to perform using the "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst to help in the process of becoming familiar with its capabilities. Please note that much of what follows in this activity would be recommended when teaching informal geometry before the formal high school course or early in the school year in that formal course.

PROCEDURE

1. Draw an acute triangle. Use "S" to change the scale if it's too small. Draw in the bisectors of the three angles of the triangle. Construct the inscribed circle.
2. Draw an acute triangle. Construct the three altitudes of the triangle. Observe the place where they meet. Now do the same for a right triangle and finally for an obtuse triangle. What can you conclude about the point where the three altitudes of a triangle meet?
3. Draw a triangle. Construct the circle which circumscribes the triangle.
4. Draw a triangle. Construct its three medians.
5. At this point you must have observed that the three altitudes, the three medians and the three angle bisectors in a triangle are concurrent. Does it appear as if the points of intersection of the altitudes, medians and angle bisectors are the same point in any one triangle? Will they ever be the same point? Experiment to determine the answer.

ACTIVITY THREE

Lab

OBJECTIVE

To illustrate the use of the "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst to develop some intuitive notions about the trigonometry of right triangles.

DESCRIPTION

The "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst will be used to have students experiment by measuring the sides of certain right triangles, forming the ratios of the sides and discovering various constants among the ratios.

PROCEDURE

1. Boot up the "GEOMETRIC SUPPOSER: TRIANGLES" by Sunburst. When the screen for drawing comes up, touch "N" for a new triangle. Use "1" to request a random right triangle. The right triangle will be labeled ABC with right angle at vert ex A.

2. Use the options offered by the software to measure the length of segment AC divided by the length of segment BC.

3. Next measure the length of segment AC divided by the length of segment AB.

4. Finally find the length of segment AB divided by the length of segment BC.

5. Next, return to the original menu by pressing <ESC> a sufficient number of times. You can't press it too much, for once you get to the original menu, additional pressing will continue to produce the same menu - hence you go no further.

6. Now press "S" for a scale change in the triangle. The angles will remain the same, but the sides have been changed in length. Thus, the first right triangle and this present one are similar.

7. Measure the same sides and calculate the same three ratios as was previously donein the first triangle. Of course you find that the ratios of corresponding sides remains the same even though the lengths of the sides have changed. A class discussion regardingthis observation is appropriate at this time.

Possible Conclusions and Extensions of the Discussion

- Since the triangles are similar, the corresponding sides are proportional.
- In right triangles, these ratios have particular names which we call trigonometric ratios of the acute angles.
- For students of high ability, you might wish to suggest a project of constructing a system similar to that of the trigonometry of right triangles - instead based upon triangles with 50, 60, 70 degree angles.

Session 6
Ethics and Technology*

Can the computer help us develop ethical behavior among students?

Ethical conduct is a skill to be learned, a way of behaving in social situations, a way of thinking about oneself, a life-long process of development. Children go through stages in acquiring pro-social behavior and reasoning about justice. Real autonomy of decision is the only way to develop judgement. All of us continually re-evaluate our conduct as new situations are presented to us. In fact, new situations can become catalysts to enable us to rethink the way we perceive things and how we react. The computer has sprung full-grown into our lives, bringing uncertainties and ambiguities, making us analyze our reactions and consider problems from a different point of view. What is right or wrong in this new situation? Are there new laws we should know about? Should new laws be written? Perhaps the computer can become a vehicle for discussion of conscience and crime in our society.

1. Serious Problems

The newspapers are full of instances where crime and computers have been combined. Child molesters are using computers to locate individuals with similar interests, to trade child pornography collections, and to provide information on potential victims. Data bank break-ins are common. The theft of chips and the illegal shipment of equipment and technology across state and national borders is such a problem in Silicon Valley that an association of law enforcement agencies (the District Attorney's Technology Transfer Association), has been created to combat it. Corporate officers have been accused of stealing trade secrets in violation of the Racketeer Influenced and Corrupt Organizations Act (RICO). Recently two men who tried to reproduce copyrighted software owned by the California State Lottery were arrested and charged with conspiracy to commit grand theft. Conservatively estimated, 20% of personal computer programs in use today are illegal copies. Clearly, there are problems.

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* This is a copy of a paper prepared by Kay Gilliland and Mattye Pollard for the *Sixth Annual Conference on Microcomputers in Education*, Arizona, March 12-14, 1986.

2. New Opportunities

The advent of the computer brings opportunities for new applications of ethical standards. It is taken for granted that driver education classes include discussion of the etiquette of the road, criminal and civil law, and moral responsibility. To what extent are we prepared to discuss these issues as they relate to computers? The National Center for Computer Crime Data is gathering statistics on the teaching of ethics, conscience, and the law in the schools. It may be that the report will show that these issues are not being widely addressed. Technology has placed these powerful devices in the hands of children as well as adults, and they are being left to flounder without direction. These computer users may inadvertently violate the rights of others, or may do so knowingly, but without having fully considered the impact of their actions.

3. Strict Requirements

To what extent will our young computer whiz kids need to be prepared to adhere to ethical standards now and in the future? Right now, a 16-year old high school student from Tallahassee faces felony charges stemming from his use of a home computer to gain access to the Florida Department of Education's computer. Computer malfunctions of various sorts can have grave consequences. In the near future courts may find computer professionals subject to malpractice liability. As our society begins to depend as much on computer programmers as on doctors, courts may hold computer professionals accountable to standards equivalent to those of the medical profession.

4. Complex and Difficult Issues

Computer ethics issues are complex and difficult. They span civil, public, labor, criminal, and international law. They cover issues of equal access, information access, and economics.

4.1. Electronic Bulletin Boards

To what extent are those who run an electronic bulletin board responsible for its contents? Must the bulletin board operator assume the burden of stewardship? Is this form of communication to be considered like a publication? How is it to be regulated?

4.2. Data Files

Is the operator of a system of files, such as credit information files, responsible for the accuracy and privacy of those files? Do we want to allow people to make up lists of individuals who have filed medical malpractice suits and sell them to doctors? Do we want lists of individuals who have sued under rent control laws made available to landlords? Who owns and is responsible for this new kind of information storage which is being transferred without the usual physical form, which is usable by people in widely separated locations, and which betrays no physical evidence when used or altered?

4.3. Copyright Laws

How are copyright laws to be interpreted? The legal right to protect the expression of ideas through copyright is so important that it is recognized in the United States Constitution. In Article I, Section 8, Clause 8, the Constitution secures to authors and inventors the exclusive right to use their creative products for a limited time, "to promote the progress of science and the useful arts." The intellectual property embodied in a computer program can be very expensive to produce. A single line of program could average ten to thirty dollars, and the cost to prepare a typical business program ranges from five to fifteen million dollars. Yet it can be copied in minutes.

The first program was accepted for copyright as early as 1964 even though the Copyright Act of 1909 did not anticipate computer technology. The Act was amended in 1976 (by the Copyright Revision Act which took effect in 1978), further amended two years later, and amended once again on November 8, 1984.

The holder of a copyrighted original expression has five exclusive rights, any one of which may be sold or transferred independently: (1) to reproduce the work in copies, excluding fair use and an archival copy; (2) to prepare derivative works based on the work; (3) to distribute copies for sale, lease, or even for free; (4) to perform the work publicly; and (5) to display the work publicly.

The program owner may make a backup copy because of the special risk of destruction of a disk by either electrical or mechanical means, but this archival copy must not be used as a second copy nor expected to replace a copy worn out through normal use. The owner may add features to the program and to the documentation, but these are not considered "original expressions" and may not be sold. The program owner may adapt the program from one language to another so long as: (1) the adaptation is made for the purpose of utilizing the program in the machine or machines the purchaser owns, and (2) the adaptation is done only in order for the owner to use the program. A BASIC program needed for both an Atari and an Apple could be adapted so that it would also work on the other machine, provided that it is not vended or made to compete with sale of the original.

The adaptation concept may help solve the questions of networking and multiple loading. There is nothing specific in the law about using one disk drive to enter a program into several machines (networking) nor about using one copy of a program for several machines (multiple loading). In some cases, disks are not designed so they can be networked or multiple-loaded, but in many cases they are. These practices appear to be legal so long as the copies are used in the machines of the owner of the program. However, many vendors claim that company policy forbids using their programs for networking or multiple loading. Christopher Williams surveyed 39 major software publishers for *Electronic Learning*. Forty-nine percent do not permit use of their programs on a classroom networking system, and those that do require that the educators check with the publishers first. Of the 20 companies who had software capable of being multiple-loaded, eight did not give permission to do so and one refused comment. A solution which has been offered is the site license – permission to make and use multiple copies for one site — but 27 of the 39 companies indicated that site licenses are not available from them. Many problems have yet to be resolved. At the request of the Judiciary Committee of the Congress, the Office of Technology Assessment is conducting a study on *Intellectual Property Rights in an Age of Electronics and Information*. The report is expected later this year.

4.4. Equal Access

Equal access is particularly important. We are in real danger of developing a two tier society: the information-rich versus the information-poor, those with the ability to access information and those who cannot. We must make sure that we protect the rights to open communication and that all segments of our population share in the benefits of computer use. Corporations can take it as their responsibility to portray women and minority persons as computer users in their advertising. They can examine their hiring procedures to make sure equal access is provided. The schools must take an active part in ensuring equal access by enabling children from all walks of life to have the opportunity, the encouragement, and the motivation to learn about and use computers. Staff development for teachers is essential. One program providing teacher inservice is EQUALS in Computer Technology. It promotes awareness of the need for women and minorities to keep their options open, provides classroom strategies and materials which can be used for this purpose, and helps teachers consider classroom logistics and practices which will encourage all students to become involved with this powerful new technology.

5. Support Needed

The individual teacher needs support to address the many ramifications of technology ethics. The public must make it clear that discussion of the etiquette of computer use, criminal and civil law, and moral responsibility should be an important part of the curriculum in our schools. Unfortunately, the opposite has been happening. The Hatch Amendment, Section 439 of the General Education Provisions Act, was passed in 1978 and regulations were issued in 1984. The Hatch Amendment, part of the Protection of Pupil Rights Law, has been used to try to discourage open discussion of ethical questions in the schools. For example, violation of the law was claimed in West Palm Beach, Florida, because of the use of a seventh grade health textbook which allegedly teaches "values clarification" and in West Alexander, Pennsylvania where objections were made to "classroom discussion of death and dying and the promotion of critical thinking in the elementary school curriculum." Such cases make it difficult to encourage teachers to help their students think through their actions and make wise choices.

6. Staff Development Needed

Thirty-eight or more states have passed laws concerning computer crime. Some people criticize this rush to judgement – lots of statutes, lots of law. Sloppy drafting can have unintended consequences. Are we really sufficiently knowledgeable to judge? Will our children be? Teachers must be given the opportunity for staff development – time to consider the issues related to technology, to review the new computer laws, and to develop appropriate, compelling, worthwhile curriculum. Abbe Moshowitz of the Department of Political Science, City College of New York, points out that weak ethical environments are a major cause of computer crime. Over-policing can be a contributing factor by encouraging weak ethical environments. This can be true in the classroom as well as in the broader environment. Students who have no opportunity to make decisions on their own may not develop sufficient inner strength and may become dependent on outside discipline to maintain their behavior. Students who have not considered and discussed the ethical questions associated with computers may fall prey to those who would mislead them.

7. Role Playing in the Classroom

Role playing in the classroom may give students the opportunities they need to think through hypothetical situations, see various points of view, and clarify their thinking regarding important issues. EQUALS has used vignettes with over six hundred teachers and many of them, in turn, have used the scenarios with their classes. The educators who took part in the activity expressed surprise at the complexity of the issues and agreed that the subject should be further explored by students and adults. EQUALS has found it important to help participants delineate each role clearly and to distinguish between the times when they are role playing and when they are expressing their own opinions. Participants are given name tags corresponding to the characters they are playing and are asked to begin by introducing themselves to the small group by using some of the information they were given about the situation. Then they are to discuss the situation by representing the point of view of that person. When the role playing has been completed, participants are asked to remove the name tags, be themselves, and discuss the situation as they perceive it themselves. What would they really do in such a situation? How could they really solve the problem? When all the groups have finished, the group reconvenes as a whole. One person from each group describes the group's scenario and reports what happened when they were playing the characters and how they resolved the situation later when they were expressing their own opinions. Teachers who used the situations in classrooms reported that it was often easier to give the groups all the same situation and even take a second day to switch roles with the same situation enabling students to think very carefully and come up with creative solutions.

DIRECTIONS:

Divide the class into groups of four students. Explain that each group will discuss some situations which might arise in relation to technology in our society. Outline the following steps to the class:

1. Each group of four students will have one situation to discuss. Each student is given a name tag and a slip of paper describing one character's view of the problem.

2. Each student puts on the name tag and introduces the character he or she represents.

3. Students discuss the situation from the points of view of the persons represented.

4. Students remove the name tags and discuss the same problem from their own points of view, attempting to find a creative solution.

5. Class reconvenes and a report is given by each group.

There are four situations: Word Processor, Tepid Pebbles, Cracking The Computer, and Gatito. Each situation has four characters. Cut apart the information for the four characters and the name tags. For four new situations, see the August/September 1984 issue of *The Computing Teacher* or the November 1984 issue of the *Mathematics Teacher*. Teachers have reported that students also like to make up their own scenarios. It is important for teachers to ensure that students play a great variety of roles and always end by discussing the situations from their own points of view and reporting on their decisions to the class.

SITUATION 1: WORD PROCESSOR (Narindar)

You are Narindar, a high school student with a new computer. Your friend Erika has come over with disks given to her by her older sister. Together with two friends, Tom and Rafer, you have discovered that the disks have essays written on the word processor for English assignments. Tom says he wants to change them a bit, print them out, and turn them in to his English class. You think this is not honest and besides, Tom would be better off if he really practiced his writing – some day he might need to be able to write well.

Discuss this issue with Tom, Erika, and Rafer.

SITUATION 1: WORD PROCESSOR (Tom)

You are Tom, a high school football star. Your father was a star before you and you feel a lot of pressure to carry on the family tradition. The school recently instituted grade requirements for athletes and you are afraid failure in English will keep you from playing. You have put great effort into becoming an outstanding athlete and you feel the new rule is unfair. Your friend's word processing disks look like life savers to you.

Discuss this issue with Erika, Narindar, and Rafer.

NARINDAR Word Processor	**TOM** Word Processor

SITUATION 1: WORD PROCESSOR (Erika)

You are Erika, a high school student meeting with Narindar, Tom, and Rafer. You know how hard Tom has worked to become a fine athlete and you sympathize with his problems in English. After all, you think, why should a football star have to spend time learning to write? You agree with Tom that he should print out the essays, revise them a little, and turn them in. You have used a word processor a lot and know that small changes are easy to make.

Discuss this issue with Tom, Narindar, and Rafer

SITUATION 1: WORD PROCESSOR (Rafer)

You are Rafer, a high school student and friend of Erika, Tom and Narindar. You have looked at the essays and heard that Tom wants to use the word processor to change them a bit and hand them in for his class. Your father is an English teacher and you are convinced that writing is an important skill, even for football players. You don't want to see your friend cheat by handing in the English essays, no matter how easy it would be to do so.

Discuss this issue with Tom, Erika, and Narindar.

ERIKA
Word Processor

RAFER
Word Processor

SITUATION 2: TEPID PEBBLES (Robert)

You are Robert, a high school student who has a new video cassette recorder. You have rented a copy of *Tepid Pebbles* and invited your friends over to see it. Barbara has brought her own VCR because she wants to copy the program. You really want to be a good friend, but you are worried about the copying since the label says not to copy and you feel responsible because you are the person who rented the film.

Discuss this issue with Barbara, Umberto, and Heidi.

SITUATION 2: TEPID PEBBLES (Barbara)

You are Barbara, a high school student and friend of Robert. You want to copy *Tepid Pebbles* and use it at home. You know it is okay to copy from the air for home use, so why not copy the rental tape? Besides, everyone does it. Copying the tape is very easy to do and you know a lot of people who would like to see the video and can't afford to rent it, so you brought your VCR over to Robert's house when you heard he had rented a copy.

Discuss this issue with Robert, Umberto, and Heidi.

ROBERT Tepid Pebbles	**BARBARA** Tepid Pebbles

SITUATION 2: TEPID PEBBLES (Umberto)

You are Umberto, a high school student whose older brother manages the store where Robert has just rented a hot new tape called *Tepid Pebbles.* You have come over to see the show. When you hear that Barbara plans to copy the tape you object and explain that if everybody copied tapes and showed them around, the rental store would lose a lot of business and might even fold.

Discuss this issue with Robert, Heidi, and Barbara.

SITUATION 2: TEPID PEBBLES (Heidi)

You are Heidi, a high school friend of Robert, Barbara and Umberto. Your mother is a municipal court judge. When you find out that Barbara intends to copy the tape, you explain that the paper Robert signed when he rented the tape was like a license agreement. It requires good faith and fair dealing just like any contract. Therefore he ought not to allow the tape to be copied.

Discuss this issue with Barbara, Robert, and Umberto.

UMBERTO
Tepid Pebbles

HEIDI
Tepid Pebbles

SITUATION 3: CRACKING THE COMPUTER (Nick)

You are Nick, a high school computer whiz. For a lark, you have cracked the high school computer and discovered how grades are stored. Now your friend Starla has received a C in physiology and she is terrified that it will ruin her chance to get a scholarship. She wants you to change the grade for her and you are sure you can do it without anyone detecting the alteration.

Discuss this issue with Starla, Sung Tae, and Rosa.

SITUATION 3: CRACKING THE COMPUTER (Starla)

You are Starla, a high school senior. You have always been a good student and you plan to get a scholarship to college and become a nurse. Physiology, however, has been very difficult and your hard-earned grade point average has suffered. Without a scholarship you are afraid you will have to go to work instead of college. You know that your friend Nick has invaded the school computer and just for this one course you want Nick to change the grade from C to B.

Discuss this issue with Nick, Sung Tae, and Rosa.

NICK Cracking The Computer	**STARLA** Cracking The Computer

SITUATION 3: CRACKING THE COMPUTER (Sung Tae)

You are Sung Tae, a high school friend of Starla, Nick, and Rosa. You are taking computer science and have been discussing the ethics of technology in class. When you hear that Starla wants Nick to change her physiology grade by invading the school computer, you object. You are sure what Starla wants Nick to do is wrong and you urge her to talk with her physiology teacher to see if there is something Starla can do to improve her grade.

Discuss this issue with Starla, Nick, and Rosa.

SITUATION 3: CRACKING THE COMPUTER (Rosa)

You are Rosa, a high school student. Your father and mother work in a hospital and you are convinced that physiology is a very important subject, especially for a nurse. When you hear that your friends want to use the computer to change Starla's grade in physiology, you don't think it would be a very good idea. You argue that misuse of the computer is cheating – just as bad as copying during an exam.

Discuss this issue with Sung Tae, Starla, and Nick.

SUNG TAE Cracking The Computer	**ROSA** Cracking The Computer

SITUATION 4: GATITO (Laurinda)

You are Laurinda, a high school student who has been into computers for years. You and your friend Keisha have combined efforts to adapt a hot new computer program, *Gatito,* to another computer language so it will play on more kids' computers. It took a lot of time to convert the program and you'd like to sell some copies.

Discuss this issue with Keisha, Luke, and Kwon.

SITUATION 4: GATITO (Keisha)

You are Keisha, a high school student, friend of Laurinda, and computer expert. You and your friend Laurinda have adapted a computer adventure game, *Gatito,* to run on a different brand of computer and you know a lot of people who want to buy copies. You feel it is only fair that you be able to make money on your work.

Discuss this issue with Luke, Kwon, and Laurinda.

LAURINDA
Gatito

KEISHA
Gatito

SITUATION 4: GATITO (Kwon)

You are Kwon, a high school student and friend of Keisha. You'd like to buy a copy of *Gatito,* but you are not sure it is legal. You talked it over with your mother, a lawyer, and she cited some cases that showed that the original copyright extends to translations. You think Laurinda should contact the original producers before she sells any copies. Your mother said to act on the principle of informed consent – if you have any doubts, ask.

Discuss this issue with Luke, Laurinda, and Keisha.

SITUATION 4: GATITO (Luke)

You are Luke, a high school student and friend of Laurinda. You know how hard she and Keisha have worked on the adaptation, but you also are greatly influenced by the discussions of ethics in your computer science class. After all, the writer of the program should receive the benefits of that original expression, no matter what computer language it is written in.

Discuss this issue with Laurinda, Kwon, and Keisha.

KWON Gatito	**LUKE** Gatito

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APPENDIX TO ETHICS AND TECHNOLOGY ROLE PLAYING SITUATIONS TAKEN FROM *THE MATHEMATICS TEACHER*, November, 1984

SITUATION 1: POLYWOG Andy

You are Andy, a high school student who has several copies of popular computer games, including Polywog. Your friend Peggy wants a copy of Polywog so that she can play the game on her computer at home. Peggy has often given you copies of her games, so you want to be a good friend and reciprocate by copying Polywog for her.

Discuss this issue with Peggy, Susan, and Eric.

SITUATION 1: POLYWOG Peggy

You are Peggy, a high school student who likes to play computer games at home. Your friend Andy just received Polywog, the hottest new computer game, for his birthday. You would like Andy to make a copy of Polywog for you. You've given Andy copies of games in the past and think it's only fair that Andy make a copy of Polywog for you.

Discuss this issue with Andy, Susan, and Eric.

SITUATION 1: POLYWOG Susan

You are Susan, a high school student who knows Andy from the Computer Club. Peggy is your best friend. Peggy asks Andy to make a copy of the computer game Polywog for her. You inform them that your mother, a software developer, wrote Polywog. You helped her with the graphics for the program. Both of you spent many hours developing and testing Polywog. Your mom's income is generated by royalties from the sales of Polywog. You feel it is unethical for Andy to copy Polywog for Peggy. Your mom would lose lots of money if students made a practice of copying computer game programs.

Discuss this issue with Eric, Andy, and Peggy.

SITUATION 1: POLYWOG Eric

You are Eric, a high school student who likes to play computer games. Your father is a lawyer. You hear Andy tell Peggy that he will give her a copy of Polywog, but Susan objects. You point out that your teacher doesn't allow copying of software, and you remind them that it is illegal to copy software. You suggest that there must be other ways that Andy could arrange for Peggy to use Polywog without making a copy for her.

Discuss this issue with Peggy, Susan, and Andy.

SITUATION 2: PASCAL COURSES Ms. Basyk, Computer Teacher

You are Ms. Basyk, a computer teacher at HiTech High School, scheduled for leave next year. You have taught Pascal for the past three years in a computer lab equipped with ten microcomputers and three Pascal language systems. Since Pascal is disk-dependent, you have made several additional copies of Pascal disks so that all ten computers can be used during your classes. You want every student to have as many hands-on experiences with Pascal as possible during class. Mr. Coball, another teacher in the department, will be teaching your class next year and does not want to use the copied disks.

Meet with Ms. Logogh, Mr. Coball, and Mr. Toepoe to discuss this issue.

SITUATION 2: PASCAL COURSES Mr. Coball, Computer Teacher

You are Mr. Coball, a computer teacher at Hi-Tech High School. Ms. Logogh, the chair of the computer department, has asked you to teach Pascal next year because Ms. Basyk will be on leave. You are excited about teaching this course, since you have taught only introductory computer courses in the past. However, a potential problem needs to be resolved. Ms. Basyk uses several pirated copies of the Pascal language disks in her class. You refuse to use the pirated disks and have asked Ms. Logogh to purchase more sets of the Pascal systems for use in next year's class.

Meet with Ms. Basyk, Ms. Logogh, and Mr. Toepoe to discuss this issue.

SITUATION 2: PASCAL COURSES Mr. Toepoe, Principal

You are Mr. Toepoe, principal of HiTech High school. Ms. Logogh, the chair of the computer department, has called a meeting to discuss a problem with you. Mr. Coball has asked her to buy several additional copies of the Pascal system to be used in his class next year. In the past Ms. Basyk has had no trouble teaching Pascal using only three copies of the language system. There isn't enough money in the budget to purchase the additional software. The school district's policy forbids copying of commercial diskettes.

Discuss this issue with Ms. Logogh, Ms. Basyk, and Mr. Coball.

SITUATION 2: PASCAL COURSES Mr. Logogh, Department Chair

You are Ms. Logogh, the chair of the computer department at HiTech High School. Ms. Basyk, who has taught Pascal at HiTech High for several years, will be on leave next year. Mr. Coball will teach the Pascal class, but he has requested that you purchase seven more copies of the Pascal language system. When you asked him why he needs more copies of Pascal, he told you that he believes it's wrong to use the pirated disks that Ms. Basyk has been using in her course. You have arranged a meeting with the two teachers and Mr. Toepoe, the principal of HiTech High.

Discuss this issue with Ms. Basyk, Mr. Coball, and Mr. Toepoe.

SITUATION 3: S.A.T. REVIEW Mr. Jacobs, Parent

You are Mr. Jacobs, the father of two children who attend Sunrise High School. Mary is a junior and Todd is a senior. The Scholastic Aptitude Test (S.A.T.) will be given in a month to college-bound juniors and seniors. Both of your children plan to attend college, so they want to review for the S.A.T. by using a review program that can be checked out from the school's media center. Unfortunately, only one copy of the review program is available. Because of the great demand for this program, each student can use the program for only one hour a week. You have asked Ms.Dawne, the media specialist, to make copies of the program so that several students, including your children, can use the program at the same time.

Meet with Ms. Dawne, Ms. Ray (the principal), and Mr. Byte (the district's computer specialist) to try to resolve this issue.

SITUATION 3: S.A.T. REVIEW Ms. Dawne, Media Specialist

You are Ms. Dawne, the media specialist at Sunrise High School. One of your responsibilities is to monitor the use of the five microcomputers located in the media center. Because of the upcoming S.A.T., many students have been requesting the *S.A.T. Review* program. Since only one copy of this program is available, each student can use it only about an hour a week. Yesterday, Mr. Jacobs, a parent, called and asked you to make more copies of the S.A.T. program. You told Mr. Jacobs that it is both illegal and unethical to make copies of commercial software.

Meet with Mr. Jacobs, Ms. Ray, and Mr. Byte to try to resolve this issue.

SITUATION 3: S.A.T. REVIEW Mr. Byte, District Computer Specialist

You are Mr. Byte, district computer specialist. Ms. Ray, principal of Sunrise High School, has called you to discuss a problem regarding computer software that has arisen at her school. Mr. Jacobs, the father of two students at Sunrise, has asked Ms. Dawne, the media specialist, to make several copies of the *S.A.T. Review* program. He wants his children to have more time to use this program to prepare for the upcoming S.A.T. Currently, the library has only one copy of the review program, so each student is allowed only one hour a week to use the program. You point out that it is illegal to make copies of commercial software. In addition, making such copies violates the district's policy on software copyright laws.

Meet with Ms. Ray, Mr. Jacobs, and Ms. Dawne to discuss this problem.

SITUATION 3: S.A.T. REVIEW Ms. Ray, Principal

You are Ms. Ray, principal of Sunrise High School. You recently received a call from Mr. Jacobs, a parent of two students who attend Sunrise. He has requested that Ms. Dawne, the media specialist, make several copies of the popular *S.A.T. Review* program so that his children have more opportunities to review for the upcoming test. You think that this request is reasonable, since it is important that college-bound students at Sunrise do well on the S.A.T. After all, Sunrise has a fine reputation for preparing students to attend the top universities in the nation. In fact, ten Sunrise students recently were finalists in the prestigious National Merit Scholarships.

Meet with Mr. Jacobs, Ms. Dawne, and Mr. Byte to try to resolve this issue.

SITUATION 4: SCHOOL BOARD MEETING C. Taylor, Superintendent

You are the school superintendent, an appointed official of the school board. You would like to see the district take a leadership position in the educational use of computers. You plan to present a policy statement to the board regarding the need to observe the copyright laws and publishers' licensing agreements concerning software. You feel that such a statement will enhance the district's image. In addition, you are proposing the development of a unit of study for educating students about the legal, ethical, and practical problems caused by copying programs that are not in the public domain.

Attend the regularly scheduled board meeting.

SITUATION 4: SCHOOL BOARD MEETING School Board Member

You are Bernard Brown, an elected school board member. You are convinced that teachers can strengthen the curriculum of the elementary schools through the use of computers. You want the needed hardware and software to be supplied for classroom use. You want to provide the greatest oportunities for the students to use computers at the least cost. Your campaign was based on a promise of an improved curriculum with no increase in spending. On the one hand, you are aware that violations of copyright laws and licensing agreements will ultimately result in higher costs and could lessen the incentives for development of good educational software. ON the other hand, the schools could stretch their money farther if the copyrights were loosely interpreted.

Attend the regularly scheduled school board meeting.

SITUATION 4: SCHOOL BOARD MEETING Parent

You are Mr. Johnson, the parent of Betty, a fifth-grade student. You have heard that the school district plans to spend $7500 on software for its elementary schools. Betty is very excited about learning to program in Logo, but the school has only five computers. You know that software can be copied, so you would rather have the district spend the bulk of its money on the purchase of more computers so sufficient hardware will be available for frequent use by students. You think the law is unclear on copying of software, and you feel that computer programs should be no different from records and videotapes. If you can copy music and motion pictures, why not software?

Go to the school board meeting to present your case.

SITUATION 4: SCHOOL BOARD MEETING Software Company Owner

You are Ms. Press, owner of a company that produces software. You have developed an excellent program to introduce Logo to elementary school students. You spent hundreds of hours developing the program, and for you, the issue of software piracy is straightforward: anyone who copies a computer program is stealing. You stand to lose your livelihood if copying by individuals and by schools reduces your sales, so you want copying prevented. You shrink-wrap your program and include a prominent warning that the person who breaks the seal agrees to a licensing contract. You provide one backup copy and state that the program must not be copied or used on more than one machine at a time. You consider that this license is a legal document and that anyone who breaks the contract is acting unethically and illegally.

Go to the school board meeting to present your case.

ICCE POLICY STATEMENT ON NETWORK AND MULTIPLE MACHINE SOFTWARE

Reprinted in The Mathematics Teacher,
NCTM
November, 1984, pp. 606-607

Just as there has been shared responsibility in the development of this policy, so should there be shared responsibility for resolution of the problems inherent in providing and securing good educational software. Educators have a valid need for quality software and reasonable prices. Hardware developers and/or vendors also must share in the effort to enable educators to make maximum cost-effective use of that equipment. Software authors, developers and vendors are entitled to a fair return on their investment.

Educators' Responsibilities

Educators need to face the legal and ethical issues involved in copyright laws and publisher license agreements and must accept the responsibility for enforcing adherence to these laws and agreements. Budget constraints do not excuse illegal use of software.

Educators should be prepared to provide software developers or their agents with a district-level approved written policy statement including as a minimum:

1. A clear requirement that copyright laws and publisher license agreements be observed;
2. A statement making teachers who use school equipment responsible for taking all reasonable precautions to prevent copying or the use of unauthorized copies on school equipment;
3. An explanation of the steps taken to prevent unauthorized copying or the use of unauthorized copies on school equipment;
4. A designation of who is authorized to sign software license agreements for the school (or district);
5. A designation at the school site level of who is responsible for enforcing the terms of the district policy and terms of licensing agreements;
6. A statement indicating teacher responsibility for educating students about the legal, ethical and practical problems caused by illegal use of software.

Hardware Vendors' Responsibilities

Hardware vendors should assist educators in making maximum cost effective use of the hardware and help in enforcing software copyright laws and license agreements. They should as a minimum:

1. Make efforts to see that illegal copies of programs are not being distributed by their employees and agents;

2. Work cooperatively with interested software developers to provide an encryption process which avoids inflexibility but discourages theft.

Software Developers'/Vendors' Responsibilities

Software developers and their agents can share responsibility for helping educators observe copyright laws and publishers' license agreements by developing sales and pricing policies. Software developers and vendors should as a minimum:

1. Provide for all software a back-up copy to be used for archival purposes, to be included with every purchase;

2. Provide for on-approval purchases to allow schools to preview the software to ensure that it meets the needs and expectations of the educational institution. Additionally, software developers are encouraged to provide regional or area centers with software for demonstration purposes. The ICCE encourages educators to develop regional centers for this purpose;

3. Work in cooperation with hardware vendors to provide an encryption process which avoids inflexibility but discourages theft;

4. Provide for, and note in advertisements, multiple-copy pricing for school sites with several machines and recognize that multiple copies do not necessarily call for multiple documentation;

5. Provide for, and note in advertisements, network-compatible versions of software with pricing structures that recognize the extra costs of development to secure compatibility and recognize the buyer's need for only a single copy of the software.

The Board of Directors of the National Council of Teachers of Mathematics has endorsed this policy statement issued by the International Council for Computers in Education regarding network and multiple machine software.

Session 6.1
The Teacher & Copyright Law

The Computer-Using Teacher and the Copyright Law

A Copy is a Copy is a Copy

It is becoming quite clear to all of us that a copy is a copy is a copy. It does not matter how ephemeral it is: a RAM copy that may be wiped out in just a few minutes after the brief lesson is over, or a copy on a diskette that can be handed to the next person. It does not matter how it is loaded: by hand into four machines or by a networking system into fifteen machines. It is still a copy. There is little question that the copyright law means that these are illegal copies.

Exclusive Right

The owner of the copyright is the only one who has the right to exploit it, and the owner may allow someone else to do so by making an assignment of the rights, a license to the rights, or a sale of the rights. The owner may decide to transfer all of the rights or only a portion of them because copyright is divisible. An owner may choose to allow multiple loading of the software but not networking. She or he may choose to license a site to copy the software or to limit use of the purchased copy to one machine at a time.

Software: A Special Case

Purchase of a copy of a work does not convey the right to reproduce it, with the exception of software. In the case of software, it is not an infringement of copyright to make a copy for archival purposes or for the purposes of adapting the software for use on another type of machine. The following exerpt amends title 17 of the United States Code and is taken from Public Law 96-517, dated December 12, 1980 (94 STAT, 3028-29).

> **SEC.10.(a) Section 101 of title 17** of the United States Code is amended to add at the end thereof the following new language:
>
> *A 'computer program' is a set of statements or instructions to be used directly or indirectly in a computer in order to bring about a certain result.*
>
> **Section 117.** Limitations on exclusive rights; Computer Programs:
>
> *Notwithstanding the provisions of section 106, it is not an infringement for the owner of a copy of a computer program to make or authorize the making of another copy or adaptation of that computer program provided:*
>
> *(1) that such a new copy or adaptation is created as an essential step in the utilization of the computer program in conjunction with a machine and that it is used in no other manner, or*
>
> *(2) that such new copy or adaptation is for archival purposes only and that all archival copies are destroyed in the event that continued possession of the computer program should cease to be rightful.*
>
> *Any exact copies prepared in accordance with the provision of this section may be leased, sold, or otherwise transferred, along with the copy from which such copies were prepared, only as part of the lease, sale or other transfer of all rights in the program. Adaptations so prepared may be transferred only with the authorization of the copyright owner.*

Copyright Notice

You can't copyright an idea, but you can copyright its expression. The copyright, according to the 1976 Copyright Act, is in force the minute the work is a reality. When the author makes a copy of the work, it must bear a notice of copyright (the symbol of word, the year of first publication, and the name of the owner):

© 1986 Jane Doe
Copr. 1986 Jane Doe
Copyright 1986 Jane Doe

The first method is preferable because it is accepted worldwide wherever copyright is recognized. The latter two are valid in the United States. If the work is published in Latin America, it must also say, "All rights reserved." Errors and omissions weaken the protection against unauthorized copying. Registration with the federal Copyright Office is not a prerequisite to copyright protection; however, a copyright owner cannot file any infringement action unless and until registration is effective.

Fair Use

"Fair Use" is provided for under Section 107 of the Copyright Act. For purposes such as criticism, comment, review, news reporting, teaching, and research, use of the copyrighted material is not an infringement. Things to be considered in making this determination include the purpose and character of use, the nonprofit nature of the use; the nature of the work itself; the amount and substantiality of the portion used; and the effect upon the potential market for, or value of, the copyrighted work. Direct classroom use of an entire program, even though it would certainly be for educational purposes, would not seem to fit the definition of "fair use".

Models for Students

There is little doubt that teachers and other educators become models for students in their decisions concerning ethical issues. If we do not follow the law, how can we expect our students to do so? We must find ways to serve our students' cognitive needs without destroying ourselves ethically.

Software Library

Software that is used sporadically by individual students can be placed in a library and checked out in the same way print materials are handled at schools. Extra care and education must be provided to avoid the library's becoming another source of difficulty with regard to copying.

Informed Consent

The solution is to deal with the software suppliers who will meet us halfway. Copyright is divisible, and therefore a software company can allow multiple loading and networking and site licensing if it chooses. We must operate under the principle of informed consent, notifying the company of our intentions before we purchase. One suggestion is to ask for the software company's policy on whatever practice we need for a particular classroom. Another equally valid method is to describe in writing, on the purchase order, exactly how the piece of software will be used. If the company chooses not to sell under those circumstances, we can do our best to find another software company. New and exciting programs are being written every day--we don't have to be dependent on a company that does not take into consideration the legitimate needs of classroom teachers. Most of all, we will not abdicate our responsibility to follow the law. The Constitutional mandate empowers Congress to grant "authors and inventors" the exclusive right to their "writings and discoveries" for limited times" in order to promote the progress of science and the useful arts." Far be it for us, as teachers, to fail to heed this provision which has worked to the benefit of society since 1776!

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Session 7
Open Forum

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Session 8 Software Evaluation

When microcomputers first became available, the process of evaluating software was relatively unimportant because very little software existed (why weed out something when nothing was available to take its place?) and most software which was available was created by the teacher using it. Teachers could create software that suited their own purposes. When software finally did become available many small publishing houses produced "home-brewed" software which met the needs of the educator who produced it but the programs were not transferable to another environment. This software met the needs of few users other than the developer and frequently had content errors, instructional errors, and even programming errors. Teachers complained about the poor quality of available software.

Certainly there exists poorly designed, inadequate software on the market today. However, there are many examples of high quality software which can provide an effective supplement to classroom instruction. As developers become more knowledgeable about the capabilities of the machine, more sophisticated applications which address content properly, are pedagogically sound (in many cases better than some actual live instruction), and contain no programming errors are being produced. The question now is "how can one make the proper selection of software when one is faced with a limited budget." Certainly software budgets cannot be wasted on poor quality software, nevertheless one expects to make some mistakes considering the avalanche of software catalogues with which schools now inundated.

The software evaluation process consists of two steps. The first is identifying "appropriate" software for a particular school environment. The second is determining whether the software selected is meeting the needs of the students.

IDENTIFYING APPROPRIATE SOFTWARE

Where should mathematics educators at all levels go to find the kind of software they want? The first step is to have a clear idea of what type of software they are looking for and what they expect the software to do for them. It is absolutely imperative that the need for the software, as well as an idea of where in the curriculum, how, and with what audience the software will be used, is absolutely identified before any software is selected. Acquiring software without having clear direction as to the objective being filled by the software almost guarantees that the software will stay on the shelf in the resource room for a considerable period of time. Furthermore, absence of specific learning objectives to be met makes the teacher fair prey for every software huckster who produces a slick sales presentation. In short, the learning objectives must drive the software selection and not the other way around.

Once the objectives are in mind one can begin looking for the software which comes reasonably close to filling those objectives. Basically, there are six places to learn about software: professional journals, professional meetings, computer journals, word of mouth or colleagues, computer stores, and local universities. Professional journals like the Mathematics Teacher and Arithmetic Teacher regularly review mathematics software. The reviews are made by classroom teachers and educators who have some familiarity with mathematics applications in computer software. The professional journals also have the highest likelihood of reviewing software which meets one's specific needs.

The second place to look is at professional meetings. Often the vendor displays provide an ample opportunity for educators to "play" with the software which opportunities to try out software may be more valuable than reading reviews. Also, it is less likely that a vendor would demonstrate poorly conceived software around professional teachers where it could receive enormous criticism. The sessions themselves often demonstrate software by teachers and other educators who have no vested sales interest in a particular piece of software. Frequently one can get a lead on some interesting software from a presenter or attendees in a particular session.

It is possible to learn about software from generic computer or computer education journals. Since they are not subject matter specific, one may have to search a while to find something acceptable, but these journals are frequently good places to find out about software trends in general.

Colleagues, particularly at other schools, may have insight into an excellent piece of software for a specific application. A local professional organization chapter's president might be knowledgeable about software or they might know someone who is using a particularly good piece of software in the mathematics area of concern.

Local computer stores may try to stock some educational software. However, high inventory costs and low software turnover create problems for some stores. Generally, stores will stock only items which have a reasonable chance of quickly selling. Therefore, they are likely to have a limited selection of only the most popular items. Some computer software stores, however, may have a larger selection and will frequently let educators preview the software for 30 days.

The local university may have selected types of software which are representative of what is available in mathematics. Because of limited budgetary constraints, it is unlikely they would have a complete library of software to fit all needs, but they may have enough in a particular subject area to identify a place to start. In some cases, a university may have an extensive software library. For example, Arizona State University and Teachers College, Columbia University have developed resources for teachers to try out software in mathematics. Furthermore, mathematics educators at a local university may be knowledgeable about software for a particular need and may be able to reference articles and reviews which could be important.

Once software has been identified, software vendors should be contacted to determine procedures for examining selected items on approval. Many vendors now give a money back guarantee if software is returned within thirty days. Unfortunately, most schools will have to back up the initial request with a purchase order first and then cancel the purchase order if the software is returned. This can create a paperwork nightmare for the school district's business officer so one should be certain the software is reasonably appropriate before requesting preview privileges.

Some publishers offer a low cost (usually under $15.00) "demo" disk of the software for educators to look at and keep. While this may be a good way to decide if one wants to look at the actual software itself, this demo disk should not be used to decide whether to purchase the software. Usually, the most impressive parts of the software are on the demo disk. The weaker parts are seldom shown. The software may still be excellent, but one cannot tell solely by examining the demo. It is similar to buying a used car where the salesman shows the shiny exterior and interior without showing the engine. The car may be in fine working order, or it may not.

EVALUATING SOFTWARE

Once the software has been received the evaluation process can begin. Selected colleagues should examine the software and check for content errors and determine whether the software can help meet the appropriate objectives. If it is tutorial or drill/practice in nature, does it handle the content appropriately? Or does it "give away" the rules the students are expected to discover?

Beforehand prepare an evaluation sheet like the one at the conclusion of this article. When the software arrives, the person checking it in can attach a sheet with names of appropriate faculty members to review it. In one school, the first teacher to use the software evaluates it. In another school the department head evaluates all software. In some schools a team is set up for evaluation. In others, a faculty member responsible for the course where the software will be used reviews the package. Different approaches work best for different schools. What is important is that each school have a procedure which works for them and that someone in the school reviews the software. Having an experienced reviewer in the school is very helpful for other teachers who use the software, particularly if it is complicated, difficult to use, curriculum extensive, or has poor documentation.

If students can go through the software within the return period, some should be asked to do so. Some schools assign students extra credit for working through software on a regular basis. Let the students try to "crash" it. Follow-up with them daily to determine where they had problems using the software. Perhaps directions or procedures were not clear. These should be checked again by a teacher.

If the software appears to meet the need then it should be promptly paid for. One should check with the publisher first about possible multiple copy discounts, if needed. Site licensing and network capabilities are becoming increasingly important issues. What about backup privileges? Will the publisher replace at no or low cost any damaged diskettes beyond the warranty period? If so, what is the turn around time to get new copies? If it takes 6 - 8 weeks to get the software back one might consider acquiring a second copy before it is needed.

After obtaining the software one should make sure that it is easily available to any faculty member who wants to use it. Certainly some faculty members may be more adept at using the computer than others and those with lesser skills may shy away from using the computer outright. Providing in-service on the software can make the implementation process somewhat easier.

The process of software evaluation consists of three preliminary components: Checking the content for content accuracy; guaranteeing the software is pedagogically sound; and determining that there are no programming errors. Finally, additional considerations such as generalizability of the software, copying privileges etc. need to be described.

CONTENT ACCURACY.

Today, content accuracy is not the problem it used to be. In the late 1970's non-mathematicians and non-mathematics teachers wrote many of the mathematics programs. As a result, some content errors crept into the software. One of the most glaring was the description of the decimal point as the "zeroth place" in a discussion of place value. Certainly this is contrary to common understanding of place value (a group of one place is equivalent to one on the place to the left). Unfortunately for this software, 10 decimal points do not make a 1. Content "bombs" like this may make the software unusable. Certainly one should attempt to return any software that is content inaccurate even if the warranty period has expired. Consider writing a note to the Mathematics Teacher or the Arithmetic Teacher to inform others.

PEDAGOGICAL QUALITY.

Pedagogical quality is frequently related to personal preference. If the teacher wants students to discover relationships, then the software either must allow the opportunity to do that or provide enough data for the students and teacher to find the discoveries in class. If the students need drill on their multiplication facts, good software allows the teacher to select the domain of multipliers and/or the range of answers. One area worth considering is how the computer lets the student type in answers. Some software requires the student to enter the answer of a multiplication problem from right to left (starting with the ones place first). Others expect it from the leftmost non-zero digit to the right. Specify which is preferred in the evaluation sheet so the unsuspecting teacher using it the next time is not surprised.

Another area to consider is how the computer handles incorrect answers. Are some incorrect answers anticipated and therefore receive different feedback than others? What if the student does not know the right answer; does it "cul-de-sac" the student over and over or does it eventually give the student hints or the right answer? Cul-de-sacing is not acceptable for free response questions but might work in multiple choice situations where there are only a fixed number of responses available.

Sometimes there are clear-cut teaching mistakes. One piece of software asks the student which digit is in the tens place:

97.92

Obviously 9 is in the tens place. But look! It is also in the tenths place as well. Conceivably a student could get the answer "correct" for the wrong reason. If frequently occurring, such problems might make this software unacceptable for your use.

Another area to check is multiple student entry points for the software. If the software is fairly extensive, it might not be possible for the student to finish working through the program in one sitting. Certainly, the student should be able to return to the software near where he or she left off, rather than working all the way through it again. The location of entry points and how to find them may be critical if the software is to meet the educational objectives. Clearly these items need to be documented. Also, record keeping may be in order here. If the software determines where the student is to return, it must keep some sort of record of the student's progress. This can create some logistical problems. The student may have to type in his name the same way he typed it in the first time and recall a "password." Furthermore, if the software is running on a floppy disk system, the student will need to use the same disk he used the last time. Teachers frequently want evidence that the student actually completes an assignment. The teacher may also want to check for error patterns, if appropriate, although some new software is beginning to do that automatically. Teachers very often find the record keeping capability interesting, but never use it because it is cumbersome or does not give the data the teacher really needs. Of course if record keeping is particularly well handled, or badly treated, this should be noted on the evaluation sheet.

Another area to check is the amount of interaction and the ability to "navigate" through the software in a coherent way. Students find linear software which progresses from start to finish with little branching, and which may contain many screens where all one does is read and then press the return key very boring. Some software companies set limits on the number of screens which can be displayed before a "real" question is called for. Unfortunately there is no "best" maxim to follow. Some software works well with eight to ten linear screen segments while other software needs a question on virtually every screen. It depends on the content and the audience. As a rule of thumb one should expect at least one question for every three screens, but this can vary considerably. Certainly primary level software must have more questions because of the shorter attention span of younger students. Frequently, good software allows the user to make selections from a menu. Often these menus are tree-structured, allowing the student to progress to another level of the menu after selecting from the first one The ESC (escape) key is a way to return to the previous menu layer. In this way it is possible to navigate to various points in the program with a minimum of keystrokes.

PROGRAMMING ERRORS.

Programming problems do not normally occur with today's software. The era of breaking words in the middle at the end of a line, scrolling off the screen, error termination notices when one types a letter if a numeric answer is required, and error messages are generally things of the past. This isn't to say that some errors won't crop up. Software publishers try to test every possible branch to make sure there are no errors, but even the most extensive field testing cannot uncover everything particularly as software becomes more sophisticated and extensive. Sometimes the software error is due to hardware difficulties. Insufficient memory, bad chips, and even a defective memory board can cause some problems. Before contacting the publisher it is a good idea to check the hardware requirements of the software and make certain they are met. If the problem still exists, try another similarly configured computer to see if the problem occurs there as well. If it is clear the problem exists when using the configuration specified by the publisher, one should contact the publisher as to what to do (Some publishers have toll-free numbers to registered owners). Most reputable publishers will want to correct any defect.

Some software expects to access a particular hardware configuration and will crash or lock-up the keyboard if it cannot find it. This can occur if the software expects to find the disk drive in a particular slot or port, or if the software expects to receive a "printer ready" signal and cannot find it. In the first instance the hardware should be reconfigured. In the second, let the user know that the printer must be on and selected before using the software.

SOFTWARE EVALUATION SHEETS.

There are many software evaluation sheets which attempt to facilitate the software evaluation process. One can find them in computer journals, in book chapters relating to software evaluation, and in documents devoted to software evaluation. (See the NCTM Guidelines for example) These evaluation sheets are generally divided into three sections: Hardware configuration, Likert scaled items dealing with software quality, and a brief description of the software. Teachers should feel free to modify existing evaluation sheets to suit their needs.

The hardware configuration sections are fairly standard. Information to be detailed includes what type of machine the software will run on; how much memory is required; what language and operating systems are needed;how many disk drives are required; and what necessary printer configurations are expected. Obviously, if all of the school's machines are the same, then simply checking the software on one system is sufficient. It might be useful here to specify any additional hardware needed (e.g. videodisc player type etc.) or if blank formatted diskettes are required and how many. (Sometimes the record keeping capability may require blank formatted diskettes in a second disk drive.)

The Likert scaled items on software quality are an attempt to give some sort of numeric rating to various aspects of the software. Unfortunately, there is not a significant amount of interrater reliability when using these items. This means that different people evaluating the same item may score that item considerably differently. Infoworld has recently attempted to give software ratings on a scale of 1 to 10. A quick review of the letters section indicates many people frequently disagree substantially with their ratings. Also, it is unclear what aLikert scaled value means. For example, on a 10 point scale what does it mean to say the graphics are a 7? Does that mean they are good? Relate to the objective? This American Bandstand approach to software evaluation probably appeals to our need to quantify, but should really be replaced by more professional tools.

The third section of the evaluation deals with software description and is the part that frequently is left blank, but is probably the most important part. It is here that the idiosyncracies of the software can be described. Perhaps this section should be expanded and even replace the second section. The evaluation instrument following this article uses that approach.

It seems to make more sense to describe what the software does; how it accomplishes this; what the graphics look like and how they contribute to the learning process; what special problems the user needs to know about (e.g. crashes if the printer isn't "ready" etc.); and special features (e.g. allows the teacher to set the problem delimiters) or content errors.

OTHER CONSIDERATIONS.

There are other considerations one should think about when selecting software. Since a software budget is often limited some teachers prefer more generic software which can be generalized across mathematics classes. Examples are (1) function plotting software which allows the user to enter functions into the computer and see them plotted on a cartesian or polar coordinate system, (2) spreadsheets which allow the teacher to design "templates" which can be used in class to extract roots, find common denominators etc., (3) symbol manipulators which allow the user to put in poly-nomials and see them multiplied, divided, factored, integrated, etc., image manipulators which allow the user to define geometric shapes and drop perpendiculars, rotate a figure around a point etc. and (5) puzzle generators which allow the teacher to enter terms and numerals in word search puzzles, crossword/number puzzles etc. If these can meet the educational needs in a variety of different classes they may be a cost effective way of spending software money.

Sometimes selecting software used in a nearby school can be a good idea if the software is complex. There would exist a group of colleagues who can provide training and help if needed. This can be particularly important if the documentation is not very clear and a help-line phone number either does not exist or is a toll-call away.

If the school needs multiple copies of the software because many students will use it simultaneously then multiple copies must be acquired. This can be very expensive even if an individual item is reasonable. Some software requires that the diskette be present in the disk drive at all times. Other software can be completely loaded into RAM up front. Teachers must be informed that it is a violation of copyright law to load the software into several machines at the same time. If one is going to run multiple systems of the same software simultaneously, one must have multiple copies of the software unless a release has been granted by the publisher.

Site licensing, backup policies, copying privileges, reputation of the publisher and/or distributor, and relationship to the curriculum are all important issues to consider. These issues are not of equal importance. The relative weight of each must depend upon the circumstances under which the software is being evaluated.

One can see that software evaluation is not a trivial task. It is, however, an extremely important one. Clearly, schools will continue to be faced with tight budgets. Administrators may demand accountability for expenditures because that is what is being demanded of them. Therefore, it is in the interests of the school and the professional staff that software evaluation is done properly. This process should result in the school acquiring good software which is effective and meets the needs of the students.

Subject Area: ______________________

Progam Title: ______________________

Publisher Address/Phone: ______________________

Warranty/Backup Policy: ______________________

____

Hardware Requirements: ______________________

____

Program Objectives: ______________________

____

____

Level of Student (grade/ability): ______________________

____

Strengths of Program: ______________________

____

____

____

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____

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Weaknesses of Program: ______________________

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____

____

Special features or characteristics:

Reviewed by:

Date:

The 1987 Educational Software Preview Guide

developed by the

EDUCATIONAL SOFTWARE EVALUATION CONSORTIUM

at the

CALIFORNIA TECC SOFTWARE EVALUATION FORUM

December 1-5, 1986

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PREFACE

THE 1987 EDUCATIONAL SOFTWARE PREVIEW GUIDE includes a list of favorably reviewed microcomputer software for use in instructing students in kindergarten through grade twelve. It is NOT a buying guide. It has been developed solely as an aid to educators in locating programs they may want to preview. The Consortium's participants recommend that all software be previewed by educators to determine its suitability for their instructional programs and students.

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KEY TO ABBREVIATIONS

SUBJECT ABBREVIATIONS

AT	Art
BE	Business Education
CS	Computers
EP	Electronic Periodicals
FL	Foreign Languages
HE	Home Economics
HL	Health
IT	Instructional Tools
KB	Keyboarding
LA	Language Arts/English
LM	Library Media Skills
MA	Mathematics
MU	Music
PR	Preschool/Early Childhood
PS	Problem Solving/Logic
SC	Science
SS	Social Studies
TE	Testing
VE	Vocational Education/Industrial Arts

GRADE LEVEL ABBREVIATIONS

P	Primary (K-3)
E	Elementary (4-6)
M	Middle (7-9)
S	Secondary (9-12)
T	Teacher

COMPUTER ABBREVIATIONS

AC	Acorn
AM	Amiga
AP	Apple
AT	Atari
CO	Commodore 64
IB	IBM PC
JR	IBM PCjr
MC	Macintosh
PE	Commodore PET
TC	TRS-80 Color
TR	TRS-80 Model III/4
VC	Commodore VIC

INSTRUCTIONAL MODE ABBREVIATIONS

AU	Authoring System
CA	Creative Activity
CP	Computer Programming
DB	Data Base
DE	Demonstration
DP	Drill and Practice
EG	Educational Game
GG	Graphics Generator
IF	Interface
IM	Instructional Materials Generator
PS	Problem Solving/Logic
SD	Spreadsheet
SH	Shell/Mini-authoring System
SI	Simulation
SK	Spelling Checker
TC	Telecommunications
TE	Test
TU	Tutorial
WP	Word Processor

PRICE NOTATION

*	in PRICE column indicates a series for which programs are also sold separately.

MATHEMATICS - ADVANCED MATHEMATICS

TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS P	E	M	S	T	PRICE
COMPUTER GRAPHING EXPERIMENTS V.2	ADD WES	AP	DP,GG,TU			Y			60.00
GRAPHS TRIGONOMETRIC FUNCTIONS FROM USER-DETERMINED PARAMETERS									
COMPUTER GRAPHING EXPERIMENTS V.3	ADD WES	AP	DP,GG,TU			Y			60.00
GRAPHS CONIC FUNCTIONS FROM USER-DETERMINED PARAMETERS									
ELECTRONIC BLACKBOARD: TRIGONOM	COMPRESS	AP	DE,GG,TU			Y			50.00
GRAPHS AND EXPLORES TRIGONOMETRIC FUNCTIONS FROM USER-DEFINED PARAMETERS									
GRAPHICAL ANALYSIS II	VERNIER	AP	GG,PS		Y	Y			29.95
PLOTS GRAPHS WITH EXPERIMENTAL DATA									
MATHEMATICS V.4: ADVANCED	MECC	AP	DP,TU			Y			39.00
NUMERICAL INTEGRATION, LIMITS OF FUNCTIONS, EQUATION GRAPHING, MATRICES, AND POLYNOMIAL EQTINS									
MICROSOFT MU-MATH	MICROSOFT	IB	PS			Y			300.00
PERFORMS ALGEBRA, TRIG, CALCULUS (DIFFERENTIATION AND INTEGRATION), AND TRANSCENDENTAL FUNCTIONS									
TRIGONOMETRY OF THE RIGHT TRIANGLE	MIC WRKSHP	AP	DP			Y			39.95
STEP-BY-STEP SOLVING OF TRIGONOMETRIC WORD PROBLEMS									

MATHEMATICS - ALGEBRA

TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS P	E	M	S	T	PRICE
ALGEBRA ARCADE	COMPRESS	AP,CO,IB	DP,EG		Y	Y			49.95
GAME FORMAT PRACTICE IN CREATING GRAPHS OF EQUATIONS ON A COORDINATE PLANE									
COMPUTER GRAPHING EXPERIMENTS V.1	ADD WES	AP	DP,GG,TU			Y			60.00
GRAPHS USER-DETERMINED ALGEBRAIC FUNCTIONS									
ELECTRONIC BLACKBOARD: ALGEBRA	COMPRESS	AP	DE,GG,TU		Y	Y			95.00
GRAPHS AND EXPLORES ALGEBRAIC FUNCTIONS WITH PARAMETRIC INPUT									
ELECTRONIC BLACKBOARD: FUNCTION	COMPRESS	AP	DE,GG,TU		Y	Y			50.00
GRAPHS AND EXPLORES FUNCTIONS FROM USER-DEFINED PARAMETERS									
EQUATIONS I	MIC WRKSHP	AP,CO,IB,PE,TR	DP,TU		Y	Y			29.95
PRACTICE SOLVING EQUATIONS OF THE FORM AX + B = C									
EQUATIONS II	MIC WRKSHP	AP,CO,IB,TR	DP,TU			Y			29.95
PRACTICE SOLVING EQUATIONS OF THE FORM AX + B = CX + D									
EXPLORING TABLES AND GRAPHS I	WEEK READ	AP	EG,GG,TU	Y	Y	Y			34.95
INTRODUCES THE USE OF GRAPHS: INCLUDES A TOOL FOR CONSTRUCTING GRAPHS FROM A GIVEN SET OF DATA									
EXPLORING TABLES AND GRAPHS II	WEEK READ	AP	EG,GG,TU	Y	Y	Y			34.95
REAL-LIFE APPLICATIONS OF TABLES AND GRAPHS: PICTURE, BAR, LINE, AND AREA GRAPHS									
FACTORING ALGEBRAIC EXPRESSIONS	MIC WRKSHP	AP,CO,IB,TR	DP,TU			Y			29.95
INSTRUCTION AND PRACTICE IN FACTORING LINEAR AND QUADRATIC EXPRESSIONS									
GRAPHING EQUATIONS	CONDUIT	AP	DE,EG,PS		Y	Y			60.00
GAME FORMAT PRACTICE IN GRAPHING OF LINEAR AND QUADRATIC EQUATIONS									

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE P	LEVELS E	M	S	T	PRICE
GREEN GLOBS AND GRAPHING EQUATIONS GAME FORMAT PRACTICE IN GRAPHING OF LINEAR AND QUADRATIC EQUATIONS	SUNBURST	AP,IB	DP,EG,PS			Y	Y		59.00
KING'S RULE, THE FORM AND TEST HYPOTHESES, RECOGNIZE NUMERICAL PATTERNS, AND DEVELOP PROBLEM-SOLVING SKILLS	SUNBURST	AP,CO,IB,TC,TR	EG,PS		Y	Y	Y		59.00
MATHGRAPHER DEMONSTRATES PROPERTIES OF FUNCTIONS AND GRAPHING CONCEPTS	HRM SOFTWR	AP,CO	TU		Y	Y	Y		69.00
MISSION: ALGEBRA PRACTICE SOLVING LINEAR EQUATIONS GENERATED AUTOMATICALLY OR BASED ON POINTS PLOTTED BY USER	DESIGNWARE	AP,AT,CO,IB,JR	DP,SI				Y		44.95
PROBLEM-SOLVING IN ALGEBRA READING, TRANSLATING, AND SOLVING WORD PROBLEMS INVOLVING LINEAR AND QUADRATIC EQUATIONS	BRITANNICA	AP,TR	DP,TU			Y	Y		299.00
QUATIONS MATH GAME, BASED ON SCRABBLE, FOR BUILDING EQUATIONS RATHER THAN WORDS	SCHOLASTIC	AP	DP,EG			Y	Y		39.95
ROYAL RULES PRESENTS MATHEMATICAL SEQUENCES FOR STUDENT STO DEDUCE RULES	SUNBURST	AP,IB	SI			Y	Y		69.00
SOLVING QUADRATIC EQUATIONS PRACTICE SOLVING RANDOM QUADRATIC EQUATIONS BY FACTORING	CBS	AP,CO,IB,JR	DP,TU			Y	Y		29.95
SPINNERS AND SLUGS EXPLORE A VARIETY OF PROBLEM-SOLVING SITUATIONS	SCOTT FORS	AP	PS,SI		Y	Y			49.95
SUCCESS W/ALGEBRA: LINEAR EQUATIONS STEP-BY-STEP SOLVING OF RANDOMLY GENERATED SIMULTANEOUS LINEAR EQUATIONS	CBS	AP,CO,IB,JR	DP			Y	Y		29.95
SUCCESS W/ALGEBRA: QUADRATIC EQUAT STEP-BY-STEP SOLVING OF RANDOMLY GENERATED QUADRATIC EQUATIONS	CBS	AP,CO,IB,JR	DP			Y	Y		29.95
TOBBS LEARNS ALGEBRA PROBLEM-SOLVING IN NUMBER RELATIONSHIPS USING A 2-BY--2 GRID	SUNBURST	AP,TR	DP,SI			Y	Y		59.00

MATHEMATICS - GEOMETRY/MEASUREMENT

SEE ALSO 'INSTRUCTIONAL TOOLS - INSTRUCTIONAL MATERIALS GENERATOR' SECTION

TITLE	PUBLISHER	COMPUTERS	MODES	P	E	M	S	T	PRICE
BUMBLE GAMES FIVE PROGRAMS INTRODUCE USE OF NUMBER PAIRS TO DESCRIBE POSITIONS IN AN ARRAY AND ON A GRID	TLC	AP,CO	DP,EG,PS	Y	Y				54.95
BUMBLE PLOT PRACTICE PLOTTING AND GRAPHING SKILLS ON A +10 TO -10 COORDINATE GRID	TLC	AP,CO	DP,EG,PS		Y	Y			54.95
CREATIVITY UNLIMITED DEVELOP FLEXIBLE AND ORIGINAL APPROACHES TO BHILD, ROTATE, AND EXPAND OBJECTS	SUNBURST	AP	CA			Y	Y		59.00
EXPLORER METROS PRACTICE ESTIMATING METRIC MEASUREMENTS	SUNBURST	AP	DP,EG		Y	Y	Y		59.00
FACTORY, THE PRACTICE VISUAL DISCRIMINATION, SPATIAL PERCEPTION, SEQUENCING, AND ORDERING SKILLS	SUNBURST	AP,AT,CO,IB,TC	EG,PS,SI		Y	Y	Y		59.00
GEOMETRIC PRESUPPOSER EXPERIMENT WITH AND HYPOTHESIZE ABOUT GEOMETRIC CONCEPTS	SUNBURST	AP	CA			Y	Y		99.00

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS					PRICE
				P	E	M	S	T	
GEOMETRIC SUPPOSER: QUADRILATERALS	SUNBURST	AP	CA,DE,DP,PS		Y	Y	Y		99.00
EXPERIMENT WITH AND HYPOTHESIZE ABOUT QUADRILATERALS									
GEOMETRIC SUPPOSER: TRIANGLES	SUNBURST	AP	CA,DE,GG,PS			Y	Y		99.00
EXPERIMENT WITH AND HYPOTHESIZE ABOUT TRIANGLES									
GEOMETRY	BRODERBUND	MC	CA,TU				Y		99.95
MANIPULATE GEOMETRIC FIGURES TO CREATE PROOFS: FULL-YEAR COURSE									
GOLF CLASSIC/BATTLING BUGS	MILLIKEN	AP,IB,JR	DP,EG		Y	Y			34.95
PRACTICE WITH ANGLES, LENGTH ESTIMATION, AND READING GRAPHIC EXPRESSIONS									
RIGHT TURN, THE	SUNBURST	AP,CO,IB,JR	EG,PS		Y	Y	Y		59.00
ROTATE AND FLIP FIGURES ON A THREE-DIMENSIONAL GRID									
STICKYBEAR SHAPES	WEEK READ	AP,AT,CO	DP,EG	Y					39.95
IDENTIFY, CHOOSE, AND NAME SHAPES: FIGURE-GROUND RELATIONSHIPS									
SUPER FACTORY, THE	SUNBURST	AP,IB,JR	CA,CP,PS,SI		Y	Y	Y		59.00
SPATIAL GEOMETRY 3-D VERSION OF 'THE FACTORY' TO EXPERIMENT WITH DESIGNS ON A CUBE									
TRAP-A-ZOID	DESIGNWARE	AP,AT,CO,IB,JR	DP,EG		Y	Y	Y		39.95
GAME FORMAT USES GEOMETRIC CONCEPTS TO TRAP ZOIDS FROM SPACE: MULTIPLE SKILL LEVELS									
VOYAGE MIMI: MAPS AND NAVIGATION	HOLT, R&W	AP	EG,PS,SI,TU		Y	Y	Y	Y	75.00
APPLY MAPPING AND NAVIGATIONAL SKILLS TO RESCUE DISTRESSED WHALES									

MATHEMATICS - NUMBER

SEE ALSO 'INSTRUCTIONAL TOOLS - INSTRUCTIONAL MATERIALS GENERATOR' SECTION

TITLE	PUBLISHER	COMPUTERS	MODES	P	E	M	S	T	PRICE
ADDITION LOGICIAN	MECC	AP	DP	Y					49.00
PROBLEMS IN A GAME FORMAT FOCUS ON WHOLE NUMBER ADDITION AND REGROUPING									
ADDITION MAGICIAN	TLC	AP,CO,IB	DP,EG,PS	Y	Y				34.95
A SET OF GAMES TO PROVIDE MATH PRACTICE IN A PROBLEM-SOLVING ENVIRONMENT									
ADVENTURES WITH FRACTIONS	MECC	AP,CO	DP,TU		Y	Y			39.00
INTRODUCES TWO METHODS OF DEALING WITH FRACTIONS, WITH GAMES FOR PRACTICE									
ARITH-MAGIC	QED	AP,CO,IB,PE,TR	EG	Y	Y	Y			35.00
THREE CLASSIC MATH PUZZLES									
CHALLENGE MATH	SUNBURST	AP,CO	DP,EG	Y	Y	Y			59.00
CALCULATING AND ESTIMATING WITH WHOLE NUMBERS AND DECIMALS IN A GAME FORMAT: EDITING OPTION									
CHARLIE BROWN'S 1,2,3,'S	RANDOM	AP	DP	Y					39.95
ANIMATED PRACTICE IN NUMBERING AND COUNTING FROM 1 TO 10									
CIRCUS MATH	MECC	AP	DP,EG	Y					49.00
SEVERAL LEVELS OF ADDITION PROGRAMS WITH GRAPHICS: EDITING OPTION									
CLOCK	HARTLEY	AP,IB	DP	Y					39.95
PRACTICE TELLING TIME AND CONVERTING DIGITAL TIME TO ANALOG TIME									
CLOCK WORKS	MECC	AP	DP	Y					49.00
PRACTICE TELLING TIME USING BOTH ANALOG AND DIGITAL CLOCKS									

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				P	E	M	S	T	
CODE QUEST DECODE CLUES TO DISCOVER OBJECTS OR USE THE MINI-AUTHOR TO CREATE YOUR OWN OBJECTS	SUNBURST	AP,AT,CO,IB,TR	EG,PS		Y	Y			59.00
COUNTERS THREE GAMES CONCRETELY PROVIDE 1-TO-1 CORRESPONDENCE FOR COUNTING, ADDITION, AND SUBTRACTION	SUNBURST	AP	DP,EG,TU	Y					59.00
COUNTING GRAPHIC INTRODUCTION TO COUNTING, BASIC ADDITION, AND SUBTRACTION	MICRO-ED	AP,CO	DP,TU	Y					34.95
COUNTING CRITTERS PRACTICE WITH BASIC NUMBER SKILLS USING NUMBERS FROM 1 TO 20	MECC	AP	DP	Y					49.00
EARLY ADDITION GRAPHIC SEQUENCES FOR PRACTICE IN SIMPLE ADDITION FACTS; EDITING OPTION	MECC	AP,AT	DP	Y					49.00
EARLY GAMES FOR YOUNG CHILDREN INTRODUCES SHAPES, LETTERS, DRAWING, ADDITION, AND SUBTRACTION	SPRINGBOARD	AP,CO,IB,JR,MC	DP,EG	Y					34.95
EJERCICIOS DE MATEMATICAS SPANISH VERSION OF 'MECC ELEMENTARY V.1'	MECC	AP	DP,EG,PS	Y	Y				39.00
ELEMENTARY V.1: MATH INCLUDES HURKE, BAGELS, AND TAXMAN PROGRAMS: EMPHASIZES LOGIC AND DRILL ON BASIC FACTS	MECC	AP	DP,EG,PS,SI	Y	Y				29.00
ENCHANTED FOREST, THE EXPLORE THE CONCEPTS OF AND, OR, NOT: IDENTIFY ATTRIBUTES OF SHAPE, COLOR, AND SIZE	SUNBURST	AP,IB	EG,SI,TU		Y	Y	Y		59.00
FACT SHEETS PRODUCE WORKSHEETS ON FACTS FOR THE FOUR BASIC ARITHMETIC OPERATIONS	HARTLEY	AP	DP,IM	Y	Y	Y		Y	49.95
GEARS PREDICTING RESULTS AND PROBLEM-SOLVING WITH GEARS AND ROTATIONS: SCIENTIFIC METHOD	SUNBURST	AP,IB,JR,TC	DP,EG,PS,SI		Y	Y	Y		59.00
GETTING READY TO READ AND ADD DRILL IN LETTER, NUMBER, AND SHAPE RECOGNITION	SUNBURST	AP,AT,CO,IB,JR	DP,EG	Y					59.00
GO TO THE HEAD OF THE CLASS PLAYERS ADVANCE BY GIVING CORRECT ANSWERS TO GENERAL KNOWLEDGE QUESTIONS	MED MAT	AP,IB,JR	EG		Y	Y	Y		39.95
INTEGERS CHOOSE OPERANDS IN ADDITION OR SUBTRACTION	JMH	AP,CO	DP,TU		Y	Y			49.95
INTEPRETING GRAPHS PRACTICE MAKING MEANINGFUL INTERPRETATIONS OF GRAPHS OF PHYSICAL PHENOMENA	CONDUIT	AP	DE,DP,EG		Y	Y	Y		45.00
JUGGLES' RAINBOW REIMBORCE THE CONCEPTS OF LEFT AND RIGHT, ABOVE AND BELOW	TLC	AP,CO	DP,EG	Y					44.95
LEARNING ABOUT NUMBERS PRACTICE AND REIMFORCEMENT FOR COUNTING, SIMPLE TIME TELLING, AND SIMPLE ARITHMETIC	C & C	AP	DP,EG	Y	Y				50.00
MAGIC CASH REGISTER CASH REGISTER SIMULATION HELPS STUDENTS PRACTICE PLACE VALUE WITH DECIMALS	AVANT GARD	AP	DP,SI	Y	Y				34.95
MASTERING MATH DIAGNOSTIC SYSTEM DETERMINES STUDENT PLACEMENT IN MECC MASTERING MATH SERIES	MECC	AP	TE	Y					29.00
MASTERING MATH WORKSHEET GENERATOR CREATES AND PRINTS WORKSHEETS	MECC	AP	DP,IM	Y				Y	39.00

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				P	E	M	S	T	
MATH ACTIVITIES COURSEWARE LV.1-8 SERIES OF PROGRAMS IN A GAME FORMAT TO REIMFORCE SKILLS IN MATHEMATICS	HOUGHTON	AP	DP,EG	Y	Y	Y			1320.00*
MATH ASSISTANT I-II ASSESSMENT TOOL TO DIAGNOSE AND IMPROVE MATH ACHIEVEMENT: EDITING OPTION	SCHOLASTIC	AP	DP,IM,TE	Y	Y				129.95*
MATH BLASTER SIX HUNDRED PROBLEMS IN THE FOUR BASIC ARITHMETIC OPERATIONS	DAVIDSON	AP,AT,CO,IB,MC	DP,EG	Y	Y				49.95
MATH CONCEPTS I-II PRACTICE ON CONCEPTS RANGING FROM BEFORE/AFTER TO ROUNDING DECIMALS	HARTLEY	AP,IB	DP	Y	Y				39.95*
MATH MAZE PRACTICE BASIC MATH SKILLS BY MANEUVERING A FLY THROUGH A MAZE	DESIGNWARE	AP,AT,CO,IB	EG	Y	Y				39.95
MATH RABBIT PRACTICE EARLY NUMBER CONCEPTS IN A SERIES OF FOUR GAMES	TLC	AP	DP,EG	Y					54.95
MATH SEQUENCES, REVISED NUMBER READINESS AND FOUR ARITHMETIC OPERATIONS WITH INTEGERS, FRACTIONS, AND DECIMALS	MILLIKEN	AP,AT	DP	Y	Y	Y			495.00*
MATH SHOP HELP THE PROPRIETORS WITH MATH PROBLEMS: INVENTORY, SALES, ETC.	SCHOLASTIC	AP,IB	EG,SI			Y			104.95
MATH WORKSHEET GENERATOR GENERATE MATH PROBLEMS AT RANDOM USING TEACHER-DETERMINED PARAMETERS	ED'L ACTV	AP,TR	DP,IM	Y	Y	Y	Y	Y	59.95
MATHJ-TEMATIQUE FRENCH VERSION OF 'MATHEMATICS ACTIVITIES COURSEWARE LV.3-6'	HOUGHTON	AP	DP,EG	Y	Y				824.00*
MATH: SOLVING STORY PROBLEMS LV.3-8 PRESENTS POLYA'S PROBLEM-SOLVING MODEL AND PRACTICE SOLVING STORY PROBLEMS	HOUGHTON	AP,IB	DP,PS	Y	Y	Y			1746.00*
MEANING OF FRACTIONS DEMONSTRATE FRACTIONS: STUDENTS PRACTICE WRITING FRACTIONS FROM DESCRIPTIONS	CAE	AP	DE,DP,TU	Y	Y				44.95
METEOR MISSION MULTIPLICATION GAME: USER CAN CREATE NEW GAME CONTENT	DLM	AP	DP,EG,IM,SH	Y	Y	Y			44.00
METEOR MULTIPLICATION PRACTICE SKILLS IN MULTIPLYING WHOLE NUMBERS: ARCADE GAME FORMAT WITH VARIABLE SPEED	DLM	AC,AP,AT,CO,IB	DP,EG		Y				44.00
MICROS FOR MICROS-ESTIMATION THREE ESTIMATION ACTIVITIES ALLOW STUDENTS TO MAKE GUESSES AND CHECK THEIR RESULTS	LAWR HALL	AP	DP,EG	Y	Y				34.95
MINUS MISSION PRACTICE SUBTRACTION OF WHOLE NUMBERS IN ARCADE GAME FORMAT WITH VARIABLE SPEED	DLM	AC,AP,AT,CO,IB	DP,EG	Y	Y				44.00
MONEY GRAPHIC PRESENTATIONS INTRODUCE AND DRILL ON MONEY CONCEPTS	GAMCO	AP,CO,TR	DP,TU	Y	Y				54.95
MONEY! MONEY! INSTRUCTION AND PRACTICE IN COUNTING MONEY AND MAKING CHANGE: EDITING OPTION	HARTLEY	AP	DP,TU	Y	Y				39.95
MONSTER MATH CORRECT ANSWERS TO BASIC MATH PROBLEMS CAUSE A MONSTER TO DISAPPEAR FROM THE SCREEN	IBM	IB	DP,EG	Y	Y				28.00
MULTIPLICATION PUZZLES DRILL ON MULTIPLICATION FACTS AND REGROUPING	MECC	AP	DP	Y	Y				49.00

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NUMBER FARM	DLM	AP,CO,IB,JR	DP,EG	Y					29.95
SIX MUSICAL GAMES USE ANIMATION TO PRESENT AND REINFORCE COUNTING AND NUMBER CONCEPTS									
NUMBER MUNCHERS	MECC	AP	DP,EG		Y	Y	Y		49.00
PRACTICE NUMBER CONCEPTS IN A GAME FORMAT									
NUMBER STUMPER	TLC	AP,IB,JR	DP,EG	Y	Y				39.95
PRACTICE ADDITION AND SUBTRACTION OF WHOLE NUMBERS (1 - 12) IN A GAME FORMAT									
PICTURE PARTS	SCOTT FORS	AP,AT	DP,EG	Y					29.95
ADDITION, SUBTRACTION, AND MULTIPLICATION PRACTICE WITH ANIMATED GRAPHICS AND SOUND									
PIECE OF CAKE MATH	SPRINGBOARD	AP,CO,IB	EG	Y	Y				29.95
FIVE GAMES SET IN A BAKERY: BASIC SKILLS PRACTICE									
POND, THE	SUNBURST	AP,CO,IB,JR,TC	EG,PS	Y	Y	Y			59.00
PROBLEM-SOLVING GAME INVOLVING PATTERN ANALYSIS									
PROBLEM-SOLVING COMP CRSWR LV. K-8	MCGRAW HILL	AP	EG,PS	Y	Y	Y			719.55*
PROBLEM-SOLVING IN AN ADVENTURE GAME FORMAT									
PROBLEM-SOLVING STRATEGIES	MECC	AP	PS,TU			Y	Y		49.00
TWO INTERACTIVE TUTORIALS INTRODUCE STRATEGIES OF TRIAL-AND-ERROR AND EXHAUSTIVE LISTING									
PUZZLE TANKS	SUNBURST	AP,CO,IB,JR,TR	EG,PS		Y	Y	Y	Y	59.00
PRACTICE MATH AND LOGIC SKILLS BY FILLING A TANK FROM A NUMBER OF SMALLER TANKS									
QUOTIENT QUEST	MECC	AP	DP		Y				49.00
DRILL ON DIVISION FACTS WITH DIVIDENDS OF UP TO FOUR DIGITS									
RATIO AND PROPORTION	ED'L ACTV	AP	DP,TU		Y	Y	Y		59.95
STEP-BY-STEP TUTORIAL WITH GRAPHIC EXAMPLES, REVIEWS, AND QUIZZES									
RATIO AND PROPORTION	JMH	AP,CO	DP,TU		Y	Y			49.95
TUTORIAL AND PRACTICE IN RATIOS AND PROPORTIONS									
READ AND SOLVE MATH PROBLEMS #1	ED'L ACTV	AP,AT,CO,IB,TR	DP,TU			Y	Y		99.95
PRACTICE CONVERSION OF WORD PROBLEMS TO NUMBER PROBLEMS									
READ AND SOLVE MATH PROBLEMS #2	ED'L ACTV	AP,CO,IBMJR,TR	DP,TU		Y				99.95
PRACTICE LINEAR CONVERSION OF WRITTEN PROBLEMS TO NUMBER PROBLEMS									
SALINA MATH GAMES	ED'L ACTV	AP,TR	DP,EG	Y	Y	Y	Y		159.00*
COVER FOUR OPERATIONS WITH WHOLE NUMBERS, FRACTIONS, DECIMALS, AND PERCENTS									
SEMCALC	SUNBURST	AP,TR	DP,PS,TU			Y	Y		95.00
A TOOL TO DEVELOP STRATEGIES FOR INTERPRETING WORD PROBLEMS IN MATHEMATICS									
SOUTH DAKOTA	ED'L ACTV	AP	PS,SI		Y	Y	Y		63.00
USER MUST EMPLOY MATH SKILLS AND CRITICAL THINKING IN A SIMULATION OF FARM ECONOMICS									
SPACE SUBTRACTION	MECC	AP	DP,EG	Y					49.00
DRILL ON SUBTRACTION FACTS AND PROBLEMS WITHOUT REGROUPING									
SPEEDWAY MATH	MECC	AP	DP	Y	Y				49.00
BASIC SKILLS QUESTIONS IN A RACE CAR FORMAT									
STAR MAZE	SCOTT FORS	AP,AT	DP,EG	Y	Y				29.95
DRILL AND PRACTICE ON DIVISION OF WHOLE NUMBERS									

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STICKYBEAR MATH 1	WEEK READ	AP,CO,IB	DP,EG	Y					39.95
ACTIVITIES TO INTRODUCE AND REINFORCE BASIC MATH CONCEPTS AND SKILLS									
STICKYBEAR MATH 2	WEEK READ	AP	DP	Y	Y				39.95
PRACTICE IN MULTIPLICATION AND DIVISION OF WHOLE NUMBERS									
STICKYBEAR NUMBERS	WEEK READ	AP,AT,CO,IB	DP,EG	Y					39.95
SIMPLE PRACTICE 1-TO-1 CORRESPONDENCE PRESENTATION OF NUMBERS FROM 1 TO 10									
SUBTRACTING DECIMALS	MIC WRKSHP	AP,AT,CO,IB,TR	DP		Y	Y			29.95
PRACTICE SUBTRACTION WITH DECIMAL NUMBERS									
SUCCESS WITH MATH: MULT FRACTIONS	CBS	AP,CO,IB,JR	DP		Y	Y			29.95
STEP-BY-STEP PRACTICE IN MULTIPLYING FRACTIONS									
SURROUNDING PATTERNS	STRAWBERRY H	AP,CO	CA,PS	Y	Y				55.00
DEVELOP SHAPES AND PATTERNS IN DIFFERENT COLORS									
SURVIVAL MATH	SUNBURST	AP,CO,TR	DP,EG,PS,SI		Y	Y	Y		59.00
INCLUDES 'HOT DOG STAND'; AN ECONOMIC SIMULATION OF SALES AT A BALL GAME									
SWEET SHOP	DC HEATH	AP,CO	DP,EG	Y					45.00
THREE GAMES FOR SINGLE-DIGIT ADDITION, SUBTRACTION, AND COUNTING									
TEASERS BY TOBBS	SUNBURST	AP,CO,IB,TC,TR	DP,EG,PS		Y	Y	Y		59.00
TWO PROGRAMS TO PRACTICE LOGICAL WAYS TO SOLVE ADDITION AND MULTIPLICATION PROBLEMS									
TEDDY'S PLAYGROUND	SUNBURST	AP	DP,EG	Y					59.00
PRACTICE IN VISUAL DISCRIMINATION AND ANALOGIES									
UNDERSTANDING WORD PROBLEMS	SVE	AP	DP,EG,TU		Y				239.00
MULTI-MEDIA APPROACH WITH FILMSTRIPS, SKILL SHEETS, AND COMPUTER PROGRAMS									
USING A CALENDAR	HARTLEY	AP	DP,TU	Y	Y				39.95
ILLUSTRATIONS, INFORMATION, AND QUESTIONS ABOUT CALENDARS, MONTHS, AND HOLIDAYS									
VOYAGE MIMI: SHALES AND ENVIRONMENT	HOLT R&W	AP	IF,PS,SI		Y	Y			370.50
PROBE KIT FOR MEASURING TEMPERATURE, LIGHT, AND SOUND: INCLUDES 'BANK STREET LAB'									
WHATSIT CORPORATION	SUNBURST	AP,CO,IB,TR	DP,EG,SI		Y	Y	Y		59.00
STUDENTS USE MATH SKILLS TO MAKE GROUP DECISIONS TO OPERATE COMPETITIVE BUSINESSES									
WORDMATH 1-2	MILLIKEN	AP	DP,TU		Y	Y			295.00
INSTRUCTION AND PRACTICE IN STRATEGIES FOR SOLVING WORD PROBLEMS									

MATHEMATICS - PROBLEM SOLVING

SEE PROBLEM SOLVING/LOGIC' SECTION

MUSIC

TITLE	PUBLISHER	COMPUTERS	MODES	P	E	M	S	T	PRICE
BANK STREET MUSICWRITER	MINDSCAPE	AP,AT,CO,IB,JR	CA	Y					49.95
CREATE MUSIC AND PRINT A HARD COPY OF THE SCORE									
DOREMI	TEMPORAL	AP	DP,TU		Y	Y	Y		75.00
TRAINING IN AURAL IDENTIFICATION OF INTERVALS OF THE MAJOR SCALES: REQUIRES DAC BOARD									

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS					PRICE
				P	E	M	S	T	
HARMONIOUS DICTATOR PRACTICE RECOGNITION OF CHORD PROGRESSIONS; REQUIRES DAC BOARD	TEMPORAL	AP	DP				Y		150.00
JAZZ DICTATOR PRACTICE AURAL IDENTIFICATION OF CHORD PROGRESSIONS IN A JAZZ STYLE; REQUIRES DAC BOARD	TEMPORAL	AP	DP		Y	Y	Y	Y	150.00
MAGIC PIANO CREATE ORIGINAL MUSIC; TWO DRILLS ON MUSIC CONCEPTS	EDUSOFT	AP	CA,DP		Y	Y	Y		49.95
MODE DRILLS VISUAL AND AURAL DRILL ON MODES; REQUIRES DAC BOARD	TEMPORAL	AP	DP		Y	Y	Y		70.00
MUSIC CARD MC1 SERIES OF PROGRAM SDEMONSTRATING THE CAPABILITIES OF THE ALF MUSIC BOARD; INCLUDES ALF BOARD	ALF PRODUCTS	AP	CA	Y	Y	Y	Y	Y	75.00
MUSIC CONSTRUCTION SET USE ICONS TO CREATE, EDIT, AND RECORD MUSIC; USES MUSICAL NOTATION	ELECTR ART	AP,AT,CO,IB	CA	Y	Y	Y	Y		14.95
MUSIC FUNDAMENTALS; BEG MUSIC I INTRODUCTION TO READING AND PLAYING MELODY	SILVER	AP	TU		Y	Y	Y		39.95
MUSIC FUNDAMENTALS; BEG MUSIC III INSTRUCTION AND PRACTICE IN EXTENDING RHYTHM AND MELODY SKILLS	SILVER	AP	TU		Y	Y	Y		39.95
MUSIC THEORY EIGHTEEN PROGRAMS TO DRILL ON TERMS, NOTATION, RHYTHM, PITCH, CHORDS, AND SCALES	MECC	AP	DP		Y	Y	Y		49.00
MUSIC; PITCH DRILL ON AURAL AND VISUAL RECOGNITION OF INTERVALS; COMPARE AUDIBLE AND VISUAL MELODIC PATTERNS	MECC	CO	DP		Y	Y	Y		29.00
MUSIC; RHYTHM DRILL ON RHYTHMIC FUNDAMENTALS; MATCH AUDIBLE AND VISUAL MELODIC PATTERNS	MECC	CO	DP		Y	Y	Y		29.00
MUSIC; SCALES AND CHORDS DRILL ON MELODY AND HARMONY; RECOGNIZE MAJOR AND MINOR SCALES	MECC	CO	DP		Y	Y	Y		29.00
MUSIC; TERMS AND NOTATIONS DRILL ON IDENTIFYING MUSICAL SYMBOLS, ENHARMONIC EQUIVALENTS, AND KEY SIGNATURES	MECC	CO	DP		Y	Y	Y		29.00
MUSICWORKS CREATE, PLAY, AND PRINT MUSICAL SCORES	SPINNAKER	MC	CA		Y	Y	Y	Y	49.95
NOTABLE PHANTOM ARCADE GAME BASED ON PIANO NOTES	DESIGNWARE	AP,CO,IB,JR	EG		Y	Y	Y		19.95
POLYWRITER NOTES ON A MIDI KEYBOARD ARE TRANSLATED INTO MUSICAL NOTATION; CAN DISPLAY, EDIT, AND PRINT	PASSPORT	AP	CA		Y	Y	Y	Y	299.95
PRACTICAL MUSIC THEORY TEXTBOOK, DISK, AND WORKBOOK DESIGNED TO INTRODUCE BASIC MUSIC THEORY	ALFRED PUB	AP,CO	DP			Y	Y		199.95
SEBASTIAN II PRACTICE IDENTIFYING ERRORS BETWEEN WHAT IS PLAYED AND THE MUSICAL SCORE; REQUIRES DAC BOARD	TEMPORAL	AP	EG		Y	Y	Y	Y	125.00
SONGWRITER COMPOSE AND REWRITE COMPLICATED MELODIES WITHOUT USING MUSICAL NOTATION	MINDSCAPE	AP,AT,CO,IB	CA,DP		Y	Y	Y		9.95
TONEY LISTENS TO MUSIC TEN LEVELS OF DISCRIMINATION; TUNES, INTERVAL, TEMPO, RHYTHM, AND NOTATION, REQUIRES DAC BOARD	TEMPORAL	AP	DE,DP,EG		Y	Y	Y		90.00

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS P	E	M	S	T	PRICE
PRESCHOOL/EARLY CHILDHOOD									
ALPHABET CIRCUS SIX MUSICAL GAMES INTRODUCE AND REINFORCE LETTER RECOGNITION SKILLS	DLM	AP,CO,IB,JR	DP,EG	Y					29.95
ASTRO GROVER PRACTICE WITH EARLY NUMBER SKILLS USING SESAME STREET CHARACTERS	CBS	AP,CO	DP	Y					29.95
BAKE AND TASTE SIMULATE THE CREATION OF BAKED GOODS AND EVALUATE THEIR QUALITY	MINDPLAY	AP,IB	SI	Y	Y	Y	Y		39.99
BODY AWARENESS PRACTICE AND REINFORCEMENT ON POSITION OF BODY PARTS, PICTURE/WORD MATCH, AND SEASONAL CLOTHES	LEARN WELL	AP	DP,EG,SH	Y	Y	Y	Y		49.95
CHARLIE BROWN'S ABC'S THE PEANUTS GANG ASSISTS STUDENTS IN PRACTICING THE ABC'S	RANDOM	AP,CO,IB	DP,EG	Y	Y	Y	Y		39.95
CLOCK PRACTICE TELLING TIME AND CONVERTING DIGITAL TIME TO ANALOG TIME	HARTLEY	AP,IB	DP	Y					39.95
COMPARISON KITCHEN SIX GAMES REINFORCE PRE-READING AND MATH SKILLS OF VISUAL PERCEPTION AND DISCRIMINATION	DLM	AP	DP,EG	Y					29.95
COUNTERS THREE GAMES PROVIDE 1-TO-1 CORRESPONDENCE FOR COUNTING, ADDITION, AND SUBTRACTION	SUNBURST	AP	DP,EG,TU	Y					59.00
CREATURE CREATOR SELECT PARTS TO CREATE CREATURES AND DESIGN DANCE SEQUENCES FOR THEM	DESIGNWARE	AP,AT,CO,IB	CA,EG,PS	Y					29.95
DINOSAURS MATCH, SORT, OR COUNT DINOSAURS VISUALLY OR AUDITORILY; COVERS HABITATS AND FEEDING BEHAVIOR	ADV ID	AP,CO,IB,JR	DP,EG	Y					34.95
EARLY GAMES FOR YOUNG CHILDREN INTRODUCE SHAPES, LETTERS, DRAWING, ADDITION, AND SUBTRACTION	SPRINGBOARD	AP,CO,IB,JR,MC	DP,EG	Y					34.95
EASY AS ABC FIVE GAMES REINFORCE UPPERCASE/LOWERCASE LETTER RECOGNITION AND ALPHABETICAL ORDER	SPRINGBOARD	AP,CO,IB,JR	DP,EG	Y					39.95
PACEMAKER ENCOURAGE MEMORY AND CREATIVE SKILLS BY CREATING AND REMEMBERING FACIAL FEATURES	SPINNAKER	AM,AP	CA,DP,EG	Y	Y				39.95
FIRST-LETTER FUN PRACTICE MATCHING BEGINNING SOUNDS OF WORDS WITH THE LETTERS THAT MAKE THOSE SOUNDS	MECC	AP	DP,EG	Y					49.00
FISH SCALES SIX GAMES TO PRACTICE MEASUREMENT SKILLS WITH MUSIC AND GRAPHICS	DLM	AP	DP,EG	Y					29.95
FUN FROM A TO Z DISCRIMINATE AMONG LETTERS AND MATCH UPPERCASE/LOWERCASE LETTERS	MECC	AP	CA,EG	Y					49.00
GERTRUDE'S PUZZLES SOLVE PUZZLES INVOLVING RECOGNITION OF COLOR AND SHAPE PATTERNS	TLC	AP,CO,IB	EG,PS	Y	Y	Y			59.95
GERTRUDE'S SECRETS DEVELOP CRITICAL THINKING SKILLS BY FINDING PATTERNS IN SHAPES AND COLORS	TLC	AP,CO,IB	EG,PS	Y	Y				59.95

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GETTING READY TO READ AND ADD DRILL IN LETTER, NUMBER, AND SHAPE RECOGNITION	SUNBURST	AP,AT,CO,IB,JR	DP,EG	Y					59.00
HOW TO WEIGH AN ELEPHANT GAMES PRESENT CONCEPTS OF WEIGHT/MASS/VOLUME, ORDERING/SEQUENCING, AND OBSERVATION/PREDICTION	LRNG TECH	AP,CO	DP,EG,PS	Y	Y				19.95
JUGGLES'RAINBOW REINFORCE THE CONCEPTS OF LEFT AND RIGHT, ABOVE AND BELOW	TLC	AP,CO	DP,EG	Y					44.95
STICKYBEAR PRINTER ALLOWS TING IN VARIOUS FORMATS ON BOTH MONOCHROME AND COLOR PRINTERS	WEEK READ	AP	CA	Y	Y	Y			39.95
STICKYBEAR READING ANIMATED ACTIVITIES INTRODUCE AND REINFORCE EARLY READING SKILLS	WEEK READ	AP,CO,IB	DP,EG	Y					39.95
STICKYBEAR SHAPES NAME, CHOOSE, AND FIND SHAPES IN THREE ANIMATED ACTIVITIES	WEEK READ	AP,AT,CO	DP,EG	Y					39.95
STICKYBEAR TOWN BUILDER MAP SKILLS INCLUDING COMPASS READING, DIRECTIONS, MAP READING, AND POSITIONING	WEEK READ	AP,CO	EG,SI	Y	Y				39.95
TONEY LISTENS TO MUSIC TEN LEVELS OF DISCRIMINATION; TUNES, INTERVAL, TEMPO, THYTHM, AND NOTATION; REQUIRES DAC BOARD	TEMPORAL	AP	DE,DP,EG		Y	Y	Y		90.00
TONK IN THE LAND OF BUDDY-BOTS PRACTICE VISUAL DISCRIMINATION OF SHAPES AND PATTERNS	MINDSCAPE	AP,AT,CO,IB,JR	EG,PS	Y	Y				29.95
WHO, WHAT, WHERE, WHEN, WHY DISTINGUISH AMONG WORDS AND THEIR CONCEPTS; EDITING OPTION	HARTLEY	AP,IB	DP	Y	Y				35.95
WORD SPINNER WORD-BUILDING GAME THAT INCREASES IN SPEED AS USER'S SKILL DEVELOPS; EDITING OPTION	TLC	AP,CO,IB	DP,EG	Y					34.95
ZANDAR THE WIZARD GAME TO PRACTICE CRITICAL THINKING AND DEDUCTIVE REASONING SKILLS	SVE	AP	EG,PS	Y	Y				99.00
KINDER KONCEPTS SERIES OF THIRTY PROGRAMS COVERING MATH AND READING READINESS SKILLS	MIDWEST	AP,CO,PE	DP,EG	Y					99.00
KINDERCOMP SIX GAMES TO HELP PREPARE CHILDREN TO READ, SPELL, AND COUNT	SPINNAKER	AP	DP,EG	Y	Y	Y	Y		39.95
LEARNING ABOUT NUMBERS PRACTICE AND REINFORCEMENT FOR COUNTING, SIMPLE TIME TELLING, AND SIMPLE ARITHMETIC	C & C SOFT	AP	DP,EG	Y	Y				50.00
LETTERS AND FIRST WORDS INTRODUCE LETTER RECOGNITION SKILLS AND SHORT WORDS; REINFORCE BASIC LANGUAGE SKILLS	C & C SOFT	AP	DP,EG	Y					60.00
LETTERS AND WORDS THREE GAMES; ALPHABETIZING, LETTER MATCHES, SIGHT WORDS; EDITING OPTION	LEARN WELL	AP,IB,JR	DP,EG,TU	Y					49.95
MEMORY CASTLE KNIGHT MUST COMPLETE A MISSION BY REMEMBERING AND PERFORMING TASKS FROM A GROWING LIST	SUNBURST	AP,CO,IB,JR,TC	EG,PS		Y	Y	Y		59.00
MEMORY MATCH CONCENTRATION-TYPE GAME TO ENHANCE MEMORY SKILLS	HARTLEY	AP,IB	DP,EG	Y					39.95
MEMORY; A FIRST STEP TEN PROGRAMS TO PRACTICE PROBLEM-SOLVING SKILLS; COMPUTER AND NON-COMPUTER ACTIVITIES	SUNBURST	AP,IB,JR	DP,PS	Y	Y				250.00*

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				P	E	M	S	T	
MICROS FOR MICROS-ESTIMATION	LAWR HALL	AP	DP,EG	Y	Y				34.95
THREE ESTIMATION ACTIVITIES ALLOW STUDENTS TO MAKE GUESSES AND CHECK THEIR RESULTS									
MOPTOWN PARADE	TLC	AP,CO,IB	EG,PS	Y	Y	Y			54.95
SEVEN GAMES TO PRACTICE LOGICAL THINKING, STRATEGY DEVELOPMENT, AND PATTERN RECOGNITION									
MUPPET WORD BOOK, THE	SUNBURST	AP	EG	Y					59.00
WORD CONCEPTS USING SESAME STREET CHARACTERS, MOUSE, TOUCH-WINDOW, OR 'MUPPET LEARNING KEYS'									
MUPPETS ON STAGE	SUNBURST	AP,CO,JR	CA,DP,EG	Y					59.00
THREE PROGRAMS TO REINFORCE LETTER, NUMBER, AND COLOR RECOGNITION; COMES WITH 'MUPPET LEARNING KEYS'									
NUMBER FARM	DLM	AP,CO,IB,JR	DP,EG	Y					29.95
SIX GAMES USE MUSIC AND ANIMATION TO REINFORCE COUNTING AND NUMBER CONCEPTS									
PAINT WITH WORDS	MECC	AP	CA,EG	Y					49.00
VOCABULARY DEVELOPMENT; DISPLAYS WORDS CHOSEN BY STUDENTS; OPTIONAL UFONICS SPEECH SYNTHESIZER									
PICTURE PARTS	SCOTT FORS	AP,AT	DP,EG	Y					29.95
ADDITION, SUBTRACTION, AND MULTIPLICATION PRACTICE WITH ANIMATED GRAPHICS AND SOUND									
PIECE OF CAKE MATH	SPRINGBOARD	AP,CO,IB	DP,EG	Y	Y				29.95
FIVE GAMES DRILL STUDENTS ON ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION									
RIGHT OF WAY	MECC	AP	DP,EG	Y					39.00
A FRIENDLY MONKEY PROVIDES PRACTICE IN TELLING RIGHT FROM LEFT									
STICKERS	SPRINGBOARD	AP,CO,IB	EG	Y	Y	Y	Y		34.95
ONE HUNDRED STICKER PICTURES HELP STUDENTS TO DISTINGUISH AND USE SHAPES									
STICKYBEAR ABC	WEEK READ	AP,AT,CO	DP,EG	Y					39.95
THREE GAMES; IDENTIFICATION, ORDER, AND MATCHING; WITH GRAPHICS AND SOUND									
STICKYBEAR NUMBERS	WEEK READ	AP,AT,CO,IB	DP,EG	Y					39.95
SIMPLE PIAGETIAN 1 -TO-1 CORRESPONDENCE PRESENTATION OF NUMBERS FROM 1 TO 10									
STICKYBEAR OPPOSITES	WEEK READ	AP,AT,CO	DP,EG,PS	Y					39.95
DISPLAY AND PRACTICE TO REINFORCE CONCEPTS OF FULL/EMPTY, UP/DOWN, IN FRONT OF/BEHIND									

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PROBLEM SOLVING/LOGIC

TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS P	E	M	S	T	PRICE
ADVENTURE CONSTRUCTION SET STUDENTS WRITE AND ILLUSTRATE THEIR OWN ADVERTURE GAMES	ELECTR ART	AM,AP,CO	CA,PS		Y	Y	Y		49.95
AGENT U.S.A. USE MAP SKILLS IN SIMULATED SPY MISSION ACROSS THE U.S.A.	SCHOLASTIC	AP,AT,CO,IB	EG,PS,SI		Y	Y	Y		39.95
ALL SORTS OF MEGGLES PRACTICE IN DECISION-MAKING SKILLS; TESTING AND RECORD KEEPING; REQUIRES UFONIC SYSTEM	JOSTENS	AP	DP,PS	Y	Y				149.00
ARROW DYNAMICS LOGIC GAME TO PRACTICE LOGICAL THOUGHT AND STRATEGY FORMATION	SUNBURST	AP	EG,PS		Y	Y			59.00
BAFFLES LOCATE HIDDEN BAGGLES BY OBSERVING HOW THEY REFLECT LIGHT RAYS	CONDUIT	AP	EG,PS,SI			Y	Y		50.00
BUMBLE GAMES INTRODUCES USE OF NUMBER PAIRS TO DESCRIBE POSITIONS IN AN ARRAY AND ON A GRID	TLC	AP,CO	DP,EG,PS	Y	Y				54.95
BUMBLE PLOT PRACTICE PLOTTING AND GRAPHING SKILLS. FROM +10 TO -10 ON COORDINATE GRID	TLC	AP,CO	DP,EG,PS		Y	Y			54.95
CAR BUILDER ALLOWS STUDENTS TO DESIGN AND REDESIGN CARS FOR DIFFERENT PURPOSES	WEEK READ	AP	PS,SI		Y	Y	Y		39.95
CHALLENGE MATH CALCULATIING AND ESTIMATING WITH WHOLE NUMBERS AND DECIMALS IN A GAME FORMAT	SUNBURST	AP,CO	DP,EG	Y	Y	Y			59.00
CHIPWITS PROGRAMMING LANGUAGE ALLOWS STUDENTS TO TRAIN A ROBOT TO WORK IN DIFFERENT ENVIRONMENTS	BRAINPOWER	AP,MC	CP,PS,SI			Y	Y		39.95
CODE QUEST DECODE CLUES TO DISCOVER OBJECTS; USE THE MINI-AUTHOR TO CREATE YOUR OWN OBJECTS	SUNBURST	AP,AT,CO,IB,TR	EG,PS		Y	Y			59.00
CREATIVE CONTRAPTIONS DESIGN AND TROUBLESHOOT RUBE-GOLDBERG CONTRAPTIONS BY APPLYING PROBLEM-SOLVING TECHNIQUES	BANTAM	AP,CO,IB	EG,PS	Y	Y	Y			39.95
DINOSAURS AND SQUIDS STRATEGIES FOR SOLVING PROBLEMS INVOLVING TWO VARIABLES	SCOTT FORS	AP	EG,PS		Y	Y			49.95
DISCOVERY LAB DESIGN AND CONDUCT EXPERIMENTS TO FIND THE BEST ENVIRONMENTAL CONDITIONS FOR ALIENS	MECC	AP	PS,SI		Y	Y	Y		49.00
DISCRIMINATION ATTRIBUTES AND RULES MULTI-DISK SET PRESENTS DISCRIMINATION AS PART OF THE PROBLEM-SOLVING PROCESS	SUNBURST	AP	EG,PS	Y	Y	Y			225.00
DRAGON'S KEEP SIMPLE PROBLEM-SOLVING ADVENTURE TO IMPROVE THINKING SKILLS	SIERRA	AP,CO	EG	Y	Y				29.95
ENCHANTED FOREST, THE IDENTIFY ATTRIBUTES OF SHAPE, COLOR, AND SIZE TO REINFORCE CONCEPTS; AND,OR,NOT	SUNBURST	AP,IB	EG,SI,TU		Y	Y	Y		59.00
EXPLORING TABLES AND GRAPHS I INTRODUCES THE USE OF GRAPHS; INCLUDES TOOL FOR CONSTRUCTING GRAPHS FROM A GIVEN SET OF DATE	WEEK READ	AP	EG,GG,TU		Y	Y	Y		34.95

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EXPLORING TABLES AND GRAPHS II	WEEK READ	AP	EG,GG,TU		Y	Y	Y		34.95
REAL-LIFTIONS OF TABLES AND GRAPHS; PICTURE, BAR, LINE, AND AREA GRAPHS									
EZ LOGO	MECC	AP	CP,PS,TU	Y	Y				49.00
TWO PROGRAMS INTRODUCE A SUBSET OF LOGO COMMANDS; SEPARATE LOGO NOT REQUIRED									
FACTORY, THE	SUNBURST	AP,AT,CO,IB,TC	EG,PS,SI		Y	Y	Y		59.00
VISUAL DISCRIMINATION, SPATIAL PERCEPTION, AND LOGIC SKILLS; DEVELOP SEQUENCE AND ORDER SKILLS									
FRIENDLY COMPUTER, THE	MECC	AP,CO,IB	CP,PS,TU	Y					49.00
SEQUENCE OF FIVE GRADED PROGRAMS TO INTRODUCE THE COMPUTER AND KEYBOARD									
FUN HOUSE MAZE	SUNBURST	AP	EG,PS		Y	Y	Y		59.00
PRACTICE IN PATTERN RECOGNITION AND IDENTIFYING MULTIPLE SOLUTIONS WITH 3-DIMENSIONAL MAZES									
GAME SHOW, THE	ADV ID	AP,CO,IB,JR	EG,IM,SH	Y	Y	Y	Y	Y	39.95
PASSWORD FORMAT GAME; USERS MAY ADD THEIR OWN SUBJECTS AND WORDS									
GEARS	SUNBURST	AP,IB,JR,TC	DP,EG,PS,SI		Y	Y	Y		59.00
PROBLEM-SOLVING WITHIN A STRUCTURE OF GEARS AND ROTATIONS; PREDICTING RESULTS; SCIENTIFIC METHOD									
GERTRUDE'S PUZZLES	TLC	AP,CO,IB	EG,PS	Y	Y	Y			59.95
STUDENTS SOLVE PUZZLES INVOLVING RECOGNITION OF COLOR AND SHAPE PATTERNS									
GERTRUDE'S SECRETS	TLC	AP,CO,IB	EG,PS	Y	Y				59.95
STUDENTS EXPLORE CRITICAL THINKING SKILLS AS THEY FIND PATTERNS IN SHAPES AND COLORS									
GLIDEPATH	HRM SOFTWR	AP	EG,SI			Y	Y		59.00
SIMULATES FLIGHT OF GLIDER OVER IMAGINARY TERRAIN									
GNEE OR NOT GNEE	SUNBURST	AP,CO,IB,TC	EG,PS	Y	Y				59.00
GAME TO DEVELOP VISUAL DISCRIMINATION AND RULE FORMATION BASED ON ATTRIBUTES									
GRIMBLE PROCESS, THE	JOSTENS	AP	EG,PS,TU	Y	Y				149.00
GAME WITH SPEECH CAPABILITY TO REINFORCE LOGICAL THINKING AND PROGRAMMING SKILLS; REQ.UFONIC SYSTEM									
HIGH WIRE LOGIC	SUNBURST	AP,IB,JR	EG,PS		Y	Y	Y		59.00
LANGUAGE-BASED CRITICAL THINKING GAME FOR DEVELOPING BOOLEAN LOGIC SKILLS									
HOT DOG STAND	SUNBURST	IB,JR,TC	EG,SI	Y	Y	Y			59.00
ECONOMIC SIMULATION OF OPERATING A HOT DOG STAND AT A FOOTBALL GAME									
HOW TO WEIGH AN ELEPHANT	LRNG TECH	AP,CO	DP,EG,PS	Y	Y				19.95
GAME APPROACH TO CONCEPTS OF WEIGHT/MASS/VOLUME, ORDERING/SEQUENCING, AND OBSERVATION/PREDICTION									
IGGY'S GNEES	SUNBURST	AP,CO	PS	Y	Y				59.00
PRACTICE DISCRIMINATION STRATEGIES TO SOLVE INCREASINGLY COMPLEX PROBLEMS									
INCREDIBLE LABORATORY, THE	SUNBURST	AP,AT,CO,IB,TC	EG,PS,SI		Y	Y	Y		59.00
DESIGN EXPERIMENTS TO DETERMINE THE COMBINATION OF CHEMICALS NEEDED TO PRODUCE EACH MONSTER									
JACK AND THE BEANSTALK	HRM SOFTWR	AP	PS	Y	Y	Y			49.00
USE THE ORIGINAL FOLKTALE TO PRACTICE SKILLS IN PLANNING, DEDUCTIVE REASONING, AND SEQUENCING									
JUGGLES' RAINBOW	TLC	AP,CO	DP,EG	Y					44.95
REINFORCES THE CONCEPTS OF LEFT AND RIGHT, ABOVE AND BELOW									

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KERMIT'S ELECTRONIC STORYMAKER USERS CREATE STORIES WITH MUPPETS AS CHARACTERS; DEVELOP VOCABULARY AND SENTENCE STRUCTURE	SIMON & SCHU	AP,CO	DP	Y					34.95
KING'S RULE, THE FORM HYPOTHESES, RECOGNIZE NUMERICAL PATTERNS, AND TEST LOGIC SKILLS	SUNBURST	AP,CO,IB,TC,TR	EG,PS		Y	Y	Y		59.00
LEARNING THROUGH LOGO BEGINNING LOGO COMMANDS AND PROCEDURES; ACTIVITY CARDS; REQUIRES APPLE LOGO	SUNBURST	AP	CP,PS		Y	Y	Y		55.00
LOGIC BUILDERS SERIES OF CHALLENGES TO IMPROVE MEMORY SKILLS	SCHOLASTIC	AP,CO	PS		Y	Y	Y		39.95
MEMORY CASTLE A KNIGHT MUST REMEMBER AND PERFORM AN INCREASING LIST OF TASKS TO COMPLETE A MISSION	SUNBURST	AP,CO,IB,JR,TC	EG,PS		Y	Y	Y		59.00
MEMORY; A FIRST STEP PUPPET DEFINES AND INTRODUCES SEQUENTIAL PROBLEM-SOLVING SKILLS; INCLUDES NON-COMPUTER ACTIVITIES	SUNBURST	AP,IB,JR	DP,PS	Y	Y				250.00*
MIND PUZZLES A SET OF GRADUATED PUZZLES AND TOOLS TO PRACTICE PROBLEM-SOLVING STRATEGIES	MECC	AP	PS,SI			Y	Y		49.00
MINDSTRETCHER SERIES TEN LOGIC PUZZLES IN GAME FORMATS; FOR ONE OR MORE PLAYERS	ISL SOFTWR	AP,CO,PE	EG,PS		Y	Y	Y		100.00*
MIRRORS ON THE MIND-INTRO TO PROGRADD WES USES LOGO-LIKE LANGUAGE TO PROVIDE EXPERIENCE WITH A PROGRAMMING LANGUAGE		AP	CP		Y	Y	Y		49.95
MIRRORS ON THE MIND-STATISTICS FOR ESTIMATING MEAN AND STANDARD DEVIATION, CORRELATION COEFFICIENT FROM GRAPH OR PLOT	ADD WES	AP	DE,PS		Y	Y	Y		49.95
MIRRORS ON THE MIND-STRATEGIES PROBABILISTIC GAMES FOR USERS TO DEFINE STRATEGIES, TEST HYPOTHESES, AND REFINE CONJECTURES	ADD WES	AP	EG,PS		Y	Y	Y		49.95
MOPTOWN HOTEL • USERS IDENTIFY ATTRIBUTE PATTERNS OF BIBBETS AND GRIBBETS IN THIS COMPETITIVE LOGIC GAME	TLC	AP,CO,IB	EG,PS		Y	Y	Y		54.95
MOPTOWN PARADE SEVEN GAMES TO PRACTICE LOGICAL THINKING, STRATEGY DEVELOPMENT, AND PATTERN RECOGNITION	TLC	AP,CO,IB	EG,PS	Y	Y	Y			54.95
ODD ONE OUT PRACTICE IN CLASSIFICATION SKILLS; COLOR GRAPHICS, ANIMATION, AND SOUND	SUNBURST	AP,CO	EG,PS	Y	Y				59.00
PINBALL CONSTRUCTION SET DESIGN AND CONSTRUCT PINBALL GAMES BY MANIPULATING COMPONENTS ON TEH SCREEN	ELECTR ART	AP,AT,CO,IB,MC	AU,EG		Y	Y	Y		14.95
PLANETARY CONSTRUCTION SET TWO ACTIVITIES HAVE STUDENTS EXPERIMENT, EXPLORE, AND CREATE PLANETS FOR SPECIFIC LIFE FORMS	SUNBURST	AP,IB	EG,PS			Y	Y		59.00
POND, THE PROBLEM-SOLVING GAME INVOLVING PATTERN ANALYSIS	SUNBURST	AP,CO,IB,JR,TC	EG,PS	Y	Y	Y			59.00
PROBLEM-SOLVING STRATEGIES STRATEGIES OF TRIAL AND ERROR, EXHAUSTIVE LISTING, AND PROBLEM-SOLVING	MECC	AP	PS,TU			Y	Y		49.00
PUZZLE TANKS PRACTICE MATH AND LOGIC SKILLS BY FILLING A TANK FROM A NUMBER OF SMALLER TANKS	SUNBURST	AP,CO,IB,JR,TR	EG,PS		Y	Y	Y		59.00
RIGHT TURN, THE ROTATE AND FLIP FIGURES ON A THREE-DIMENSIONAL GRID	SUNBURST	AP,CO,IB,JR	EG,PS		Y	Y	Y		59.00

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS					PRICE
				P	E	M	S	T	
ROBOT ODYSSEY DESIGN A ROBOT TO ESCAPE FROM A MAZE; USE WIRES, LOGIC GATES, AND CIRCUIT CHIPS	TLC	AP,IB	CA,CP,EG,PS		Y	Y	Y		49.95
ROBOT PROBE LAND ON A STRANGE PLANET; THEN USE SIMPLE COMMANDS TO PROGRAM A ROBOT TO PICK UP FOUR PROBES	SUNBURST	TR	CP,EG,PS,SI	Y	Y	Y			59.00
ROCKY'S BOOTS DESIGN CIRCUIT TO RECOGNIZE SPECIFIC ATTRIBUTES; USES AND, NOT, OR, FLIP-FLOP GATES	TLC	AP,CO,IB	EG,PS,SI,TU		Y	Y	Y		64.95
ROYAL RULES FORM AND TEST HYPOTHESES, RECOGNIZE NUMERICAL PATTERNS, AND DEVELOP PROBLEM-SOLVING SKILLS	SUNBURST	AP,IB	PS		Y	Y	Y		69.00
SAFARI SEARCH IMPROVE PROBLEM-SOLVING AND INFERENCE SKILLS THROUGH A SERIES OF CHALLENGING ACTIVITIES	SUNBURST	AP,IB	EG,PS		Y	Y	Y		59.00
SEMCALC TOOL TO DEVELOP STRATEGIES FOR INTERPRETING WORD PROBLEMS IN MATHEMATICS	SUNBURST	AP,TR	DP,PS,TU				Y	Y	95.00
SPECTRUM; PATTERNS AND PROGRAMS LOGIC GAME USING HIDDEN PATTERN OF COLORED BARS; INTRODUCES FUNDAMENTAL PROGRAMMING SKILLS	SUNBURST	AP	EG,PS			Y	Y		59.00
STICKYBEAR OPPOSITES CONCEPTS OF FULL/EMPTY, UP/DOWN, IN FRONT OF/BEHIND	WEEK READ	AP,AT,CO	DP,EG,PS	Y					39.95
STICKYBEAR SHAPES IDENTIFYING, CHOOSING, AND NAMING SHAPES; FIGURE-GROUND RELATIONSHIPS	WEEK READ	AP,AT,CO	DP,EG	Y					39.95
STICKYBEAR TOWN BUILDER PRACTICE MAP SKILLS WHILE BUILDING UP TO TWENTY TOWNS	WEEK READ	AP,CO	DP,EG	Y					39.95
STORY TREE THREE INTERACTIVE STORIES, WITH WORD PROCESSING FOR CREATING ORIGINAL BRANCHING STORIES	SCHOLASTIC	AP,CO,IB	CA,SH,WP	Y	Y	Y	Y		59.95
SUPER FACTORY, THE 3-D SPATIAL GEOMETRY PROGRAM VERSION OF "THE FACTORY" TO EXPERIMENT WITH DESIGNS ON A CUBE	SUNBURST	AP,IB,JR	CA,CP,PS,SI		Y	Y	Y		59.00
TEASERS BY TOBBS TWO PROGRAMS TO PRACTICE LOGICAL WAYS TO SOLVE ADDITION AND MULTIPLICATION PROBLEMS	SUNBURST	AP,CO,IB,TC,TR	DP,EG,PS		Y	Y	Y		59.00
TEN CLUES MINI-AUTHORING PROGRAM STRESSING CRITICAL VS. VARISABLE ATTRIBUTES	SUNBURST	AP	PS,SH		Y	Y	Y		59.00
TIC TAC SHOW MINI-AUTHORING SYSTEM ALLOWS TEACHERS OR STUDENTS TO DEVELOP HOLLYWOOD SQUARES GAMES	ADV ID	AP,CO,IB,JR	AU,EG,IM,SH	Y	Y	Y	Y		39.95
TIN 'N FLIP DISCRIMINATION SKILLS; SIMILARITIES AND DIFFERENCES IN PATTERNS AND ORIENTATIONS	SUNBURST	AP,IB	EG,PS		Y	Y	Y		59.00
TK; SOLVER FORMULA PROCESSOR/TOOL FOR TECHNICAL AND SCIENTIFIC APPLICATIONS	UNIVERSAL	AP,IBMJR,MC	PS				Y	Y	200.00
TONK IN THE LAND OF BUDDY-BOTS PRACTICE DISCRIMINATION OF PATTERNS, NOTING SIMILARITIES AND IDFFERENCES	MINDSCAPE	AP,AT,CO,IB,JR	EG,PS	Y	Y				29.95
TRADING POST GAME FOR TWO STUDENTS TO REINFORCE VISUAL DISCRIMINATION, RULE FORMATION, ANALYSIS, AND PLANNING	SUNBURST	AP,CO,IB,JR,TC	EG,PS	Y	Y	Y			59.00
TREASURE HUNT PRACTICE ENCYCLOPEDIC REFERENCE SKILLS USING BOTH PAPER AND COMPUTER FORMAT	GROLIER	AP	EG,PS		Y	Y			54.95

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS					PRICE
				P	E	M	S	T	
PROBLEM SOLVING/LOGIC									
TRIBBLES A PROBLEM-SOLVING TUTORIAL AND SIMULATION TO INTRODUCE THE SCIENTIFIC METHOD	CONDUIT	AP	EG,PS,SI,TU				Y		40.00
TRIVIA MACHINE TRIVIA GAME HELPS TO DEVELOP DATA BASE THINKING SKILLS AND KEYWORD SEARCHING SKILLS	MECC	AP	DB,EG,PS,SI		Y	Y	Y		49.00
VIDEOWORKS ENABLES USER TO DRAW AND ANIMATE OBJECTS TO CREATE FILMS; FULL EDITING BY FRAME	SPINNAKER	MC	CA,GG		Y	Y	Y	Y	99.95
WHAT'S MY LOGIC GAME PRACTICE IN PATTERN RECOGNITION	MIDWEST	AP,CO	EG,PS			Y	Y		49.95
WHATSIT CORPORATION USE MATHJ SKILLS TO MAKE GROUP DECISIONS TO OPERATE COMPETITIVE BUSINESSES	SUNBURST	AP,CO,IB,TR	DP,EG,SI		Y	Y	Y		59.00
WHERE IN WORLD IS CARMEN SAN DIEGO USING 'FODOR'S GUIDE TO THE USE,' SEARCH THE U.S.A. TO SOLVE A CRIME AND CAPTURE THE CRIMINAL	BRODERBUND	AP	EG,SI		Y	Y	Y		44.95
WHERE IN WORLD IS CARMEN SAN DIEGO USING 'THE WORLD ALMANAC,' SEARCH THE WORLD TO SOLVE A CRIME AND CAPTURE THE CRIMINAL	BRODERBUND	AP,CO,IB	EG,SI		Y	Y			39.95
WILDERNESS; SURVIVAL ADVENTURE STUDENTS ROLE PLAY AIR CRASH SURVIVOR IN WILDERNESS SETTING	ELECTR TRANS	AP,IB	EG,PS		Y	Y	Y		50.00
WIZARDRY; PROVING GRNDS/MTH OVERLOAD TRAIN THE EXPLORERS YOU TAKE ON THIS GRAPHICS ADVENTURE GAME; MANY DIFFICULTY LEVELS	SIRI-TECH	AP,IB,JR	EG,PS			Y	Y		49.95
WRITING ADVENTURE STUDENTS WRITE AND EDIT THEIR OWN STORIES; PROOFREADING FEATURE CHECKS FOR COMMON ERRORS	DLM	AP,CO	CA,WP	Y	Y	Y			59.95
ZANDAR III & IV; IN SEARCH GAME TO IMPROVE CRITICAL THINKING AND DEDUCTIVE REASONING SKILLS	SVE	AP	EG,PS	Y	Y				99.00*
ZANDAR THE WIZARD GAME TO IMPROVE CRITICAL THINKING AND DEDUCTIVE REASONING SKILLS	SVE	AP	EG,PS	Y	Y				99.00
ZORK I & II TEXT-ONLY ADVENTURE GAMES THAT REQUIRE READING, MAPPING, AND PROBLEM-SOLVING SKILLS	INFOCOM	AP,AT,CO,IB,TC	EG,PS		Y	Y	Y		44.95*
INSTRUCTIONAL TOOLS - GRAPHICS GENERATOR									
BLAZING PADDLES TOOL, FOR CREATING COMPUTER ART; INCLUDES GRAPHICS LIBRARY	BAUDVILLE	AP,AT,CO	CA,GG		Y	Y	Y	Y	49.95
CERTIFICATE MAKER ALLOWS DESIGN AND PRINTING OF PROFESSIONAL-LOOKING CERTIFICATES	SPRINGBOARD	AP,CO,IB	GG		Y	Y	Y	Y	49.95
CLIP ART COLLECTION V.1 COLLECTION OF GRAPHIC EMAGES FOR 'PRINT SHOP'	SPRINGBOARD	AP,GG,IB	GG		Y	Y	Y	Y	29.95
EASY GRAPH PRODUCE PICTOGRAPHS, PIE CHARTS, AND BAR GRAPHS; INCLUDES INSTRUCTIONAL MATERIALS	GROLIER	AP,CO,IB,JR	GG,TU	Y	Y	Y	Y		49.95

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE P	LEVELS E	M	S	T	PRICE
FANTAVISION	BRODERBUND	AP	CA,GG		Y	Y	Y	Y	49.95
'TWEENING' CREATES UP TO 64 ANIMATED SEQUENCES FOR EACH PICTURE DRAWN BY USER									
FONTRIX 1.5	DATA TRANSFM	AP	GG			Y	Y	Y	95.00
HIGH RESOLUTION GRAPHICS UTILITY PROGRAM									
FULLPAINT	ANN ARBR SWK	MC	CA,GG	Y	Y	Y	Y	Y	99.00
DRAWING PROGRAM SIMILAR TO 'MACPAINT' BUT WITH SCROLL BARS AND OTHER ADVANCED FEATURES									
KOALAPAINTER	PTI-KOALA	AP	CA,GG	Y	Y	Y			19.95
GRAPHICS USED WITH NURSERY RHYMES; REQUIRES KOALAPAD									
MACDRAFT	INNOVATIVE	MC	GG			Y	Y	Y	269.00
INTERACTIVE DRAFTING PROGRAM FOR 2- AND 3- DIMENSIONAL APPLICATIONS									
MACDRAW	APPLE	MC	CA,GG	Y	Y	Y	Y	Y	195.00
RAPHIC DEVELOPMENT TOOL TO CREATE STRUTURED GRAPHICS AND DRAWINGS									
MACPAINT	APPLE	MC	CA,GG	Y	Y	Y	Y	Y	125.00
GENERAL-PURPOSE GRAPHIC DEVELOPMENT TOOL									
MACPROJECT	APPLE	MC	GG				Y	Y	195.00
PROJECT PLANNER ALLOWS DISPLAY OF PROJECT FLOW AND RESPONSIBILITIES									
MICROSOFT CHART	MICROSOFT	MC	GG				Y	Y	125.00
BUSINESS GRAPHICS PACKAGE WORKS INTERACTIVELY OR WITH ;MULTIPLAN' DATA									
MOUSE PAINT	APPLE	AP	GG	Y	Y	Y			99.00
GRAPHICS GENERATION PROGRAM; INCLUDES MOUSE									
NEWSROOM	SCHOLASTIC	AP,CO,IB,JR	CA,GG,IM		Y	Y	Y	Y	59.95
DESKTOP PUBLISHING PROGRAM FOR FLYERS AND NEWSLETTERS; INCLUDES INSTRUCTIONAL SUPPORT MATERIALS									
NEWSROOM	SPRINGBOARD	AP,CO,IB	CA,GG,IM		Y	Y	Y	Y	59.95
DESKTOP PUBLISHING PROGRAM FOR FLYERS AND NEWSLETTERS									
PAGEMAKER	ALDUS	MC	GG			Y	Y	Y	495.00
FULL-FUNCTION 16-PAGE DESKTOP PUBLISHING SYSTEM ALLOWS USER TO FULLY FORMAT INDIVIDUAL PAGES									
PC STORYBOARD	IBM	IB	GG			Y	Y	Y	275.00
GRAPHICS PRESENTATION PROGRAM WITH ANIMATION AND SPECIAL EFFECTS									
PFS; GRAPH	SCHOLASTIC	AP,IB	GG				Y	Y	119.95
GENERATE PIE, BAR, AND LINE CHARTS FROM USER-ENTERED DATA OR FROM 'PFS; FILE' DATA									
PRINT SHOP	BRODERBUND	AP,AT,CO,IB	CA,GG	Y	Y	Y	Y	Y	49.95
CRATE SIGNS, POSTERS, GREETING CARDS, AND BANNERS; MANY CHOICES OF GRAPHICS AND FONTS									
PRINT SHOP COMPANION	BRODERBUND	AP	CA,GG	Y	Y	Y	Y	Y	39.95
CREATE GRAPHICS FOR USE WITH 'PRINT SHOP'									
PRINT SHOP GRAPHICS LIBRARY 3	BRODERBUND	AP	CA,GG		Y	Y	Y	Y	24.95
GRAPHICS FOR BUSINESS, INTERNATIONAL SYMBOLS, MYTHOLOGY, FANTASY, AND A ZOO OF ANIMALS									
PROFESSIONAL SIGN MAKER	SUNBURST	AP,IB	GG		Y	Y	Y	Y	59.00
PRODUCE GRAPHIC LETTERS FOR SIGNS, OVERHEAD TRANSPARENCIES, ETC.									

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TITLE	PUBLISHER	COMPUTERS	MODES	GRADE LEVELS P	E	M	S	T	PRICE
PUZZLES AND POSTERS DESIGN AND PRINT WORD SEARCHES, CROSSWORD PUZZLES, MAZES, AND POSTERS	MECC	AP,CO,IB,TR	CA,EG,GG,IM	Y	Y	Y	Y	Y	39.00
TAKE 1 ACCEPTS PREVIOUSLY CREATED GRAPHICS INTO A SLIDE SHOW FOR PRESENTATION	BAUDVILLE	AP	CA,GG		Y	Y	Y	Y	59.95

INSTRUCTIONAL TOOLS - SPREADSHEET

TITLE	PUBLISHER	COMPUTERS	MODES	P	E	M	S	T	PRICE
APPLEWORKS INTEGRATED PROGRAM INCLUDES A WORD PROCESSOR, DATA, BASE, AND SPREADSHEET	APPLE	AP	DB,SD,WP			Y	Y	Y	175.00
EDUCALC DESIGNED-FOR-EDUCATION SPREADSHEET INCLUDES TUTORIAL AND INSTRUCTIONAL SUPPORT MATERIALS	GROLIER	AP,CO,IB,JR	PS,SD,TU		Y	Y	Y		64.95
EXCEL FULL-FUNCTION SPREADSHEET WITH GRAPHICS	MICROSOFT	MC	SD				Y	Y	395.00
LOTUS 1-2-3 INTEGRATED SPREADSHEET, DATA, BASE, AND WORD PROCESSOR	LOTUS	IB	DB,SD,WP				Y	Y	495.00
MICROSOFT MULTIPLAN FULL-FUNCTION SPREADSHEET	MICROSOFT	IB,MC	SD			Y	Y	Y	195.00
MICROSOFT WORKS INTEGRATED PROGRAM INCLUDES A WORD PROCESSOR, DATA BASE, SPREADSHEET, AND TELECOMMUNICATIONS	MICROSOFT	MC	DB,SD,TC,WP			Y	Y	Y	295.00
SIDEWAYS ALLOWS SIDEWAYS PRINTING OF SPREADSHEET DATA	FUNK	AP,IB	SD			Y	Y	Y	69.95
SUPERCALC3A FULL-FUNCTION SPREADSHEET	COMP ASSOC	AP	GG,SD				Y	Y	195.00
TK; SOLVER FORMULA PROCESSOR/TOOL FOR TECHNICAL AND SCIENTIFIC APPLICATIONS	UNIVERSAL	AP,IB,JR,MC	PS				Y	Y	200.00

INSTRUCTIONAL TOOLS - TELECOMMUNICATIONS

TITLE	PUBLISHER	COMPUTERS	MODES	P	E	M	S	T	PRICE
APPLE ACCESS II PRODOS-BASED COMMUNICATIONS PROGRAM	APPLE	AP	TC			Y	Y	Y	75.00
ASCII EXPRESS SMART TERMINAL COMMUNICATIONS PACKAGE	UNITED	AP,IB	TC			Y	Y	Y	129.95
INFORMATION CONNECTION DEMONSTRATES USE OF ON-LINE DATA BASES; INCLUDES COMMUNICATIONS SOFTWARE	GROLIER	AP,CO,IB,JR	TC,TU		Y	Y	Y	Y	74.95
MACTERMINAL COMMUNICATIONS PROGRAM	APPLE	MC	TC			Y	Y	Y	125.00
MICROSOFT WORKS INTEGRATED PROGRAM INCLUDES A WORD PROCESSOR, DATA, BASE, SPREADSHEET, AND TELECOMMUNICATIONS	MICROSOFT	MC	DB,SD,TC,WP			Y	Y	Y	295.00
PC-TALK SMART TERMINAL COMMUNICATIONS PACKAGE	FREEWARE	IB	TC				Y	Y	35.00
RED RYDER V.9 FULL-FUNCTION COMMUNICATIONS PACKAGE	FREESOFT	MC	TC				Y	Y	40.00

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ALPHABETICAL LIST OF TITLES

1987 EDUCATIONAL SOFTWARE PREVIEW GUIDE - TITLES AND PRICES

TITLE	PUBLISHER	COMPUTER	CURR AREA	PRICE
ADDITION LOGICIAN	MECC	AP	MA	49.00
ADDITION MAGICIAN	TLC	AP,CO,IB	MA	34.95
ADJECTIVES	HARTLEY	AP,IB	LA	39.95
ADVENTURE CONSTRUCTION SET	ELECTR ART	AM,AP,CO	LA,PS	49.95
ADVENTURES WITH FRACTIONS	MECC	AP,CO	MA	39.00
AFRICA	ED'L ACTV	AP,TR	SS	69.00
AGEMT U.S.A.	SCHOLASTIC	AP,AT,CO,IB	PS,SS	39.95
AIR POLLUTION	EME	AP,IBMJR,TR	SC	37.00
ALCOHOL 'THE PARTY'	MARSHWARE	AP	HE,HL	49.95
ALEXANDER	GESSLER	AP,IB,JR	IT	245.00
ALGEBRA ARCADE	COMPRESS	AP,CO,IB	MA	49.95
ALL SORTS OF MEGGLES	JOSTENS	AP	CS,PS	149.00
ALPHABET CIRCUS	DLM	AP,CO,IB,JR	LA,PR	29.95
ALPHABETIC KEYBOARDING	SW PUB	AP,IBMJR,TR	BE,KS	89.50
ALPHABETIZATION	MILLIKEN	AP	LA	75.00
AMERICA COAST-TO-COAST	CBS	AP,CO,IB	SS	49.95
AMERICAN HISTORY ACHIEVEMENT I	MIC WRKSHP	AP	SS	49.95
ANAGRAMAS HISPANGAMERICANOS	GESSLER	AP	FL	29.95
ANALOGIES ADVANCED	HARTLEY	AP,IB	LA	39.95
ANALOGIES TUTORIAL	HARTLEY	AP,IB	LA	49.95
APPLE ACCESS II	APPLE	AP	IT	75.00
APPLE LOGO	APPLE	AP	CS	100.00
APPLE LOGO II	APPLE	AP	CS	100.00
APPLE SUPER PILOT/LOGO	APPLE	AP	IT	175.00
APPLE WRITER IIE	APPLE	AP	IT	149.00
APPLEWORKS	APPLE	AP	IT	175.00
ARBPLOT	CONDUIT	AP	IT	125.00
ARCHEOLOGY SEARCH	MCGRAW HILL	AP,TR	SS	180.00
ARITH-MAGIC	QED	AP,CO,IB,PE,TR	MA	35.00
ARROW DYNAMICS	SUNBURST	AP	CS,PS	59.00
ASCII EXPRESS	UNITED	AP,IB	IT	129.95
ASTRO GROVER	CBS	AP,CO	PR	29.95
AUTOMATED ACCOUNTING	SW PUB	AP,CO,IB,TR	BE	64.50
BAFFLES	CONDUIT	AP	PS,SC	50.00
BAKE AND TASTE	MINDPLAY	AP,IB	PR	39.99
BALANCE OF POWER	MINDSCAPE	AM,IB,MC	SS	59.95
BANK STREET FILER	BRODERBUND	AP,CO	IT	69.95
BANK STREET MUSICWRITER	MINDSCAPE	AP,AT,CO,IB,JR	MU	49.95
BANK STREET SCHOOL FILER	SUNBURST	AP	IT	99.00
BANK STREET SPELLER	SCHOLASTIC	AP	IT	64.95
BANK STREET STORYBOOK	MINDSCAPE	AP,CO,IB,JR	AT,LA	49.95
BANK STREET WRITER	BRODERBUND	AP,CO,IB	IT	69.95
BANK STREET WRITER	SCHOLASTIC	AP,AT,CO	IT	69.95
BANK STREET WRITER II	SCHOLASTIC	AP	IT	69.95
BANK STREET WRITER III	SCHOLASTIC	AP,IB	IT	79.95
BANK STREET WRITING ACTIVITIES	SCHOLASTIC	AP,AT,CO,IB	LA	59.95
BASICS OF BASIC	FOCUS	AP,IB	CS	79.00
BATAILLE DE MOTS	GESSLER	AP,CO,IB	FL	49.95
BATALIA DE PALABRAS	GESSLER	AP,CO,IB	FL	49.95
BEYOND THE RISING SUN; DISCOV JAPAN	ED'L ACTV	AP	SS	63.00
BIOFEEDBACK MICROLAB	HRM SOFTWR	AP,CO	SC	350.00
BIRDSEED	EDUTECH	AP	SC	95.00
BLAZING PADDLES	BAUDVILLE	AP,AT,CO	AT,IT	49.95
BLAZING THE BASIC TRAIL	SUNBURST	AP	CS	59.00
BODY AWARENESS	LEARN WELL	AP	PR,SC	49.95
BODY IN FOCUS	CBS	AP,CO,IB	SC	49.95

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BODY TRANSPARENT	DESIGNWARE	AP,CO,IB,JR	HL,SC	44.95
BOPPIE'S GREAT WORD CHASE	DLM	AP,CO,IB,JR	LA	29.95
BOTANICAL GARDENS	SUNBURST	AP	SC	59.00
BUILDING READING SKILLS; CIRCUS/ZOO	JOSTENS	AP	LA	149.00
BUILDING READING SKILLS; FARM/JUNGL	JOSTENS	AP	LA	149.00
BUILDING READING SKILLS; SHORT/LONG	JOSTENS	AP	LA	149.00
BUMBLE GAMES	TLC	AP,CO	MA,PS	54.95
BUMBLE PLOT	TLC	AP,CO	MA,PS	54.95
CALENDAR SKILLS	HARTLEY	AP,IB	LA	39.95
CALLIOPE	INNOVISION	MC	IT	59.95
CAPITALIZATION PLUS	MIC WRKSHP	AP	LA	44.95
CAR BUILDER	WEEK READ	AP	PS,SC,VE	39.95
CARDIOVASCULAR FITNESS LAB	HRM SOFTWR	AP,CO,IB	HL,SC	175.00
CAREER DIRECTIONS	JEFFERSON	AP,CO,IB	VE	89.00
CARTELS AND CUTTHROATS	STRAT SIMS	AP,CO,IB,JR	SS	39.95
CATGEN	CONDUIT	AP,IB	SC	75.00
CATLAB	CONDUIT	AP	SC	75.00
CELL GROWTH AND MITOSIS	IBM	IB	SC	69.95
CERTIFICATE MAKER	SPRINGBOARD	AP,CO,IB	IT	49.95
CHALLENGE MATH	SUNBURST	AP,CO	MA,PS	59.00
CHARIOTS, COUGARS, AND KINGS	HARTLEY	AP	LA	39.95
CHARLIE BROWN'S 1, 2, 3'S	RANDOM	AP	MA	39.95
CHARLIE BROWN'S ABC'S	RANDOM	AP,CO,IB	PR	39.95
CHEM LAB SIMULATIONS 1	HIGH TECH	AP,AT	SC	100.00
CHEM LAB SIMULATIONS 2	HIGH TECH	AP	SC	100.00
CHEMICAL NOMENCLATURE/BAL EQUATIONS	BERGWALL	AP	SC	59.00
CHEMICALS OF LIFE 1; STRUCTURE	IBM	IB	SC	52.00
CHEMICALS OF LIFE II; WATER, CARB	IBM	IB	SC	52.00
CHIPWITS	BRAINPOWER	AP,MC	CS,PS	39.95
CIRCUS MATH	MECC	AP	MA	49.00
CLASSIFICATION	MECC	AP,IB,JR	LA	29.00
CLASSIFICATION OF LIVING THINGS	ED'L ACTV	AP	SC	59.95
CLIP ART COLLECTION V.1	SPRINGBOARD	AP,CO,IB	IT	29.95
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ABBREV	PUBLISHER
ACTIVE LEARN	Active Learning Systems
ADD WES	Addison-Wesley Publishing Co.
ADV ID	Advanced Ideas, Inc.
ALDUS	Aldus
ALF PRODUCTS	ALF Products, Inc.
ALFRED PUB	Alfred Publishing Co., Inc.
ANN ARBOR SW	Ann Arbor Softworks
APPLE	Apple Computer, Inc.
ASHTON TATE	Ashton-Tate
AVANT GARD	Avant-Garde
BANTAM	Bantam Electronic Publishing
BATTERIES	Batteries Included
BAUDVILLE	Baudville
BEAGLE BRO	Beagle Brothers
BERGWALL	Bergwall Electronic Publishers
BORLAND	Borland International
BRAINPOWER	BrainPower, Inc.
BRITANNICA	Encyclopaedia Britannica
BRODERBUND	Broderbund Software
BYTES OF LRN	Bytes of Learning, Inc.
C & C SOFT	C & C Software
C.C. PUB	C.C. Publications
CAE	CAE Software, Inc.
CBS	CBS Interactive Learning
COMMODORE	Commodore Computer Systems Div.
COMP ASSOC	Computer Associates International
COMPRESS	COMPress
CONDUIT	CONDUIT
CORONADO	Coronado Publishers, Inc.
CREATIVE TEC	Creative Technologies
CYBERTRONICS	Cybertronics International, Inc.
DATA COMM	Data Command
DATA TRANSFM	Data Transforms
DAVIDSON	Davidson & Associates
DC HEATH	D.C. Heath & Co.
DESIGNWARE	DesignWare, Inc.
DIDATECH	Didatech Software
DLM	Developmental Learning Materials
EARTHWARE	Earthware Software Services
ED'L ACTV	Educational Activities, Inc.
EDUSOFT	EduSoft
EDUTECH	EduTech, Inc.
ELECTR ART	Electronic Arts
ELECTR TRANS	Electric Transit
EMC	EMC Publishing

EME	Educational Materials & Equipment
FOCUS	Focus Media, Inc.
FREESOFT	Freesoft Co.
FREEWARE	Freeware
FUNK	Funk Software
GAMCO	Gamco Industries
GESSLER	Gessler Educational Software
GROLIER	Grolier Electronic Publishing
HANDS-ON	Hands-on-Training Co.
HARTLEY	Hartley Courseware, Inc.
HIGH TECH	High Technology Software Products
HOLT R&W	Holt, Rinehart and Winston
HOUGHTON	Houghton Mifflin Company
HRM SOFTWR	HRM Software
IBM	IBM
INFOCOM	Infocom, Inc.
INNOVATIVE	Innovative Data Design
INNOVISION	Innovision
INST/COMM	Instructional/Communications Tech.
INTELLECTUAL	Intellectual Software
INTERLEARN	Interlearn
ISL SOFTWR	Island Software
JEFFERSON	Jefferson Software
JMH	JMH Software of Minnesota, Inc.
JOSTENS	Jostens Publications
KRELL	Krell Software Corp.
LANGENSCHEID	Langenschiedt Pubs., Inc.
LAWR HALL	Lawrence Hall of Science
LCSI	Logo Computer Systems, Inc.
LEARN WELL	Learning Well
LINGO FUN	Lingo Fun, Inc.
LIV TEXT	Living Video Text, Inc.
LOTUS	Lotus Development Corp.
LRNG TECH	Learning Technologies, Inc.
MARSHWARE	Marshware
MCGRAW HILL	McGraw-Hill Book Co./School Div.
MECC	MECC
MED MAT	Media Materials, Inc.
MIC WRKSHP	Microcomputer Workshops/CBS
MICRO P&L	Micro Power & Light
MICRO-ED	MICRO-ED, Inc.
MICROPRO	MicroPro International Corp.
MICROSOFT	Microsoft Corp.
MIDWEST	Midwest Software
MILLIKEN	Milliken Publishing Co.
MINDPLAY	Mindplay, Inc.
MINDSCAPE	Mindscape, Inc.
NEWSWEEK	Newsweek Magazine
NYSTROM	Nystrom
PASSPORT	Passport Designs, Inc.
PRO DESIGN	Program Design, Inc.

PTI-KOALA	PTI-Koala
QED	Quality Educational Designs
QUANTUM	Quantum Technology
RAND MCNLY	Rand McNally & Co.
RANDOM	Random House School Division
RESRCH DSGN	Research Design Associates
SCHOLASTIC	Scholastic, Inc.
SCOTT FORS	Scott, Foresman & Co.
SCWRIP	South Coast Writing Project
SENSIBLE	Sensible Software
SIERRA	Sierra On-Line, Inc.
SILVER	Silver Burdett & Ginn
SIMON & SCHU	Simon & Schuster Software
SIRI-TECH	Sir-Tech
SPINNAKER	Spinnaker Software
SPRINGBOARD	Springboard Software, Inc.
STRAT SIMS	Strategic Simulations, Inc.
STRAWBERRY H	Strawberry Hill Knowledge Software
STYLEWARE	Styleware
SUNBURST	Sunburst Communications, Inc.
SVE	Society for Visual Education, Inc.
SW PUB	South-Western Publishing Co.
TELOS	Telos Software Products
TEMPORAL	Temporal Acuity Products, Inc.
TERRAPIN	Terrapin, Inc.
TLC	Learning Company, The
TOM SNYDER	Tom Snyder Productions
TYC	Teach Yourself by Computer
UNITED	United Software Industries
UNIVERSAL	Universal Tech. Systems
VERNIER	Vernier Software
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WALT DISNY	Walt Disney Software
WEEK READ	Weekly Reader Family Software
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314-423-2190

Freeware
P.O. Box 862
Tiburon, CA 94920
415-435-0770

Funk Software
222 Third St.
Cambridge, MA 02142
617-497-6339

Gamco Industries
P.O. Box 1911
Big Spring, TX 79721
915-267-6327

Gessler Educational Software
900 Broadway
New York, NY 10003
212-673-3113

Grolier Electronic Publishing
Sherman Turnpike
Danbury, CT 06816
212-696-9750

Hands-on-Training Co.
4021 Allen School Rd.
Bonita, CA 92002
619-292-3639

Hartley Courseware, Inc.
123 Bridge St.
Dimondale, MI 48821
517-646-6458

High Technology Software Products
P.O. Box 60406
Oklahoma City, OK 73146
405-524-4359

Holt, Rinehart and Winston
383 Madison Ave.
New York, NY 10017
212-872-2000

Houghton Mifflin Company
P. O. Box 683, Dept. 217
Hanover, NH 03755
817-649-5254

HRM Software
175 Tompkins Ave.
Pleasantville, NY 10570
800-431-2050

IBM
P.O. Box 2150
Atlanta, GA 30055
404-238-3000

Infocom, Inc.
125 Cambridge Park Dr.
Cambridge, MA 02140
617-492-1031

Innovative Data Design
2280 Bates Ave., Suite A
Concord, CA 94519
415-680-6818

Innovision
P.O. Box 1317
Los Altos, CA 94023
415-964-2885

Instructional/Communications Tech.
10 Stepar Pl.
Huntington Station, NY 11746
516-549-3000

Intellectual Software
562 Boston Ave.
Bridgeport, CT 06610
203-335-0906

Interlearn
P.O. Box 342
Cardiff, CA 92007
619-433-3655

Island Software
P.O. Box 300, Dept. K
Lake Grove, NY 11755
516-585-3755

Jefferson Software
723 Kanawha Blvd. East
Charleston, WV 25301
800-468-4227

JMH Software of Minnesota, Inc.
P.O. Box 41308
Minneapolis, MN 55441
612-424-5464

Jostens Publications
600 W. University Dr.
Arlington Heights, IL 60004
800-323-7577

Krell Software Corp.
Flowerfield Bldg. #7, Suite #1D
St. James, NY 11780
516-751-5139

Langenschiedt Pubs., Inc.
46-35 54th Road
Maspeth, NY 11378
718-784-0055

Lawrence Hall of Science
MCEP, Univ. of Calif.
Berkeley, CA 94720
415-642-5133

Learning Company, The
545 Middlefield Rd., Suite #170
Menlo Park, CA 94025
415-328-5410

Learning Technologies, Inc.
4255 LBJ Freeway, Suite #131
Dallas, TX 75244
214-991-4958

Learning Well
200 S. Service Rd.
Roslyn Heights, NY 11577
800-645-6564

Lingo Fun, Inc.
P.O. Box 486
Westerville, OH 43081
614-882-8258

Living Video Text, Inc.
2432 Charleston Rd.
Mountain View, CA 94043
415-964-6300

Logo Computer Systems, Inc.
555 W. 57th St., Sutie #1236
New York, NY 10019
212-765-4780

Lotus Development Corp.
55 Cambridge Parkway
Cambridge, MA 02142
617-494-9114

Marshware
P.O. Box 8082
Shawnee Mission, KS 66208
816-523-1059

McGraw-Hill Book Co./School Div.
8171 Redwood Hwy.
Novato, CA 94947
415-898-5585

MECC
3490 Lexington Ave. North
St. Paul, MN 55126
612-481-3550

Media Materials, Inc.
2936 Remington Ave.
Baltimore, MD 21211
301-235-1700

Micro Power & Light
12810 Hillcrest Rd., Suite #120
Dallas, TX 75230
214-239-6620

MICRO-ED, Inc.
P.O. Box 444005
Eden Prairie, MN 55344
612-929-2065

Microcomputer Workshops/CBS
225 Westchester Ave.
Port Chester, NY 10573
914-937-5440

MicroPro International Corp.
33 San Pablo Ave.
San Rafael, CA 94903
415-499-1200

Microsoft Corp.
P.O. Box 97017
Redmond, WA 98073
800-426-9400

Midwest Software
35881 Smithfield
Farmington, MI 48024
313-477-0897

Milliken Publishing Co.
P.O. Box 21579
St. Louis, MO 63132
314-991-4220

Mindplay, Inc.
82 Montvale
Stoneham, MA 02180
617-438-5454

Mindscape, Inc.
3444 Dundee Rd.
Northbrook, IL 60062
312-480-7667

Newsweek Magazine
444 Madison Ave.
New York, NY 10022
212-350-4974

Nystrom
333 Elston Ave.
Chicago, IL 60618
800-621-8086

Passport Designs, Inc.
625 Miramontes St.
Half Moon Bay, CA 94019
415-726-0280

Program Design, Inc.
P.O. Box 4779
Greenwich, CT 06830
203-355-0906

PTI-Koala
269 Mt. Hermon Rd.
Scotts Valley, CA 95066
408-438-0946

Quality Educational Designs
P.O. Box 12486
Portland, OR 97212
503-287-8137

Quantum Technology
30153 Arena Dr.
Evergreen, CO 80439
303-674-9651

Rand McNally & Co.
P.O. Box 7600
Chicago, IL 60680
312-673-9100

Random House School Division
201 East 50th St.
New York, NY 10022
212-572-2075

Research Design Associates
P.O. Box 848
Stony Brook, NY 11790
516-928-5700

Scholastic, Inc.
4460 Black Ave., #J
Pleasanton, CA 94566
415-462-8250

Scott, Foresman & Co.
1900 East Lake Ave.
Glenview, IL 60025
312-729-3000

Sensible Software
210 S. Woodward, Suite #229
Birmingham, MI 48011
313-258-5566

Sierra On-Line, Inc.
P.O. Box 485
Coarsegold, CA 93514
209-683-6858

Silver Burdett & Ginn
250 James St.
Morristown, NJ 07960
800-631-8081

Simon & Schuster Software
1 Gulf & Western Plz.
New York, NY 10023
212-333-7000

Sir-Tech
6 Main St.
Ogdensburg, NY 13669
315-393-6633

Society for Visual Education, Inc.
1345 Diversey Parkway
Chicago, Il 60614
312-525-1500

South Coast Writing Project
Grad. School of Ed., UCSB
Santa Barbara, CA 93106
805-961-4422

South-Western Publishing Co.
5101 Madison Rd.
Cincinnati, OH 45227
513-271-8811

Spinnaker Software
1 Kendal Square
Cambridge, MA 02139
800-323-8088

Springboard Software, Inc.
7807 Creekridge Cir.
Minneapolis, MN 55435
612-944-3912

Strategic Simulations, Inc.
1046 N. Rengstorff Ave.
Mountain View, CA 94043
415-964-1355

Strawberry Hill Knowledge Software
11961 88th Ave., Rm. 202
Delta, B.C. V4C 3C9 Canada
604-594-5947

Styleware
5250 Gulton, Suite #2E
Houston, TX 77081
713-668-0759

Sunburst Communications, Inc.
39 Washington Ave.
Pleasantville, NY 10570
800-431-1934

Teach Yourself by Computer
2128 W. Jefferson Rd.
Pittsford, NY 14534
716-427-7065

Telos Software Products
3420 Ocean Park Ave.
Santa Monica, CA 90405
213-450-2424

Temporal Acuity Products, Inc.
300 120th Ave., NE. Bldg.1
Bellevue, WA 98005
206-462-1007

Terrapin, Inc.
222 Third St.
Cambridge, MA 02142
617-492-8816

Tom Snyder Productions
123 Mt. Auburn St.
Cambridge, MA 02138
617-876-4433

United Software Industries
8399 Topanga Canyon Blvd.
Canoga Park, CA 91304
303-671-0033

Universal Tech. Systems
1220 Rock St.
Rockford, IL 61101
815-963-2220

Vernier Software
2920 S.W. 89th St.
Portland, OR 97225
503-297-5317

Wadsworth Electronic Pub. Co.
10 Davis Dr.
Belmont, CA 94002
415-595-2350

Walt Disney Software
4563 Colorado Blvd.
Los Angeles, CA 90039
818-956-3005

Weekly Reader Family Software
245 Long Hill Rd.
Middletown, Ct. 06457
203-638-2400

Word Perfect Corp.
288 West Center St.
Orem, UT 84057
800-321-4566

Session 9S
Algebra

45 Minute Class
45 Minute Lab
45 Minute Class

OBJECTIVE: To see how the computer can assist in teaching about polynomials and roots. Short programs are like formulas, except that they offer step-by-step teaching advantages that formulas lack. During lab, short programs will be examined. During class, longer programs for teaching more advanced algebra will be run.

PRELIMINARIES: One thing must be stated up front about using short programs as teaching tools in a classroom, and that is that a discussion of the mathematics embodied in a program should always precede running the program. Every time.

Key mathematical points should be written and left on the chalkboard. This is will help make the computer a catalyst, not a competitor.

Users of these programs must control them with the usual DOS commands: RUN, LIST, CATALOG, Control-Reset, and Control-S.

Using programs to assist in teaching about factoring, unfactoring, quadratic formula, general polynomials, and the fundamental theorem of algebra.

This session is subdivided into topics with time frames suggested for presenters. Within each time frame, any time that remains may be used examining the problems and solutions.

COEFFICIENTS, or "UNFACTORING" (10 minutes)

Transparency 6. Compare the formula

(X-R)(X-S) = X^2 - (R+S)X + RS

with Lines 30-50 of Program P3. This is the formula that should be left on the chalkboard, along with a numerical example, such as (X-2)(X-3) = X^2 - 5X + 6 .

Transparency 7. Compare the formula

(X-R)(X-S)(X-T) = X^3 - (R+S+T)X^2 + (RS+RT+ST)X - RST

with Lines 30-60 of Program P4. This formula and a numerical example should be left on the chalkboard.

Transparency 8. Extend the patterns already noted above.

Run Program P3 using R,S = 1,1 and -1,-1 and -1,1
Run Program P4 using R,S,T = 1,1,1; -1,-1,-1; and 1,2,3
Run Program P7 using R,S,T,U = 1,1,1,1; -1,-1,-1,-1; 0,0,0,0; and 12,23,34,45

REAL ROOTS (FACTORING) (10 minutes)

Transparency 9, top half. Run Program P6. When prompted for X, input -3. Follow very rapidly with X=-2, -1, 0, 1, 2, and 3. Then seek the root that must lie someplace between 1 and 2. (It's 1.75.)

Bottom half. Run Program P5. Input A,B = 0,9

Emphasize that root-finding is essentially factoring. Thus, Programs P6 and P7 can be called "factoring programs," and Programs P3-P5 can be called "unfactoring programs." The latter can be and should be used to check the former.

QUADRATIC FORMULA (10 minutes)

Transparency 10. Discuss each line, emphasizing that the program, from Line 30 to Line 70, is essentially the quadratic formula, laid out in five logical steps. These steps reinforce students' understanding of the formula, as already studied in their textbook.

Transparency 11. Program P2 can generate a sequence of solved quadratic equations, so that students can examine patterns, as contrasted to isolated one-at-a-time cases. Use P2 to confirm that the parabolas shown, from top to bottom, have 0, 0, 1, 2, and 0 real roots (X-intercepts). Note that they all have the same number V (for Vertex) at Line 30, in agreement with their vertical alignment on the transparency.

- END OF CLASS; BEGINNING OF LAB -

WAYS TO USE SHORT PROGRAMS (10 minutes)

Transparency 12. Paraphrase it, referring to examples just examined during the lab. Concerning items C and D, note that students who are programmers should certainly modify or write some programs as a way of learning mathematics. Many of them can be encouraged to determine their own directions within algebra. Others will want suggested programs, such as the following:

1. Write a program that does to five numbers what Program P5 does to four numbers.
2. As a sequel to Program P2 (Quadratic Formula), find the cubic formula in a book, and translate it into a program.
3. After solving Problem 5 modify Program P7 to find the least real number that "works."

HORNER'S METHOD (10 minutes)

Algorithmic mathematics is becoming increasingly important, and Horner's Method is possibly the best example of an algorithm that can fit naturally into a high school algebra course.

Transparency 13. Step through Lines 30-50 of Program P8 and then Lines 30-60 of P9.

Emphasize that for a computer, this way to evaluate a polynomial is remarkably faster and more accurate than evaluating a user-defined function.

Run Program P10, using any A,B,C,D,E and several X.

SURVEY OF ADVANCED ROOT-AND-POLYNOMIAL PROGRAMS (10 min)

Some fundamental algebraic topics, such as the Fundamental Theorem of Algebra, are mentioned minimally in textbooks, largely because much computation is needed for proper development. Now we have computers, and these topics need to be recognized. They need to be included in new curricula that emphasize mathematical structure, problem solving, and the integration of computer-use with mathematics.

Transparency 14. Programs listed here are available on MATHDISK THREE and other sources.

FUNDAMENTAL THEOREM OF ALGEBRA (15 minutes)

Transparency 15. Two programs enable this theorem to take a more fundamental place in high school algebra. BAIRSTOW-HITCHCOCK finds all roots, and COEFFICIENTS does the reverse, by forming the polynomial that has any given numbers as its roots.

Run Program 93 on MATHDISK THREE. (Type RUN COEFFICIENTS and tap RETURN.) Find the polynomial having roots 1+2i and 1-2i. (Answer: Z^2 - 2Z + 5)

Run Program 92. (Type RUN BAIRSTOW-HITCHCOCK and tap RETURN.) Find the roots of Z^2 - 2Z + 5.

Next, tackle a more venturesome example: In the secret document in which Sir Leonard Bairstow introduced his method (in 1914, concerning aeroplane stability), he wrote that the following equation "presented some difficulties":

Z^8 + 20.4Z^7 + 151.3Z^6 + 490Z^5 + 687Z^4 + 719Z^3

+ 150Z^2 + 109Z + 6.87 = 0

Use Program 92 to find the eight roots (input P,Q = 0,0). (Following are the approximate roots:

.00283024 + .41326658i and its conjugate
-.66775314 + 1.3215789i and its conjugate
-5.6085071 + 1.8748821i and its conjugate
-7.7857585
-.06738137)

Tell participants that if they really want to check that these roots do indeed produce Bairstow's historic polynomial, they may do so on their own time, using the program COEFFICIENTS.

LAGRANGE POLYNOMIALS (15 minutes)

Lagrange polynomials - that is, polynomials that pass through prescribed points, is another important topic that computer power now makes accessible to students.

Two points determine a line; that is, a 1st degree polynomial. Three points that aren't on a line determine a parabola, or 2nd degree polynomial. Does this pattern extend to 4 points, and to 5 points?

Transparency 16 (the 3-point case).

Transparency 17 (the 4-point case).

Run Program 102. (Type RUN LAGRANGE and tap RETURN.) When prompted, input N=4, and input these points:

-6,10 -2,10 2,10 6,40

Rerun, using N=6 and these points:

-5,0 -3,0 0,0 3,0 5,0 6,30

Any time that remains should be spent examining the advanced algebra problems and solutions.

ALGEBRA PROBLEMS
Using Short Programs P2 - P7

1. Use Programs P3-P5 to expand these products:

   a. (X-1)(X-2) =

   b. (X+1)(X+2) =

   c. X(X+1)(X+2) =

   d. (X+2)(X-0)(X+1) =

   e. X(X+1)(X+2)X =

   f. (X+1)^2 =

   g. (X+1)^3 =

   h. (X+1)^4 =

   i. (X-1)^4 =

   j. (X+2)^3 =

2. The polynomial initially evaluated by Program P6 is

   Y(X) = 4X^3 - 3X^2 - 3X - 7 .

   Quickly input X = -3, then X = -2, then X = -1, and so on, up to X = 10. Then experiment to find a root of Y(X).

   Answer: ____________________

3. Use Program P7 to confirm the root found in Problem 2. Then change the polynomial at Line 20 as follows:

   (X-2)*(X-4)*(X-6) + 1

   Find all three roots of this polynomial:

   Roots: ____________________

   ____

   ____

4. With the help of Program P4, write down the coefficients of the polynomial (X-2)(X-4)(X-6) + 1 .

Coefficient of X^3: ____________________
Coefficient of X^2: ____________________
Coefficient of X : ____________________
Coefficient of 1 : ____________________

Use Program P4 to check the roots found in Problem 3

Coefficient of X^3: ____________________
Coefficient of X^2: ____________________
Coefficient of X : ____________________
Coefficient of 1 : ____________________

5. Experiment using Program P7 to find the greatest integer N for which the polynomial (X-2)(X-4)(X-6)+N has three real roots.

Answer: N = _______________

6. Use Program P2 (Quadratic Formula) to find the roots of these polynomials:

X^2 - 4X + 3 Roots: __________________________

X^2 - 4X + 4 Roots: __________________________

X^2 - 4X + 5 Roots: __________________________

X^2 - 4X Roots: __________________________

7. The sum of two certain numbers is 639. Their product is 8750. What are they?

Numbers: _______ and ________

8. Find the X-intercepts of these parabolas. (Write NONE if appropriate. Graphing is recommended.)

Y = (X-2)(X-4) X-intercepts: _______________

Y = (X-2)(X-4) + .5 X-intercepts: _______________

Y = (X-2)(X-4) + 1 X-intercepts: _______________

Y = (X-2)(X-4) + 2 X-intercepts: _______________

SOLUTIONS AND NOTES

for use with ALGEBRA PROBLEMS

1. a. X^2 - 3X + 2
   b. X^2 + 3X + 2
   c. X^3 + 3X^2 + 2X
   d. X^3 + 3X^2 + 2X
   e. X^4 + 3X^3 + 2X^2
   f. X^2 + 2X + 1
   g. X^3 + 3X^2 + 3X + 1
   h. X^4 + 4X^3 + 6X^2 + 4X + 1
   i. X^4 - 4X^3 + 6X^2 - 4X + 1
   j. X^3 + 6X^2 + 12X + 8

Objective: To illustrate that each selection of roots produces, or "belongs to," a polynomial.

Follow-up: Emphasize that the number of roots equals the degree of the polynomial.

Perhaps also show Pascal's Triangle and its connection with parts f-i above.

2. Root = 1.75

Objective: To gain experience with the notation Y(X) and its interpretation as a "rule."

Follow-up: Many students will have found the root of Y(X) by repeatedly taking X to be halfway between two values - call them X1 and X2, for which Y(X1)<0 and Y(X2)>0.

If that's how they did it, then they discovered the method that the computer uses in Program P7. This discovery should be very positively recognized by the teacher.

3. Roots: 1.8850925, 4.2541017, and 5.8608059

Objective: To see that each polynomial "produces" a set of roots that determine the polynomial (except for constant multiples).

Follow-up: Emphasize that it is relatively easy to produce a polynomial from its roots, but that the reverse is much harder. In fact, for most polynomials of degree >2, the only practical way to find close approximations to the roots is with a computer!

4. Coefficients found by inputting 2,4,6 to Program P4 and adding 1 to the constant term: 1, -12, 44, and -47

Coefficients found by inputting the roots (from Problem 3) to Program P4: the same, except for round off errors of less than one millionth.

Objective: To confirm the three roots found in Program 3.

Follow-up: Sketch the graphs of

P(X) = (X-2)(X-4)(X-6) and Q(X) = P(X)+1

for the class. Then they can see clearly how each of the roots 2,4,6 moves over slightly to become a root of Q(X), as a natural consequence of "lifting" the graph of P(X) upward one unit to form the graph of Q(X).

Explicitly, the root 2 of P(X) moves over to the root 1.8850925 of Q(X); 4 moves to 4.2541017, and 6 moves to 5.8608059, as the graph of P(X) slides up to become the graph of Q(X).

5. N = 3

Objective: To extend the graphical analysis introduced by Problem 4; in particular, to consider continuous change of N, and how this accounts for all the kinds of roots that a cubic polynomial can have: 3 real roots that are distinct, 3 real roots with at least one of them repeated, or 1 real root and two nonreal roots).

Follow-up: Depending on the value of N, the polynomial

Q(X) = (X-2)*(X-4)*(X-6) + N

can have 3 real roots or only 1 real root.

Explain how this observation generalizes: the numbers 2,4,6 can be replaced by any three real numbers, and the classification of the resulting roots can still easily be accounted for by counting X-intercepts as the value of N changes - that is, as a graph is moved continuously up or down.

An overhead projector provides an excellent means of illustration. Keep the graph of (X-2)*(X-4)*(X-6) fixed on the screen, and move the same graph, draw on another transparency, slowly up or down from the original position. Be sure that students see how each individual root (i.e., X-intercept), moves to the left or right - or disappears, as you move the graph.

6. Roots: 1 and 3; -2 and -2; 2+i, 2-i, 0, and 4

Objective: To see that Program P2 gives the same answers as the Quadratic Formula, as printed in the textbook.

Be sure to LIST Program P2, so that students can see that the program embodies the Quadratic Formula, written out in a logical, "in-time" sequence of steps.

Follow-up: The sliding-transparency method suggested as a follow-up to Problem 5 is recommended here, also. Put P(X) = X^2 - 4X and Q(X) = P(X) + N. Leave the graph of P(X) fixed on the screen, and create the graphs of Q(X) for N = 3, 4, and 5 by sliding a second graph of P(X) upward to positions 3, 4, and 5 units above the graph of P(X).

Be sure students see that originally there are two roots. As N approaches 4, the two roots move toward a single point. At N=4, they have reached that point - a repeated root, and as N increases past 4, the graph no longer touches the X axis, so that there are no longer any real roots.

7. The numbers: 14 and 625

Objective: To reinforce students' understanding of connection between roots (14 and 625) and coefficients (639 and 8750).

Follow-up: Note that Program P3 serves as a check on Program P2 in case the two roots are real. Note that Programs P6 and P7 could be used instead of P2, but only to find real roots.

8. X-intercepts: 2 and 4; 2.29289322 and 3.770710678; -3; and NONE

Objective and Follow-up: essentially the same as for Problem 5.

ADVANCED ALGEBRA PROBLEMS

These problems require the use of programs on MATHDISK THREE or a comparable collection of programs. Run all the polynomial-related programs before attempting to solve these problems. The reason for this preliminary work is that part of solving the problems is to determine which programs are needed for each problem.

After solving each problem, figure out what programs can be used to check your solution. Consider such checking to be an essential part of your work.

1. Make up a polynomial having roots as prescribed.

   a. 6 real roots
   b. 4 real roots and 2 nonreal roots
   c. 2 real roots and 4 nonreal roots
   d. 6 nonreal roots.

   (Nonreal means a complex number a+bi having b not equal 0. After you have written down your polynomials, be sure to check that they really do have the roots you think they have.)

   Polynomials: a. ______________________
   b. ______________________
   c. ______________________
   d. ______________________

2. Factor the polynomial

   X^6 + 6X^5 + 14X^4 + 18X^3 + 14X^2 + 6X + 1

   as a product of three quadratic polynomials that have all their coefficients real.

   Product: ______________________________

3. All eight roots of X^8 - 1 have absolute value 1. That is, they all have distance 1 from the origin. Four of them are not also roots of X^4 - 1. List those four.

   Roots: ____________
   ____
   ____
   ____

4. Only one fourth degree polynomial passes through the points (-2,18), (-1,-5.25) (1,3.75), (3,6.75), and (4,66). Find its roots, and list them in order from least to greatest.

   Roots: __________, __________, __________, __________

   Determine approximately the lowest point of the graph of this polynomial.

   Lowest point: (X,Y) = ________________________

5. Add 3 to each of the X coordinates of the five points of Problem 4. Write down these new five points. Only one fourth degree polynomial passes through them. Explain how one could easily obtain its graph from the graph of the polynomial of Problem 4.

   Explanation: ____

6. Add 3 to each of the Y coordinates of the five points of Problem 4. Write down these new five points, and continue as in Problem 5.

   Explanation: ____

7. For each of the following lists of numbers, write down a polynomial having these numbers as roots and having all coefficients real.

   Example: For 1, i we have X^3 - X^2 + X - 1 (or any polynomial multiple of this)

   a. 2, i Polynomial: ____________________
   b. 1,-1,i,-i Polynomial: ____________________
   c. (i+3)^2 Polynomial: ____________________
   d. 1, 2 Polynomial: ____________________
   e. i, 2i Polynomial: ____________________
   f. 1/(2+i) Polynomial: ____________________

8. Determine a polynomial that crosses the graph of $Y = |10X|$ at seven different points, all having X coordinates between -4 and 4.

Polynomial: ______________________________

9. Determine a polynomial P(X) having $P(3) > 29$ and $|P(X)| < 4$ for all integers X between -3 and 3.

Polynomial: ______________________________

10. Only one cubic polynomial P(X) coincides with SQR(X) at $X = 0, X = 1, X = 2$, and $X = 4$. Evaluate:

a. P(4) =
b. P(4) - SQR(4) =
c. P(3) =
d. P(3) - SQR(3) =
e. P(5) =
f. P(5) - SQR(5) =

SOLUTIONS AND NOTES

for use with ADVANCED ALGEBRA PROBLEMS

1. There are many correct answers, including these:

   a. (X - 1)^6
   b. (X - l)^4 times (X^2 + 1)
   c. (X - 1)^2 times (X^2 + 1)^2
   d. (X^2 + 1)^3

   Objective: To illustrate that the nonreal roots of a polynomial having all real coefficients occur in complex conjugate pairs.

   Follow-up: Be sure to reverse the problem; that is multiply out the polynomials using COEFFICIENTS (Program 93 on MATHDISK THREE) and then find the roots using BAIRSTOW-HITCHCOCK (Program 92).

   What students should remember most about this problem is that they started with roots to make a polynomial, and then they started with the polynomial and recovered those same roots.

2. Product:

   (X^2 + X + 1)(X^2 + 2X + 1)(X^2 + 3X +1)

   Method: Find the six roots using BAIRSTOW-HITCHCOCK. Group them in complex conjugate pairs. Each pair is input to the program COEFFICIENTS to find a quadratic factor. The three such quadratic factors are as printed above.

   Objective: Same as for Problem 1.

   Follow-up: Use this problem to illustrate the following fact about every polynomial having all real coefficients:

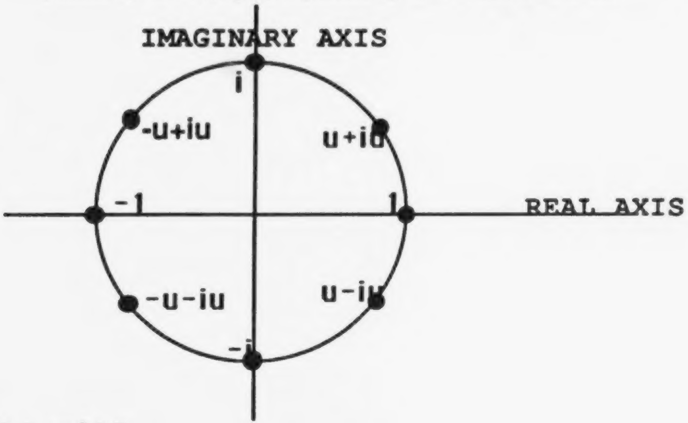
   It can be factored all the way down to linear and (irreducible) quadratic factors.

3. Roots: 1, -1, i, -i, and u+iu, u-iu, -u+iu, -u-iu, where u denotes the number SQR(1/2).

   Objective: To illustrate that an 8th degree polynomial must have 8 roots; to lead students to think of complex numbers as geometric points in a two-dimensional plane.

Follow-up: On the chalkboard, graph these eight numbers on the unit circle. Note that for each nonreal root, its conjugate is also a root. Note the geometry of conjugacy: symmetry about the real axis.

Graph:



4. Roots: -1.5, 0, 2, and 2.5
Lowest point: (-.92, -5.33) approximately

Method: Use LAGRANGE (Program 102) to find the coefficients. Then use BAIRSTOW-HITCHCOCK to find the roots.

Objective: To illustrate the fact that a polynomial is determined by any N+1 points on its graph, where N is the degree of the polynomial.

Follow-up: Use the graph provided by LAGRANGE to show where the roots and lowest point are located. Observe that four roots are interspersed by three "turnarounds" (maxes or mins), and that this relationship between roots and turnarounds is characteristic of polynomials having only distinct real roots.

5. Explanation: Move the old graph upward 3 units to produce the new graph.

Objective: Same as for Problem 4, and to illustrate vertical shifting.

Follow-up: Note that the equation for the new polynomial Q(X) is produced from the equation of the old polynomial P(X) as follows: Q(X) = P(X) + 3. Note too, however, that the roots of Q(X) are not easily obtainable from those of P(X). Not at all!

6. Explanation: Move the old graph to the left 3 units to produce the new graph.

Objective: Same as for Problem 4, and to illustrate horizontal shifting.

Follow-up: Note that the equation for the new polynomial R(X) is produced from the equation of the old polynomial P(X) as follows: R(X) = P(X+3). Also, the roots of R(X) are easily created from the roots of P(X): just add 3 to each root of P(X).

7. Polynomials:
   a. X^3 - 2X^2 + X - 2 (or any polynomial multiple of this)
   b. X^4 - 2X^3 + 3X^2 + 2X + 2
   c. X^2 - 3X + 2
   d. X^2 - 16X + 100
   e. X^4 + 5X^2 + 4
   f. 5X^2 - 4X + 1

Method: For each nonreal root a+bi, use COEFFICIENTS (Program 93) to form the quadratic,

[X - (a+bi)][X - (a-bi)]

and to multiply it by other such quadratic or linear factors.

Objective: To solidify students' understanding that the roots of a polynomial immediately give the factors of the polynomial.

8. Polynomial: (1/6)X^6 - (5/2)X^4 + (37/3)X^2 (et al)

Method: Use LAGRANGE (Program 102) to find that this is the (only) sixth degree polynomial passing through the seven points (X, |10X|), for X = -3,-2,-1,0,1,2,3 .

Objective: To illustrate how polynomials can approximate other functions.

Follow-up: Use LAGRANGE to find polynomial approximations to other nonpolynomial functions.

9. Polynomial: (1/4)X^5 - (5/4)X^3 + X (et al)

Method: Use LAGRANGE (Program 102) to find a polynomial passing through (0,0), (-1,0),(1,0), (-2,0), (2,0), and (3,30).

Objective: To illustrate in yet another way how "flexible" the set of polynomials is.

Follow-up: State that for many lists of requirements for a function to do (e.g., pass-through points, roots, rates of increase, experimental curves to approximate) a polynomial can be found that will satisfy the requirements, or come close to satisfying them.

10. Polynomial: .0631132767 times X^3
- .48223305 times X^2
+ 1.41911977 times X

Values:
P(4) = 2 P(4) - SQR(4) = 0
P(3) = 1.6213 P(3) - SQR(3) = -.1108
P(5) = 2.9289 P(5) - SQR(5) = .6928

Objective: Same as for Problem 8.

Follow-up: Show that if P(X) coincides with SQR(X) at a larger number of suitably chosen points, then P(X) more closely approximates SQR(X).

The above paragraph suggests that there should be a way to measure how close one curve is to another curve over an interval. The program DISTANCE BETWEEN CURVES (Program 147 on MATHDISK FOUR) can be easily used by students to measure how far a polynomial is from a nonpolynomial.

LIST OF TRANSPARENCIES FOR SESSION 9S

ALGEBRA

6. TWO ROOTS (PROGRAM P3)
7. THREE ROOTS (PROGRAM P4)
8. FOUR ROOTS (PROGRAM P5)
9. ROOT SEARCHERS (P6 AND P7)
10. QUADRATIC FORMULA (P2)
11. PARABOLAS
12. WAYS TO USE SHORT PROGRAMS
13. HORNER'S METHOD (PROGRAMS P8 AND P9)
14. ADVANCED ROOT-AND-POLYNOMIAL PROGRAMS
15. INTRODUCING THE FUNDAMENTAL THEOREM OF ALGEBRA
16. POLYNOMIAL THROUGH 3 POINTS
17. POLYNOMIAL THROUGH 4 POINTS

TWO ROOTS (PROGRAM P3)

FORMULA: $(X - R)(X - S) = X^2 - (R + S)X + RS$

PROGRAM:

```
10 HOME
20 INPUT "INPUT R,S = "; R,S
30 A1 = R + S
40 B = R*S
50 A = - A1
60 PRINT "(X-R)(X-S) HAS COEFFICIENTS  1, "A", "B
70 PRINT: GOTO 20
```

Transparency 6.

THREE ROOTS (PROGRAM P4)

FORMULA: $(X - R)(X - S)(X - T) = X^3 - (R + S + T)X^2 + (RS + RT + ST)X - RST$

PROGRAM:

```
10  HOME
20  INPUT "INPUT R,S,T = "; R,S,T
30  A1 = R + S + T
40  B = R*S + R*T + S*T
50  C1 = R*S*T
60  A = -A1: C = -C1
70  PRINT "(X-R)(X-S)(X-T) HAS COEFFICIENTS  "
80  PRINT "1,  "A",  "B",  "C
90  PRINT:  GOTO 20
```

Transparency 7.

FOUR ROOTS (PROGRAM P5)

FORMULA: $(X - R)(X - S)(X - T)(X - U)$

$$= X^4 - (R + S + T + U)X^3 + (RS + RT + RU + ST + SU + TU)X^2 - (RST + RSU + RTU + STU)X + RSTU$$

PROGRAM:

```
10  HOME
20  INPUT "INPUT R,S,T,U = "; R,S,T,U
30  A1 = R + S + T + U
40  B = R*S + R*T + S*T + R*U + S*U + T*U
50  C1 = R*S*T + R*S*U + R*T*U + S*T*U
60  D = R*S*T*U
70  A = -A1: C = -C1
80  PRINT "(X-R)(X-S)(X-T)(X-U) HAS COEFFICIENTS  "
90  PRINT "1,  "A",  "B",  "C",  "D
100  PRINT:  GOTO 20
```

Transparency 8.

ROOT SEARCHERS (PROGRAMS P6 AND P7)

PROGRAM P6

```
10 HOME
20 DEF FNY(X) = 4*X^3 - 3*X^2 - 3*X - 7
30 INPUT "INPUT X = "; X
40 GOSUB 80
50 PRINT "Y("X") = "; FNY(X)
60 GOTO 30
70 END
80 VTAB (PEEK(37)): PRINT "                    ":
   VTAB(PEEK(37)): RETURN
```

PROGRAM P7

```
10 HOME
20 DEF FNY(X) = 4*X^3 - 3*X^2 - 3*X - 7
30 INPUT "INPUT A,B = "; A,B
40 X = (A+B)/2: Y = FNY(X)
50 C = FNY(A): D = FNY(B)
60 PRINT "Y("X") = "; Y
70 IF Y*C > 0 THEN A = X: GOTO 40
80 B = X: GOTO 40
```

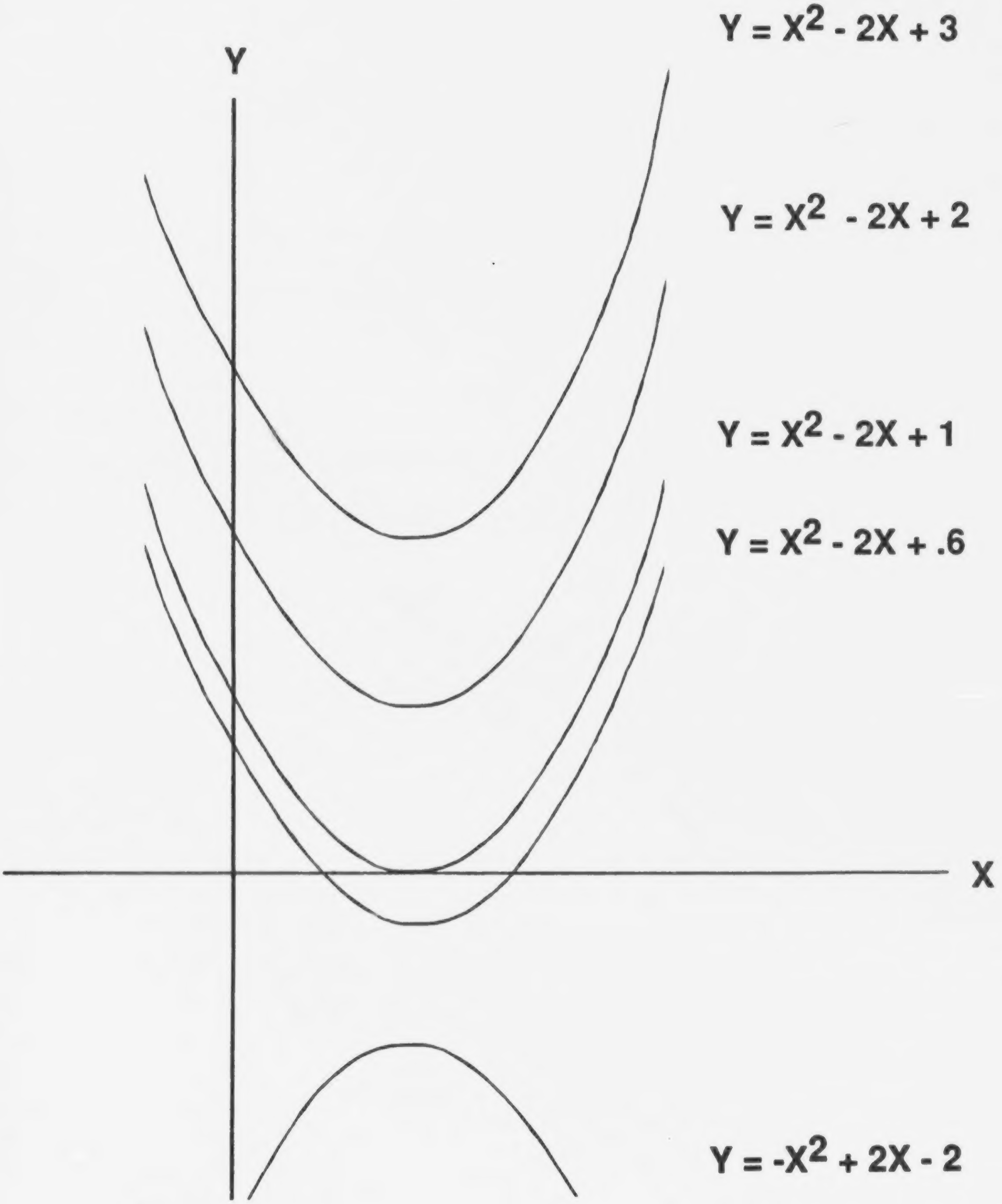
Transparency 9.

QUADRATIC FORMULA (PROGRAM P2)

```
10  HOME
20  INPUT  "INPUT  A,B,C  =  "; A,B,C
30 D  = B*B - 4*A*C : V = -B/(2*A)
40  IF D<0 THEN W=SQR(-D)/(2*A)
50  IF D=0 THEN PRINT "ONE REAL ROOT:  ";V
60  IF D>0 THEN PRINT "TWO REAL ROOTS:  ";
               (-B - SQR(D))/(2*A) :  PRINT "   AND  "
               (-B + SQR(D))/(2*A)
70  IF D<0 THEN PRINT "COMPLEX CONJUGATE ROOTS:  "
               V" + i("W")  AND  V" - i("W")"
80  PRINT:  GOTO 20
```

Transparency 10.

PARABOLAS



Transparency 11.

WAYS TO USE SHORT PROGRAMS

A. AS TOOLS DURING CLASSROOM LECTURING

TO ENHANCE TEXTBOOK COVERAGE OF TOPICS

TO FINISH WORK BEGUN ON CHALKBOARD OR OVERHEAD

TO GENERATE PATTERNS FOR DISCOVERY MAKING

B. AS TOOLS FOR DOING HOMEWORK

TO CHECK ANSWERS TO NON-COMPUTER PROBLEMS

TO HELP SOLVE PROBLEMS THAT REQUIRE COMPUTATION

C. AS STARTING MATERIALS FOR PROGRAM MODIFICATION FOR PURPOSES OF MATHEMATICAL EXPERIMENTATION

D. AS MODELS FOR ORIGINAL MATHEMATICAL PROGRAM-WRITING

Transparency 12.

HORNER'S METHOD (PROGRAMS P8 AND P9)

PROGRAM P8

```
10 HOME
20 INPUT "INPUT A,B,C = "; A,B,C
30 INPUT "X = "; X
40 Y = A*X + B
50 Y = Y*X + C
60 PRINT "Y("X") = "; Y
70 PRINT : GOTO 30
```

PROGRAM P9

```
10 HOME
20 INPUT "INPUT A,B,C,D = "; A,B,C,D
30 INPUT "X = "; X
40 Y = A*X + B
50 Y = Y*X + C
60 Y = Y*X + D
70 PRINT "Y("X") = "; Y
80 PRINT : GOTO 30
```

Transparency 13.

ADVANCED ROOT-AND-POLYNOMIAL PROGRAMS

1. MULTIPLY LINEAR FACTORS (REAL)
2. MULTIPLY LINEAR FACTORS (COMPLEX, TOO)
3. COMPLEX-NUMBER CALCULATOR
4. BAIRSTOW-HITCHCOCK (FIND ALL REAL & COMPLEX ROOTS
5. POLYNOMIAL EVALUATOR (AT ALL REAL & COMPLEX NUMBERS
6. POLYNOMIAL GRAPHER (MORE THAN ONE AT A TIME)
7. POLYNOMIAL MULTIPLICATION (ANY REASONABLE DEGREES)
8. POLYNOMIAL DIVISION (ANY REASONABLE DEGREES)
9. POLYNOMIAL COMPOSITION (ANY REASONABLE DEGREES)
10. LAGRANGE POLYNOMIALS (PASSING THROUGH ANY REASONABLE NUMBER OF POINTS)

Transparency 14.

INTRODUCING *THE FUNDAMENTAL THEOREM OF ALGEBRA*

IN THE BEGINNING ARE THE NATURAL NUMBERS 1, 2, 3, ... THEY FORM A SET N WHICH CONTAINS NO SOLUTION TO THE EQUATION 2 + X = 1.

N IS ENLARGED TO THE SET I OF ALL INTEGERS, WHEREIN 2 + X = 1 HAS A SOLUTION, BUT 2*X = 1 HAS NO SOLUTION.

I IS ENLARGED TO THE SET Q OF QUOTIENTS OF INTEGERS, WHEREIN 2*X = 1 HAS A SOLUTION, BUT X^2 = 2 HAS NO SOLUTION.

Q IS ENLARGED TO THE SET R OF REAL NUMBERS, WHEREIN X^2 = 2 HAS A SOLUTION, BUT X^2 = -1 HAS NO SOLUTION.

R IS ENLARGED TO THE SET C OF COMPLEX NUMBERS, WHEREIN X^2 = -1 HAS A SOLUTION.

AT THIS POINT, THE NEED FOR ENLARGEMENT STOPS. *EVERY POLYNOMIAL EQUATION CAN BE SOLVED IN THE SET C.* THIS SWEEPING FACT ABOUT THE COMPLEX NUMBERS IS CALLED *THE FUNDAMENTAL THEOREM OF ALGEBRA.*

Transparency 15.

POLYNOMIAL THROUGH 3 POINTS

GIVEN THREE POINTS (A,B), (C,D), AND (E,F) IN THE XY PLANE, THERE IS A 2ND DEGREE POLYNOMIAL WHOSE GRAPH PASSES THROUGH THESE POINTS:

$$\begin{aligned} L(X) = {} & B(X - C)(X - E)/[(A - C)(A - E)] \\ & + D(X - A)(X - E)/[(C - A)(C - E)] \\ & + F(X - A)(X - C)/[(E - A)(E - B)] \end{aligned}$$

WITH $X = A$, YOU CAN QUICKLY SEE THAT $L(X)$ REDUCES TO $B + 0 + 0$, AND SIMILARLY FOR $X = C$ AND $X = E$.

Transparency 16.

POLYNOMIAL THROUGH 4 POINTS

GIVEN FOUR POINTS (A,B), (C,D), (E,F), AND (G,H) IN THE XY PLANE, THERE IS A 3RD DEGREE POLYNOMIAL WHOSE GRAPH PASSES THROUGH THESE POINTS:

$$
\begin{aligned}
L(X) \;=\; & B(X-C)(X-E)(X-G)/[(A-C)(A-E)(A-G)] \\
& + D(X-A)(X-E)(X-G)/[(C-A)(C-E)(C-G)] \\
& + F(X-A)(X-C)(X-G)/[(E-A)(E-C)(E-G)] \\
& + G(X-A)(X-C)(X-E)/[(G-A)(G-C)(G-E)]
\end{aligned}
$$

NOTE HOW EASY IT IS TO VERIFY THAT L(X) REALLY DOES PASS THROUGH THE FOUR GIVEN POINTS.

Transparency 17.

Session 10E Spreadsheet Examples

Show all the different ways you could spend your $15 in Tina's Toy Store. Buy no more than two of the same kind (Grade 3, McGraw-Hill, 1981).

Two Possible Answers:

TOY	HOW MANY	PRICE	TOTAL
Frisbee	1	$1.00	$1.00
Kite	1	$3.00	$3.00
Baseball	1	$2.00	$2.00
Baseball Bat	1	$5.00	$5.00
Model Plane	1	$4.00	$4.00
		Total Bill:	$15.00

TOY	HOW MANY	PRICE	TOTAL
Frisbee	2	$1.00	$2.00
Kite	0	$3.00	$0.00
Baseball	2	$2.00	$4.00
Baseball Bat	1	$5.00	$5.00
Model Plane	1	$4.00	$4.00
		Total Bill:	$15.00

Jill bought a pizza, large coffee, and 2 small milks. She spent $3.90. What kind of pizza did she buy?
(Grade 4, McGraw-Hill, 1981)

Pizza	Quantity	Small	Quantity	Large	Item Total
Cheese		$2.00		$2.75	$0.00
Onion		$2.25		$3.05	$0.00
Pepper	1	$2.50		$3.35	$2.50
Pepperoni		$2.75		$3.65	$0.00
Sausage		$3.00		$4.00	$0.00
Beverages					
Pop		$ .50		$ .65	$0.00
Milk	2	$.40		$ .55	$ .80
Coffee		$ .45	1	$ .60	$ .60

Total:
$3.90

John is a bank teller. Help him find the total for each day and the whole week.
(Grade 4, McGraw-Hill, 1981)

	Mon.	Tues.	Wed.	Thurs.	Fri.
Bills	$7,107	$3,820	$3,954	$8,396	$13,761
Coins	$2,283	$ 246	$1,463	$3,168	$ 5,329
Total/Day	$9,390	$4,066	$5,417	$11,564	$19,090
Week's Total:	$49,527				

The local pet store is selling animals for the following prices: doberman $112.99, dachshund $59.96, setter $62.99, poodle $49.97, afghan $149, angora kitten $159.98, siamese kitten $75.89, 6 guinea pigs $21.89, lovebird $25.89, and cockatiel $89.00. Imagine you buy a puppy, a kitten and 6 guinea pigs. What is the least you can pay? (Harper and Row, Grade 5, 1981)

Animal	Price	Quantity	Subtotal
Doberman	$112. 99		$0.00
Dachshund	$ 59. 56		$ 0.00
Setter	$ 62.98		$ 0.00
Poodle	$ 49.97	1	$49.97
Afghan	$149.00		$ 0.00
Angora kitten	$159. 98		$ 0.00
Siamese kitten	$75. 89	1	$75.89
Lovebird	$ 25.89		$ 0.00
Cockatiel	$ 89.00		$ 0.00
6 Guinea pigs	$ 21.89	1	$21.89
		Total:	$147.75

George Simpson saved $6000 to buy a new Arrow Compact car. George ordered his car with all of the optional features. Using the following information, find out how much money he will have left after paying for the car.

The advertized price for the Arrow Compact is $3842, plus $115 for dealer preparation, $85 for transportation to the dealer, and $202 for state and local taxes.

The optional features are as follows:

Bucket seats	$545.00 plus $27.25 tax
AM/FM Radio	$ 75.00 plus $ 3.75 tax
CB Radio	$125.00 plus $ 6.25 tax
Deluxe trim	$375.00 plus $18.75 tax

(Grade 6, McGraw-Hill, 1981)

Price Advertized	$3,842.00
Additional Costs	
Dealer Preparation	$ 115.00
Transport to Dealer	$ 85.00
State and Local Taxes	$ 202.00
Base Price:	$4,244.00

Optional Features	Price	Tax	Quantity	Subtotal
Bucket Seats	$545.00	$27.25	1	$572.25
AM/FM Radio	$ 75.00	$ 3.75	1	$ 78.75
CB Radio	$125.00	$ 6.25	1	$131.75
Deluxe trim	$375.00	$18.75	1	$393.75
		Total:		$5,420.00
		Amt. Paid:		$6,000.00
		Amt. Left:		$ 580.00

Session 10M Applications

Signficant Differences

Objectives

To be able to explain what the term significant difference means and to be able to determine a way to find out if two groups are "significantly" different on some measure.

Description

The student will collect data on two groups using the same measure (independent variable). Then the student will put the data into the computer and determine the likelihood of an observed difference occurring by chance. If the chance of the difference occurring is small (say somewhere in the neighborhood of 10% or less) then it is probable that the difference found is a real difference - that is, it did not occur by chance.

Procedure

Load the program "2-Group Differences". Select a problem to solve. It should have two groups, the same type of ratio data collected on each member of each group. (Ratio data means that a difference between 2 scores of say 10 units is the same regardless of whether the difference is between 80 and 90 or 150 and 160 etc.)

The purpose of this session is to demonstrate software which uses Monte Carlo techniques and other statistical approaches to illustrate key ideas in data collection and evaluation.

Example of a problem: Do boys like mathematics better than girls? Do girls like mathematics better than boys? Or do they like it about the same?

Ask everybody to rate how they like mathematics on a scale of 1 to 10 with a 10 being really like it, 5 feeling so-so, and a 1 being really hate it.

Explain that if we just look at averages we don't know whether a difference is real or not. Certainly it is extremely unlikely that both boy and girl averages would be identical. (If we flip a fair coin 1000 times it is not reasonable to expect it to come out "heads" exactly 500 times) But is the difference a real difference or would we expect it by chance alone? For example, suppose the average for a boys is 7.4 and the average for girls is 7.5. Could you say they were different? What if the girl average was 7.6? Then could you say they were different? How high would the girl average have to be before you would feel confident that the difference really mattered? Unless you know what the chances are of getting a difference like that, it is impossible to tell how large the girl average would have to be in order to be different.

What the computer program does is assign every score someone actually received a different whole number. If I have 30 boys and 25 girls, each score will get a unique whole number between 1 and 55. (30+25=55) The computer then draws 30 numbers randomly with replacement from the pile of 1 to 55. If the first number drawn is 40, the score corresponding to 40 is recorded as the first boy score. The computer does this until all 30 boys have received scores. It now draws 25 numbers (with replacement) for the 25 girls. The computer has now simulated the boy group and girl group as if they came out of the same population of 55 different scores. (Notice that any boy and any girl may receive any one of the possible 55 scores) The computer then computes the boy and girl averages and finds the difference between them. If this difference is greater than the actual difference between the original data of boys and girls it is counted as a success. Otherwise it is counted as a failure. The program has completed a single trial. It now goes back and samples for 30 boys and 25 girls again. If the difference is greater than the difference in the actual averages it is counted as success, otherwise it counts as failure. This process is repeated as many times as you care to sample. The program records a real-time display of the frequency and percent (probability) of successes. If this number is small then the difference probably did not occur by chance alone. In other words it is probably a real difference.

(Another way of looking at this is assuming that one had a pool of scores corresponding to the combined set of scores of the boys and girls, what is the probability that they would separate themselves out by chance alone into two piles at least as far apart in average as they did when they were surveyed? What the computer program is doing is combining the scores into a single pool, then randomly dividing them into two piles with 25 in one pile and 30 in another. The computer then looks to determine if the difference between the averages is at least as great as the difference between the original boy/girl pile. If the computer generates data that is usually closer together than the boy and girl piles then it is quite probable that the boys and girls are in fact really different.)

Repeat the experiment with only three boys and three girls. Rerun the program several times. Notice how unstable the results are. Repeat the experiment with 10 boys and 10 girls. Do you have more confidence in the sample of 3 and 3 or 10 and 10? Why?

Repeat the experiment with 10 boys and 10 girls using 5 trials. Repeat using 100 trials. Do you have more confidence in the 5 trial sample or the 100 trial sample? Why?

In order to have confidence that your results are accurate what should you do to the number in your sample?

What should you do to the number of trials?

Have the students conduct their own survey and compare two groups on some independent measure. Let them draw conclusions as to whether the results are significantly different.

NOTE: If a lot of data is going to be collected there is the possibility that errors could occur during the data entry process. It is difficult to change data already entered through INPUT statements in BASIC. It would probably be a good idea to either establish a data file or place the data directly into the program through READ/DATA statements instead of through INPUT statements. This is fairly easy to do and would make a good programming exercise for beginning programmers.

Monte Carlo Techniques

Objective

Use Monte Carlo techniques to solve problems in probability theory.

Description

The program presents the scenario that a couple plans on having exactly three children. What are the chances that all three children will be boys assuming that the sex of an individual child is an independent event? The program also gives the student an opportunity to adjust the family size from 2 to 7 children.

Procedure

Load the program "No. of each sex in families". Study the screens carefully until you get to the screen which allows you to adjust family size.

Run the program with a family of size 2. Repeat the experiment at least 2 more times. Record the results in the table.

Run the program with a family of size 4. Repeat the experiment at least 2 more times. Record the results in the table.

Run the program with a family of size 7. Repeat the experiment at least 2 more times. Record the results in the table.

Compute the averages for each family size. Plot the results in a graph with the x-axis as the family size and the y-axis as the number of families with children all of the same sex (either boys or girls) out of 100. If you like, use the program DATA FIT. Predict what is the probability of the 5 children families having children all of the same sex?

Monte Carlo Techniques

Objective

Use Monte Carlo techniques to solve probability problems. This problem asks how many boxes of cereal one would have to purchase in order to collect all 6 coupons assuming one of the six coupons was randomly assigned to each box.

Description

The program Premium uses the random number generation capability of the computer to simulate purchases of cereal boxes. The student will see that the greater the number of trials the more confidence one can have in the results. The student will also see that the random nature of events demonstrates that repeated trials do not yield the exact same results time after time. That there is some variation in the outcomes is expected by chance alone. (keep in mind that it is extremely unlikely that you will get exactly 500 heads if you toss a fair coin 1000 times)

Procedure

Load the program "Premium". Discuss the cereal box with the six coupons described on the back. Assuming that they are randomly distributed among all of the boxes manufactured, how can we find out how many boxes, on the average will you have to buy to get all 6.

Simulate the purchase of each box with a roll of the die. Let each number on the die correspond *a priori* to a coupon. Each purchase is a roll. Record how many rolls it takes to get all 6 coupons. Record the results in the table and repeat the experiment 10 times. Average your results to get a good estimate.

Now run the program PREMIUM. When given the first choice select 2 "See the simulation of purchases by multiple contestants." You repeated the experiment 10 times with the die. The computer is asking whether you wish to repeat it anywhere from 2 to 100 times. (NOTE: the more times the computer repeats the experiment the slower the response. You may wish to compile the program before you run it.) Select 10 contestants. It should then tell you what the average number of boxes needed, the least number by at least 1 contestant and the most by at least 1 contestant. Does this come close to your results?

If you wish to try this again type Y when requested. Otherwise type N.

Now you get a chance to run the experiment by changing the number of possible coupons on the box. You can vary the number of different coupons from 2 to 9. Specify 2 coupons. The computer reruns the experiment with 2 coupons 10 times and gives the least number, most needed and the average for all 10 trials. Record the results in the table and repeat the experiment with 2 coupons at least three more times. Record the results.

Change the number of coupons to 4. Run the experiment at least 4 times. Record the results.

Change the number of coupons to 7. Run the experiment at least 4 times and record the results.

Plot the averages on a graph with the number of coupons on the x-axis and the average number of boxes needed along the y-axis. Try to draw a "best-fit" line by trial and error. Use the line to predict about how many boxes you would have to purchase on the average to get all 5 coupons in a 5 coupon giveaway. (See note below)

No. of boxes needed to get all 5 coupons:_______

Check your guess by running the computer experiment with 5 coupons. Rerun the experiment for a total of at least 4 trials.

Use your plot to answer the following question: If the cereal company would like the giveaway to convince the typical customer to buy about 16 boxes, on average, how many different coupons would they need in their cereal boxes (assuming random distribution and no trading)?

Note: If you like you may use the program DATA FIT to help you determine a "best-fit" line. First press X to exit the program, then, from the menu select the program called DATA FIT. This program first asks you for the x scale:. Press Ctrl-reset to break out of the run mode.

Type: LIST 1670,1679

Now type: 1670 DATA your data points go here with the first x-coordinate, first y-coordinate, second x-coordinate, second y-coordinate, third x-coordinate,third y-coordinate, etc.

If you need more than one line you can use 1671 to 1679. Naturally if any data already exists in these lines you need to remove them by typing the line number and then pressing RETURN.

Type the equation for the "best-fit" line in line 300. It needs to be Y in terms of X. RUN the program. The "closeness factor" is the average of the vertical distance the line is from each of your data points. (Your data points are shown as squares on the graph.) If you want to overlay the line to see if you are improving, press O when given the choice. Insert a new equation on line 300 and then RUN the program.

An Application of Ratios and Percents

Objective

To practice the concepts of ratio and percent with real data.

Description

This spreadsheet template has all fifty states with their populations in 1960, 1980 and their area in square miles. The purpose of this program is to give students an opportunity to ask questions and answer them and to more fully develop their concepts of ratio and percent. This data gives students practice with these concepts without getting bogged down in the computation. The program also gives students an opportunity to practice their spreadsheet skills and gives them a reason for learning how to manipulate data, labels, and formulas with replication inside a spreadsheet.

Procedure

Load the spreadsheet template called "US Geography". Notice that there are three columns of data. Use the sorting procedures in Appleworks to determine which were the most populous states in 1980. First make a prediction. Then check. Were you right?

Were these the same states which were the most populous in 1960?

Which were the least populous states in 1980? Make a prediction and then check. Were you right?

Which were the least populous states in 1960? Were these the same states which were the least populous in 1980?

How would you compute the percent growth of each state between 1960 and 1980? Establish the formula and then replicate it for each state.

Now predict which state grew the most in that period of time. Sort the states by percent growth (most first). Was your prediction accurate?

What types of states experienced the greatest amount of growth during the 20 year time period? What do they have in common? Why do you think these states grew as much as they did relative to the others?

Did any state decrease in population? Which one? Why do you think it decreased while the others all increased?

Notice Nevada's percent of increase. Explain what that number means in your own words.

Now we will compute population density. Population density is the average number of people per square mile. How would you generate that data for each state?

(Helpful hint: you might want to round off the population density and percent increase to two decimal places. It would make the table easier to read.)

Predict which states would have the greatest population density? Check your answer with the data. Were you right?

What types of states have the lowest population density. (Less than 100 people per square mile). What do these states have in common?

If you extract the 7,500,000 people in Chicago and reduce the square miles of the state of Illinois by the 1600 in the Chicago metropolitan area, how much does it change Illinois' population density? What kind of state would you say the rest of Illinois is outside of Chicago?

Session 10S

Using Computer Graphing as a Tool

Using Computer Graphing as a Tool
(for High School Mathematics Students)

By

Bert Waits and Franklin Demana
Department of Mathematics
The Ohio State University

Overall Objectives of Computer Graphing in High School Algebra II, Analytic Geometry and Trigonometry

(1) To study the behavior of functions and relations including conics, parametric equations, and polar equations.

(2) To gain geometric intuition about a wide variety of graphs of functions and relations important in the future study of calculus and statistics.

(3) To graphically determine the number of real solutions to equations and systems of equations. To solve equations, systems of equations, and inequalities graphically with accuracy equal to any numerical approximation method.

(4) To determine relative maximum and minimum values of functions graphically with accuracy equal to any numerical approximation method.

(5) To graphically explore the solution of "real world" problem situations that are normally not accessible to high school students.

COMPUTER GRAPHING IS A FAST, FUN, AND EFFECTIVE TOOL FOR EVERY MATHEMATICS STUDENT TO USE TO EXPLORE MATHEMATICS AND SOLVE PROBLEMS.

Definitions:

(1) A **graphing utility** is any device that will draw the graph of a function or relation. The graphing utility may be microcomputer based or calculator based (*Casio fx-7000G* or Sharp EL-5200 or HP 28C).

(2) A **viewing rectangle [L, R] by [B, T]** (see Figure 1) is a rectangular portion of the coordinate plane given by $L \le x \le R$ and $B \le y \le T$. The **default** or **standard viewing rectangle** is **[-10, 10] by [-10, 10]**. The Apple 2 graphing program you will use with these worksheets first opens up in the default or standard viewing rectangle.

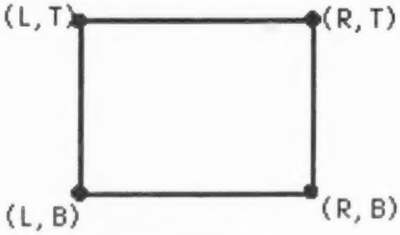


Figure 1

(3) **Zoom in** is a process of framing a small rectangular area in a given viewing rectangle, making it the new viewing rectangle, and then quickly replotting the graph in this new viewing rectangle. Zoom in is very useful for solving equations, systems of equations, inequalities, and for determining maximum and minimum values of functions.

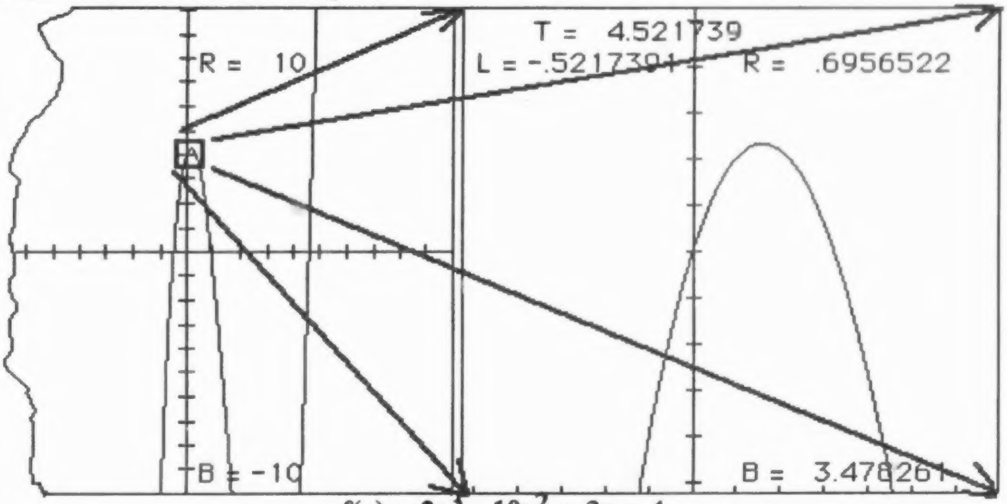
(4) **Zoom out** is a process of increasing the absolute value of the viewing rectangle parameters. It is important to be able to zoom out in *both* the horizontal and vertical directions at the same time, in the horizontally direction *only* or in the vertical direction *only*. The zoom out process is useful for examining limiting, end behavior, of functions and relations and for determining "complete" graphs.

(5) A **complete graph** is the entire graph displayed in an appropriate viewing rectangle (like $x^2 + y^2 = 16$ in [-10, 10] by [-10, 10]) or a *portion* of a graph displayed in an appropriate viewing rectagle which shows all of the important behavior and features of the graph (like $f(x) = x^3 - x + 15$ in [-10, 10] by [-10, 30]). It is possible to create a function for which you cannot determine *one* viewing rectangle which gives the complete graph. Thus several viewing rectangles may be needed to describe the complete graph.

(6) The **error** in using a point (x, y) in the viewing rectangle [L, R] by [B, T] to approximate any point (a, b) in the viewing rectangle is *at most* R - L for x and T - B for y. Of course, there are better error bounds possible by overlaying a lattice in a viewing rectangle or by using scale marks appearing in a viewing rectangle.

The Apple 2 function grapher *Advanced.grapher:*
This is the single variable function grapher designed by Bert Waits and Frank Demana of The Ohio State University that you will be using with these worksheets. This grapher has the following features which *you* can control.

(1) Enter any function that can be written in BASIC syntax. This means you can graph almost any function you can think of (use key E).
(2) Enter any viewing rectangle (use key A).
(3) automated zoom out (use key B).
(4) automated zoom in (use Key C).



$f(x) = 2x^3 - 10x^2 + 3x + 4$

ZOOM IN

(5) Overlay a lattice (grid) (use key D).
(6) Enter a second function called function G (use key F).
(7) Overlay the function G (use key G). Keys F and G may be used to overlay as many functions as you like to the original graph of the function F (entered by using key E).
(8) Change plotting speed (plot speed and plot resolution are inversely related). The default plot speed is very fast and has good resolution (use key M).
(9) Change plot mode to use points or line segments (use key J).
(10) Graph the inverse relation (use key I).
(11) Overlay and move a "cross hair" anywhere in the viewing rectangle and determine the coordinates of a desired point in the viewing rectangle (use key H).
(12) Replot the graph (use key K) in the current viewing rectangle.
(13) Overlay and move a vertical line anywhere in the viewing rectange and determine the x coordinate of the vertical line. (use key L).
(14) Overlay and move a horizontal line anywhere in the viewing rectange and determine the y coordinate of the horizontal line. (use key O).
(15) Replot in the default or standard viewing rectangle. (use key P).

Notes:

(1) It is expected that you will work through the worksheets in numerical order. More explanation is given in the first three worksheets.

(2) To stop the plot at any time, press the ESC key.

(3) Sometimes when plotting functions in viewing rectanges which contain discontinuities of the function, the program crashes or does strange things and "locks up". Usually the program can recover by pressing the CONTROL key and the RESET key simultaineously and then the K key (sometimes twice in succession). For example, the program may crash or "lock up" when graphing $f(x) = x^{1/2}$ in the [-10, 10] by [-10, 10] viewing rectangle. However, a CONTROL and RESET action followed by pressing the K key will yield a correct plot. If all else fails, reboot and choose a more appropriate viewing rectangle.

NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 1

Consider the following function:

$$f(x) = (x^3 - 10x^2 + x + 50)/(x-2)$$

Objectives: Graphically determine the "end behavior" of the function. That is, the behavior of the function values f(x) as x Æ ∞ and as x Æ - ∞. Determine the complete graph of y = f(x).

Method: Graph the function in appropriate, large viewing rectangles. Use zoom out.

Activities and questions to explore:

1. Run *Advance.grapher* (Boot the disk, press the spacebar to avoid reading the directions, then press the #1 key to load and run *Advance.grapher*). The default function F you will see is the sine function. Next enter the desired function (the function given above) by pressing the E key. Use BASIC syntax (^, *, /, etc.). Then press RETURN to obtain its graph in the default viewing rectangle.
2. Can you determine what happens to the function values as | x | gets large from this, the default, viewing rectangle?
3. Press the A key to enter the new viewing rectangle [-20, 20] by [-200, 200]. Press RETURN after entering each of L, R, B, T (in that order). Do you now "see" the complete graph? Why?
4. Press the J key and then the K key. Explain any difference(s) observed.
5. Explore the speed key (Key M). Notice that plotting speed and plot resolution are inversely proportional. Use the M key. Choose a speed. Then press the K key.
6. Press the P key to obtain the default plot. Press the D key. What do you observe?
7. Press the B key to Zoom out by a factor of 10 in the horizontal direction and by a factor of 1000 in the vertical direction. You are first asked to enter the horizontal factor. Press RETURN. Next you are asked for the vertical factor. Do you see why these two zoom factors were chosen?
8. Press the F key and install the function $y = x^2$ then press RETURN.
9. Overlay the function $y = x^2$ by pressing the G key. You do not need to press RETURN.
10. What can you conclude about the "end behavior" of f? Why?

Remarks: Think about the many possible generalizations and conjectures students and teachers can make *and* test using computer graphing and zoom out.

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NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 2

Consider the following function:

$$f(x) = (x^3 - 10x^2 + x + 50)/(x-2)$$

Objectives:

(a) Determine graphically the number of solutions to the equation $f(x) = 0$. Estimate the solutions.

(b) Solve the equation $f(x) = 0$ graphically with error less than 0.000001

Method: Look for all of the x-intercepts (zeros), that is, determine where the graph of the function f crosses the x-axis. Use zoom in to determine accurate solutions by traping the solution(s) in very small viewing rectangles.

Activities and questions to explore:

1. From your experience with Worksheet #1, you should be able to determine a viewing rectangle that displays all of the zeros of f. How many solutions (x-intercepts) are evident?
   Estimate the solutions. See #3, worksheet 1.
2. Press the C key to use zoom in to the determine each solution to the desired accuracy. Use the A, Z and left and right arrow keys to move a "crosshair" to determine the upper left corner of the new viewing rectangle. Press the space bar to "set" the upper left corner. After pressing the space bar, notice that the "crosshair" disappeares. Then use the same keys to actually draw a rectangle on the screen. Choose a rectangle which contains the desired zero. Press the spacebar again to draw the new graph in the zoom in rectangle.
3. Continue to use zoom in until you find a viewing rectangle with the specified error for each zero.

Remarks: Check that -2 is an exact zero of f. Use this fact to factor the numerator of f and then find the other two zeros algebraically. Compare with #3 above.

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NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 3

Consider the following function:

$$f(x) = (x^3 - 10x^2 + x + 50)/(x-2)$$

Objectives: Determine graphically the coordinates of all absolute and local maximum and minimum points accurate to .000001.

Method: Graph the function in an appropriate viewing rectangle(s) and zoom in on all extrema.

Activities and questions to explore:

1. Consider your experience with this function from Worksheets 1 and 2. Graph the function in the viewing rectangle [-10, 10] by [-100, 100].
2. Press the D key. Now *estimate* the coordinates of the extremum point.
3. How many extrema (and what kind) are there? Explain you answer.
4. Press the K key. Press the L key. Move the vertical line by pressing the left and right arrow keys. Press the space bar to "read" the x-coordinate. Press any key and then the ESC key to return to the main menu.
5. Press the O key. Move the horizontal line by pressing the "A" (up) and "Z" key (down). Press the space bar to "read" the y-coordinate. Press any key and then the ESC key to return to the main menu.
6. Use zoom in to determine the coordinates of any extrema to the desired accuracy.
7. After using zoom in a few times press the H key to "read" both of the coordinates of a point. Move a "+" or "crosshair" using the A, Z and left and right arrow keys to the desired location and then press the space bar to obtain the coordinates. The coordinates are displayed in the lower right corner, x first, then y.

Remarks: As you zoom in at an extremum point the graph will appear flat. To highlight the extremum point, use rectangles that are longer than they are tall. Solve this problem analytically using calculus. Comments?

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Computer Graphing Worksheet 4

Consider the inequality $|x^3 - 15x + 1| < 2x - 3$.

Objectives: Determine graphically the solution to the inequality $|x^3 - 15x + 1| < 2x - 3$. Give the answer in interval notation where the endpoints have error less than 0.000001.

Method: Graph *both* sides of the inequality in an appropriate viewing rectangle(s) Determine all values of x where the graph of the left side is **below** (<) the graph of the right side. Use zoom in to determine the endpoints.

Activities and questions to explore:

1. Press the E key. Enter the left side of the inequality as the function F. Note: enter **|x|** as **abs(x)**.
2. Press the F key. Enter the right side of the inequality as the function G.
3. Press the G key to overlay the graph of the right side function to the existing graph of the left side function.
4. Change to speed 1. Press the A key to graph in the viewing rectangle [-6,7] by [-20, 30]. Press the G key to overlay the graph of the function G. Comments?
5. Use zoom out to confirm the number of points of intersection of F and G suggested by the viewing rectangle in #4.
6. Next use zoom in to find the points of intersection to the desired degree of accuracy. You will have to press the G key each time after using zoom in.
7. Write the answer in interval notation using the information in part 5. Remember to choose the interval(s) where the graph of the left side is **below** (<) the graph of the right side.

Remarks: This inequality is beyond the algebraic skills of most college calculus students. However, high school students with the aid of computer graphing can quickly and effective solve this problem. Try to solve this problem analytically. Comments?

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NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 5

Consider the two functions

(a) $\mathbf{f(x) = x \sin x}$
(b) $\mathbf{g(x) = x \sin(1/x)}$

Objectives: Explore the behavior of these functions. Determine their "end behavior".

Method: Graph the functions in a variety of different viewing rectangles.

Activities and questions to explore:

1. Press the E key and enter the function in part (a), $f(x) = x \sin x$. Note enter **sin x as sin(x).** Use the A key to graph f in the [-50, 50] by [-50, 50] viewing rectangle or use the B key to zoom out by a factor of 5. What is limit of f(x) as x approaches ∞ (or $-\infty$)?
2. Press the F key and install the function $y = x$. Press the G key. What are the solution(s) to the *system of equations* $y = x \sin x$ and $y = x$?
3. Press the E key and enter the function in part (b), $f(x) = x \sin(1/x)$. BEFORE PRESSING THE RETURN KEY, **cover your computer screen with a large piece of thick paper. PRESS THE RETURN KEY. Wait about 10 seconds. Press the A key then remove the paper from the screen.** Enter in the [-0.2, 0.2] by [-0.2, 0.2] viewing rectangle. Change to speed = 1 (best possible resolution) by pressing the M key. Press the J key and then the K key. Do this again. Which type of plot do you prefer. Why?
4. Based on your graph found in #3 in the [-0.2, 0.2] by [-0.2, 0.2] viewing rectangle, what do you think the limit of $g(x)= x \sin(1/x)$ equals as x approaches ∞ (or $-\infty$)?
5. Press the B key to zoom out in both directions by a factor of ten. (Start with the same viewing rectangle in #4 above). Now what do you think the limit of g(x) equals as x approaches ∞ (or $-\infty$)?
6. Replot the graph of the function in part (a), $f(x) = x \sin x$, in the default viewing rectangle (Using the E key). Press the I key. Comments?

Remarks: Computer generated graphs can be misleading and they need to be carefully interpreted. For example:

(1) Draw the graph of $f(x) = (1-\cos(x^6))/x^{12}$ in the [-0.3, 0.3] by [0, 1] viewing rectangle. Comments? What is the limit of f(x) as x approaches 0? (It is 1/2).
(2) Draw the graph of $f(x) = \sin(63x)$ in the [-10, 10] by [-10, 10] viewing rectagle. Is the displayed graph correct? What is the period of $y = \sin(63x)$?

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NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 6

Consider the following "real world" problem situation:
A box with no lid is formed by cutting out equal squares from the four corners of a 32" by 75" piece of cardboard and folding up the sides. The volume of the box is a function of the size of the square removed. If x is the side length of the removed square, then the volume V = LWH or $V = x(32 - 2x)(75 - 2x)$. Here H = x.

Objectives: Determine graphically the dimensions of the box with maximum possible volume. The dimensions should have error less than 0.000001.

Method: Determine the complete graph of V, the values of x which represent the problem situation, and the maximum value(s) of V for those values of x which represent the problem situation.

Activities and questions to explore:

1. What restrictions must be placed on x so that V given above represents the problem situation?
2. Press the E key to enter the volume function.
3. Do you need a larger viewing rectangle?
4. Determine a viewing rectangle that displays a complete graph of the volume function. How many zeros are there?
5. How many local maximum and mimimum values does the function V have? Which, if any, are solutions to the problem situation? Why?
6. Use zoom in to determine the value(s) of x to the desired accuracy that gives the box(es) with maximum volume.
7. Write the answer to the problem. L = ?, W = ? and H = ?
8. What is the maximum volume? How accurate is your answer?

Remarks: This problem is usually reserved for calculus students. However, Algebra II students can quickly and effectively solve this problem using a graphing utility. Observe the meaning of **extraneous solutions** graphically. Solve this problem analytically using calculus. Comments?

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NCTM Computer Inservice Summer of 1987
Computer Graphing Worksheet 7

Consider the following "real world" problem situation:
Pam can afford to pay $200 per month for 48 months for a installment car loan. Pam wants a car that requires a $7000 loan. What APR loan rate does she need to find?

Background: This so-called **installment loan** is an example of an ***annuity***. Home mortgage loans and retirement annuities are other common examples of annuities. The APR rate is the *annual precentage rate* as defined by Federal Truth in Lending Law. The true monthly annuity interest rate is the APR rate divided by 12.

Let **R** be a periodic annuity payment (similar to a monthly car or monthly home mortgage payment) made for **n** periods (months) at periodic (monthly) interest rate **i** (decimal).

Let **S** denote the future value (n periods from now) of this annuity and **A** denote the present value (value today) of this annuity.

It is a fundamental principal (compound interest) from the mathematics of finance that S and A are related by the equation

$$S = A(1 + i)^n \qquad (1)$$

It is also true that S is the accumulated future value of the total of the n payments of R dollars each, that is,

$$S = R(1 + i)^{n-1} + R(1 + i)^{n-2} + \ldots.. + R(1 + i) + R \qquad (2)$$

Thus, by summing this finite geometric series, we have

$$S = R((1 + i)^n - 1)/i \qquad (3)$$

It then follows from (1) that

$$\mathbf{A = R(1 - (1 + i)^{-n})/i} \qquad (4)$$

Equation (4) is the **Present Value** annuity formula (**mathematical model**) we need to solve the above problem.

Objectives: Determine graphically the true monthly annuity interest rate i with error less than 0.0001, given that A= 7000, R = 200 and n = 48 in Equation (4). Find the APR rate need by Pam.

Method: Graph *both* sides of the equation

$$7000 = 200(1 - (1 + i)^{-48})/i.$$

Look for a common point or points of intersection of the two graphs.

- OVER -

Activities and questions to explore:

1. Enter the two functions. Note that the independent variable must be denoted by x.
2. What values of x (i) make sense in this problem situation.
3. Considering your answer to part #2 and the fact that one graph involves the number 7000, what would be a reasonable first viewing rectangle to choose to graphically study this problem? (One reasonable answer is [0,1] by [0,10000].) Be sure you understand why this is reasonable.
4. How many points of intersection are there? Can you find a more appropriate viewing rectangle?
5. Why is the function on the right hand side of the equation a decreasing function? Interpret this in financial terms.
6. Use zoom in to find x (i) to the desired degree of accuracy.
7. Why does the function (the right side of the equation) appear not to have a vertical asymptote at $x = 0$? What type of discontinuity occurs at $x = 0$?

Remarks: Notice that this problem has no "closed form" solution. (Try solving Equation (4) for i.) Graphically the solution is easy once a good viewing rectangle is found. Solve this problem using Newton's method from calculus.

NOTE: Sometimes the lower right text window may flash "on and off" and the program will "lock up". If this happens, press the CONTROL and RESET keys and then the K key. This will usually restore the program.

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Session 11E Spreadsheet Exercises

1. Find the total price for the equipment bought in each problem.
   (McGraw-Hill, Grade 3, 1981)

   1. 2 knives and 1 compass
   2. 1 compass and 3 flashlights
   3. 1 knife, 1 flashlight, and 1 rain hat
   4. 1 flashlight, 1 compass, 1 rain hat, and 1 knife
   5. 4 flashlights, 4 compasses, 1 knife

2. The airline sold 300 tickets altogether for the flight to Dallas. They sold 43 first-class regular tickets and 76 coach-class regular tickets. The remaining tickets were discount tickets. How many discount tickets were there?
   (Grade 4, McGraw-Hill, 1981)

3. The Atlas Motor Company is shipping 1170 standard autos, 1080 vans and 2058 compacts by train. Each tri-level, open freight car holds 15 standard autos, 12 vans, or 21 compacts. Use a spreadsheet to find out how many freight cars are needed.
   (Harper and Row, Grade 5, 1981)

4. There are five math tests this quarter. On the first four, Ann's scores were 76, 86, 95, and 99. What is the lowest score she can have on the fifth test and have an "A" test average? An "A" is 90% to 100%.
   (Harper and Row, Grade 5, 1981)

5. Imagine you have $300 to spend at the pet shop. The prices of the animals are the following: doberman $112.99, dachshund $59.96, setter $62.98, poodle $49.97, afghan $149, angora kitten $159.98, siamese kitten $75.89, 6 guinea pigs $21.89, lovebird $25.89, and cockatiel $89.00. Choose a puppy, kitten, and bird. Find your total bill and the amount of change you would receive.
(Harper and Row, Grade 5, 1981)

6. A family had the following bills from the City Gas and Power Company over the period of one year: January $66, February $62, March $43, April $20, May $15, June $15, July $12, August $10, September $15, October $20, November $22, and December $43.
(Grade 6, McGraw-Hill, 1981)

1. What was the difference between the highest and lowest bill?
2. What is the median and mode amount of the monthly bills?
3. What would this family have paid each month under a level payment plan?

7. You need to keep track of the income from a newspaper route of those that paid you and those that have not. Write a spreadsheet similar to the one below to keep track of the amounts owed and the total amount earned from all of the accounts.

NAME	WEEK 1	WEEK 2	WEEK 3	WEEK 4	TOTAL
Jones	2.50	2.50	0	0	5.00
Rodriguez	2.50	2.50	2.50	2.50	10.00
Marshall	2.50	2.50	2.50	0	7.50
				Total:	22.50

8. Track the daily highs and lows for a one week period. At the end of the week determine the averages for the daily high, daily low, and overall average. The spreadsheet might look something like the one below.

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	DAY 6	DAY 7
85	87	83	84	85	82	85
62	60	55	60	62	55	60

AVERAGE HIGH: 84.43
AVERAGE LOW : 59.14
AVERAGE OVERALL: 71.79

9. A budget is a list of what we plan to spend on our house or business in a specified period of time. Below is a budget for a family's expenses. Write a spreadsheet that predicts the total dollar value needed for next year if you increase the amount needed by 6.5%.

Rent	$300
Clothing	$120
Entertainment	$ 25
Car and Gas	$300
Insurance	$ 80
Food	$100
Taxes	$ 30

10. Create a spreadsheet that will compute how much is needed to purchase uniforms for the band for the new school year. Compare that total amount needed by changing the dollar value for different uniforms.

11. Create a spreadsheet that will compute the wind chill factor when you enter the temperature, humidity, and wind velocity.

12. A realtor friend of yours needs a quick way to compute how much space a house has before it can be placed on the market. Use a spreadsheet to compute the total square footage of a house.

13. Use a spreadsheet to assist in converting fahrenheit temperatures to centigrade. Use a chart to show all the temperatures from 0 - 115 degrees fahrenheit.

14. You are writing a report on the number of hours of TV each class member watches TV each day. Create a spreadsheet that will tally the total number of hours each student watches a week and average hours viewed by all of the students. You may want to separate the sheet between boys and girls to find out if the averages are different. Answer such questions as:

1. How many hours a week do boys watch TV?
2. How many hours a week do girls watch TV?
3. How many hours do both boys and girls watch?
4. Which watches more? Why?

15. Create a spreadsheet that keeps track of a bowler's scores and averages for a period of 10 weeks. The bowler plays three games once a week. Compute the weekly and overall averages for each of the ten weeks.

16. Population predictions can be easily made using a spreadsheet. With one change an entire spreadsheet may be recalculated. Create a spreadsheet that contains the population of 5 of the major cities in the United States. Use the latest census figures available. Find out what the population would be for each of these cities if the rate of increase is 5%. What would the population be for the same cities if the rate of increase is 8%?

17. EASY PATTERN
#4576

Fabric Width	Size 8	Size 10	Size 12
Jacket			
45 inches	1 1/4 yd.	1 3/8 yd.	1 1/2 yd.
60 inches	1 1/8 yd.	1 1/8 yd.	1 1/4 yd.
Hood (Optional)			
45 inches	1/2 yd.	5/8 yd.	3/4 yd.
60 inches	3/8 yd.	3/8 yd.	1/2 yd.

(McGraw-Hill, 1987)

Session 11M
Algebra

Concept of Slope

Objective

To verify that the rise/run in the graph of the equation of the form:

$$y=mx+b$$

is equal to the coefficient of x (b).

Description

The function plotter program is used to graph a linear function of the form y=mx+b. The student can select any two points on the graph and compute the ratio of the rise to the run. The student will see that regardless of the points selected s/he always gets about the same answer (within round-off or data collection error) and this answer is the value of m in the equation.

Procedure

Load the program "Function Plotter". Put the center at 0,0 and the scale of 1. When the graph comes up press ESC to terminate execution. Then press E for erase. Now put in your new equation at line 300 then press RETURN and type RUN. The equation is

```
300  Y=2*X-1
```

Pick any two points on the graph. Identify one point as point 1 and the other as point 2. Let X1 correspond to the x-coordinate of the first point, Y1 as the y-coordinate of the first point, X2 as the x-coordinate of the second point, and Y2 as the y-coordinate of the second point.

The purpose of this session is to demonstrate the relationships between slope and coefficient, and y-intercept and the constant in a linear equation.

The RISE corresponds to the distance (either up or down--the rise can be negative) one travels from the first point to the second. The RUN corresponds to the distance across (horizontally either right or left, either positive or negative--the run can also be negative) one travels from the first point to the second.

For example, suppose you select as point 1 the point (2,3) and as the second point (4,7). Verify that each of these points makes the equation true by substituting in the first point 2 for x and 3 for y and in the second point 4 for x and 7 for y.

RISE = Y1 - Y2

3 - 7

-4

RUN = X1 - X2

2 - 4

-2

Rise/Run = (-4)/(-2) = 2

Would you get the same results if you were to reverse the order of the points. i.e. point 1 becomes point 2 and the original point 2 now becomes point 1? Try it and see.

Select two other points on the graph. They do not have to be in quadrant I nor do they have to be in the same quadrant.

What is Rise/Run = ?

Try it with two more points. Is the Rise/Run the same?

Erase the graph, construct the linear equation of your choice but try to keep the coefficient between -2 and 2. If you have trouble determining the coordinates of a particular point, turn the pointer on by pressing P when this option is offered. Move the small pointer around the screen using the up/down/right/left arrow keys (Ctrl-J,Ctrl-K on the II+). Notice that the coordinates of the point are printed near the bottom of the screen. You may not be able to get exactly on the point you want but you should be able to get close.

Y-Intercepts of Linear Equations

Objective

To determine the relationship between the constant term in a linear equation and value of the y-intercept.

Description

The student will use the plotting program to observe the relationship between the constant term and the y-intercept.

Procedure

Run the program "Function Plotter". When it asks for x-center type in 0. When it asks for y-center type in 0. When it asks for scale type in 1. As soon as it starts plotting press ESC to stop it and follow the directions.

What you have typed in is the coordinate of the center of the screen (0,0) and how much each unit is worth (1). If it is possible to put the origin on the screen, then it is displayed. If the origin lies off the screen, a scope is displayed which gives the relative position of plotted points. The center of the scope and the scale indicating how much each unit is worth is displayed on the bottom of the graph. You may stop the plot any time by pressing ESC. You are then given the option to overlay a new graph, erase and start over, plot more points (one time only), or call up a pointer. The pointer appears on the screen and the coordinates of the point identifies are displayed on the screen bottom. The pointer is moved by pressing the arrow keys. Press ESC to exit the pointer. Changing the scale to a smaller number has the effect of magnifying the graph (examining it under a microscope). Adjusting the scale to a larger number is equivalent to moving away from the graph. If you want the big picture select a larger scale, if you want to observe what is happening around a point select a smaller scale. When you press ESC and want to insert a new function always make sure that it is written as a BASIC program line 300 where Y= an proper combination of Applesoft functions.

Plot each of the following and sketch on a sheet of paper:

$y=x+2$

$y=x+4$

$y=x+5$

Predict what $y=x+3$ looks like. Check your answer by plotting with the computer program.

Predict what y=x+0 looks like. Check your answer by plotting with the computer program.

Predict what y=x+7 looks like. Check your answer by plotting with the computer program.

Plot each of the following and sketch on a separate sheet of paper:

y=x+-2

y=x+-3

y=x+-5

Predict what y=x+-4 looks like. Check your answer by plotting with the computer program.

In the table place the coordinate of the point on each line which crosses the y-axis. Next to these coordinates place the corresponding constant term for the equation. What do you notice?

In your own words, what effect does changing the constant term have on the graph?

Plot each of the following:

y=2*x+1

y=2*x+5

y=2*x+-4

Predict what y=2*x+3 looks like. In your own words what "direction" is the graph going? Is it parallel to the others? Check by plotting the equation on the computer.

Plot y=-.2*x+4 Predict what y=-.2*x+1 looks like. Which direction is the graph going? Is it parallel to the first? Check by plotting the equation on the computer.

Coefficients of Linear Graphs

Objective

To determine the relationship between the coefficient of x in a linear equation and the slope of the graph.

Description

The student will use a plotting program like PLOTS to draw the equations given. The student will discover the relationships between the various terms in the equation and the appearance of the graph.

Procedure

Load the plotting program.

Plot each of the following and sketch on a separate piece of paper.

y=1*x

y=2*x

y=3*x

y=4*x

What happens as the coefficient gets larger?

Predict what y=2.5*x looks like. Check your results by plotting it.

Predict what y=0*x looks like. Check your results by plotting it.

Plot each of the following:

y=-1*x

y=-2*x

y=-3*x

y=-4*x

What happens as the coefficient gets smaller?

Predict what y=-2.5*x looks like.

Plot y=2*x. Then try to plot a perpendicular to the graph. What did you get as the equation? (Hint: use a protractor to check)

Plot y=(1/3)*x. Try to plot a perpendicular to the graph. What did you get as the equation?

Can you predict what the equation of the perpendicular would be of the equation: y=4*x. Check your results by plotting with the plotting program.

Session 12 Research Summary

Review of Research

Computers in Mathematics Education, K-12

Until three years ago, it was possible to state that little research had been published on microcomputer uses in mathematics education. That is definitely no longer the case! While most of the research is still in the form of doctoral dissertations, it has begun to overflow into journals.

The majority of the research in this review pertains to microcomputers. However, some summaries draw on research with other computers, and it may be that some of the research semmingly conducted with microcomputers was actually done with other computers. How important that distinction is must be answered by the reader.

Research Summaries

We know from the research of the past 25 years that computers can be used effectively in mathematics education in each of their various applications. Jamison et al. (1974) noted that findings of 'no significant difference' predominated, but some studies did report savings in student time, an index of success. Edwards et al. (1975) concluded that usual instruction supplemented by computer-assisted instruction (CAI) is more effective than usual instruction alone. Vinsonhaler and Boxx (1972) similarly found substantial improvement in mathematics achievement when CAI was used.

Begle (1979) concluded that the findings on computer use

> have been positive, although the percentage of studies finding no significant differences has been larger among the studies involving high cognitive level objectives than among those concerned only with computation .... The results of these studies have not been spectacular. In almost every case of a significant difference, the students using a computer orientation performed better than the control students. However, about the same number of studies reported no significant differences. (pp. 117-188)

In a meta-analysis of 40 mathematics studies, Burns (1982; Burns and Bozeman, 1981) concurred that a mathematics instructional program supplemented with CAI--either drill and practice or tutorial--was significantly more effective in fostering student achievement than a program using only traditional instruction. Bracey (1982) similarly concluded that CAIU is effective in developing mathematics skills. Clements (1984) suggested that programming may affect cognitive style, rather than cognitive ability, while Bright (1983) noted that CAI programs seem to have been constructed to increase academic learning time, and this increase may result in improved efficiency.

Status Surveys

Mathematics has been the predominant subject area in which microcomputers are used (Becker, 1985b; Judd, 1982; Mondy, 1980; Nakafuji, 1986; Price, 1983) and are considered indispensable (Fuchs, 1983). However, Becker reported that a minority (40%) of computer-using teachers in secondary schools are mathematics teachers or computer specialists; indeed, more business teachers (30%) use computers than mathematics teachers (20%). Microcomputers are used in the classroom to a greater extent when teachers use them more often on their own time (Craig, 1986).

Courses most frequently offered were problem solving, computer literacy, and programming (Mayer, 1980; Nakafuji, 1986), although Becker (1985b) indicates that the major uses were for CAI (about half), programming (about one-fourth), problem solving and discovery learning, and word processing. Perceptions of high mathematics and science requirements prompted students to take computer courses (Cole and Hannafin, 1983). However, computers are not well integrated into the curriculum (Smith, 1985).

Most commercial software was for arithmetic, with the emphasis on drill and practice (Becker, 1985a; Blum, 1983; Willson, 1983). The longer a school had a microcomputer, the more it was used for teaching programming and the less for drill and practice (Becker, 1985a). The greatest need for additional microcomputer applications was in mathematics, especially problem solving (Burke, 1983).

In 1983, a majority of schools with microcomputers had fewer than five; fewer than 10% had as many as 15 (enough to serve about half the students in one classroom at a time) (Becker, 1985a). High schools were twice as likely as elementary schools to own microcomputers, and had nearly four times as many (twice as many per student).

In 1985, a majority of elementary schools had five or more computers; more than 5700 elementary schools had 15 or more. Half of the secondary schools had 15 or more computers (the median is 28 computers in large schools and 20 overall). Only 6% of those having computers had only one; 7% of secondary and 15% of elementary schools lack computers (Becker, 1985b). There is clearly variance by location; thus, students in 62% of the mathematics departments in Indiana used microcomputers (Green, 1983).

Where a single teacher dominated the organization of computer use, above-average students had disproportionate access (Becker, 1985a).

When the second national assessment of mathematics was conducted in 1978, few students had had experience using or programming computers (Carpenter et al., 1980). The number of students with access to computers in school doubled between 1978 and 1982, to almost one-fourth of the 13-year-olds and one-half of the 17-year-olds. Students aged 17 who completed a course in computer science and who said they knew how to program a computer also doubled (Carpenter et al., 1983).

Teacher training in microcomputing was the highest priority of teachers for inservice education (Overdorf, 1984).

Computer Literacy

Computer literacy has been the focus of a number of studies; cited are those with particular connections to mathematics instruction.

- A hierarchy consisting of nine concept clusters was devised as the framework for a microcomputer literacy curriculum for intermediate grades (Koontz, 1983).
- Use of microcomputer-assisted drill and practice on computation significantly improved both the affective and cognitive computer literacy of students, but no difference in acquisition of mathematical skills was found compared with a group using an individualized kit in grade 5 (Steele, 1982; Steele et al., 1983, 1984).
- Both a drill-and-practice program and programming instruction improved computer literacy in the affective domain for fifth graders, but only the first improved it in the cognitive domain (Battista and Steele, 1984).
- Students in grades 5-8 gained significantly in computer literacy during a five-week summer session (Ford et al., 1982).
- Mathematics achievement accounted for more variation in problem-solving skills than did computer literacy skills in an analysis of national assessment data for 17-year-olds (Al-Orainy, 1985).
- Sex did not affect scores on a scale assessing cognitive dimensions of computer literacy, but significant sex differences were found for two of six attitudinal variables (Jones, 1984). Ninth-grade males sextyped computers and computing as a male domain more than females did.

- Many students cannot be considered computer literate. For instance, although most students in the third national assessment of mathematics recognized that computers store instructions and information, many did not realize that computers require special languages or that they are suited for doing repetitive, monotonous tasks. One-third of the 13-year-olds and one-fifth of the 17-year-olds believed that computers have minds of their own (Carpenter et al., 1983).

Programming

Studies on the effects of teaching computer programming vary by focus and by result.

- Some studies have found no evidence that students taught computer programming (most often with BASIC) achieve better than those not taught to program (Cheshire, 1981; Collenback, 1983; Jordan, 1985; Milojkovic, 1984; Woods, 1984).

- In one study, fifth graders with no access to computers achieved a high mean gain in problem-solving scores than did those who used computer programming (Reding, 1982).

- Generally, no sex differences were found; when they were, makes scored better than females.

- Mathematics (especially algebra) was among the factors correlated with success in programming in BASIC (Jones, 1986) and in COBOL (Allen, 1985).

- Both direct group instruction and directed independent study appear to be effective means of teaching computer programming (Kasilus, 1983).

- The same processes were used by secondary school students in computer programming and in problem solving: heuristics, subgoals, looking back techniques, trial and error, and regular patterns of analysis and synthesis (Wells, 1981).

- Instruction in computer programming in either BASIC or Logo appeared to have a significant effect on the ability of middle school students at teh concrete level of development to analyze problems (Dvarskas, 1984).

- Students in grades 10 and 11 having instruction in BASIC alone had significantly high achievement than those taught both Logo and BASIC (Calamari, 1984). [Becker (1985a) reported that 84% of the schools he surveyed taught programming only in BASIC.]

A number of studies have focused on the use of Logo. While some reported no significant differences between students given or not given instruction with Logo (Hines, 1985; Horner, 1984), others reported success with using Logo to teach geometric shapes (Assaf, 1986; Pateman, 1986), problem solving (Bamberger, 1985; Evans, 1985), and spatial skills (Roberts, 1985).

Computer-Assisted Instruction (Tutorial)

A large number of studies have involved totorial CAI. As would be expected, the findings are mixed concerning achievement.

- Use of CAI produced higher achievement (and some affective benefits as well)than conventional instruction in many cases (Abegglen, 1985; Abram, 1984;Carmen and Kosberg, 1982; Englebert, 1984; Hallett, 1985; Hawley, 1986; Knerr, 1982; Lawrence, 1986; Levy, 1985; Mason, 1985; Merrell, 1985; Merritt, 1983;Mevarech, 1985; Mevarech and Rich, 1985; Miller, 1984; Mills, 1980; Modisett, 1980; Oden, 1982; Romero, 1980; Shu, 1984; Todd, 1986; Warner, 1981; West, 1985; Williams, 1985; Wright, 1984).
- In at least one case, achievement was significantly related to time on the computer, but gains were not reatined (Hawley, 1984).
- Adaptive and advisement strategies required greater instructional time, with no associated gain in achievement. The linear design required less time (Goetzfried and Hannafin, 1985).
- No significant differences between CAI and conventional instruction were found in other cases (Burrowes, 1983; Carmen adn Kosberg, 1982; Gallitano, 1984; Gifford, 1980; Gleason, 1986; McDermott and Watkins, 1983; Millman, 1985; Saunders and Bell, 1980; Spiegel, 1986; Taylor et al., 1984; von Stein, 1982).
- Significant differences favored the non-computer groups in one study (Signer, 1982).
- Computer-monitored sessions led to more time spent in reteaching than during traditional sessions, but time was not clearly associated with higher achievement (Stepnoski, 1985).

A variety of related factors was considered as well.

- Local teacher evaluation was the most effective in predicting the effectiveness of six CAI programs (Day 1985).
- Students simultaneously enrolled in academic and computer mathematics courses achieved slightly better than students enrolled only in an academic mathematics course (Payne, 1980).
- Age, sex, attitudes, and sibling rank each significantly affected achievement (Scurlock, 1985).
- Significant relationships were found among mathematics achievement, field dependence/independence, and reflective/impulsive cognitive styles. High mathematics achievers tended to be field independent and more impulsive than lower mathematics achievers (Berenson, 1986).
- Secondary school students seemed to be visual-cue-oriented and college students were verbal-definition-oriented (Schonemann, 1984).

The cost of CAI has been considered ever since it was first used in the 1960s.

- CAI costs more than conventional instruction (Hawley, 1986; Modisett, 1980), but is lower when cost-effectiveness (based on achievement) is considered (Ash, 1986; Hawley, 1986). Ash found that the computation cost-effectiveness index of 2.840 for traditional instruction ws more than three times larger than that for CAI (.819), while for concepts the index was 1.086 for traditional instruction and 1.273 for CAI. Hawley reported that the cost of CAI was $.24 per day per student more than the cost of traditional instruction in grade 3, and $.30 more in grade 5. He concluded that when based on cost per unit of gain, or on the value placed on mathematics attainment and computer literacy by school boards and parents, CAI can be considered more cost-effective than traditional instruction.

Use of the computer as a medium for diagnosis was also explored (Hopkins, 1985; Neves, 1982; Stone, 1985; Trifiletti, 1980; Woerner, 1980), with indication that it may be more effective than the teacher in this role.

Other variables explored include feedback. Lee (1985) reported no significant differences for three types of feedback, while children receiving competitive feedback increased attributions to ability but not to effort (Lewis, 1986). Noonan (1984) found that knowledge of correct responses was better than knowledge of results.

Problem solving has also been explored. Extensive computer problem solving enhanced understanding of mathematical topics, according to Hersberger (1983) and Nelson (1986). However, retarded students had comprehension difficulties in recalling appropriate schemata for developing an effective solution strategy (Judd, 1985). Language experience techniques can help students comprehend and solve problems (Grabe and Grabe, 1985).

Additional studies concerned historical background (Solomon, 1986), teacher student interaction (Garrity, 1980), and the use of graphics (Rambally, 1983).

Drill and Practice

Most of the studies on computer-assisted drill and practice focused on the elementary level, where such software is widely used. Of 12 studies, eight reported that no significant differences in achievement were found between groups having computer-assisted drill and practice or non-computer drill and practice (Bukatman, 1982; Easterling, 1983; Foster, 1983; Fuson and Brinko, 1985; Kleiman et al., 1981; Marchionini, 1982; Powell-Rahlfs, 1985) or a tutorial CAI program (Blazejewski, 1984; Powell-Rahlfs, 1985).

In four of the 12 studies, computer-assisted drill and practice resulted in high achievement (Carrier et al., 1985; Davidson, 1985; Haus, 1983; Menis et al., 1980).

No significant difference in attitudes was reported in three studies (Griswold, 1984; Marchionini, 1982; Powell-Rahlfs, 1985).

Other researchers have used drill-and-practice programs to explore other variables. No significant differences were found for different forms of feedback in two studies (Clark, 1983; Dalton and Hannafin, 1985). One study found that both knowledge or results with a corrective procedure and no feedback produced significantly higher achievement than knowledge of results alone (Bumgarner, 1984).

In two studies, no significant differences were found for different types of reinforcement (Brawley, 1985; Dalton and Hannafin, 1984).

Paced drills produced higher correct response rates and better learning than unpaced or match-pace drills (Humphrey, 1984).

The scrambled presentation group in grade 3 had greater accuracy immediately and three weeks later, but the ordered presentation group had greater recall nine months later (Mich, 1986).

Students in grades 4-6 with an internal locus of control learned better with the computer, while those with an external locus learned better from traditional instruction (Bukatman, 1982).

Mentally handicapped students in grades 7-12 improved on computation skills taught with drill-and-practice software, but no differences were observed for LD and EH students (Whitman, 1986).

Games

Eight studies considered the effects of various mathematics games played on microcomputers.

- Games were reinforcing and motivating for most pupils in a third grade (Moore, 1984). Using games as rewards, setting time limits, and playing with a peer were found to serve as extrinsic reinforcers.
- The computer game group responded correctly to twice as many items on the speed test of addition basic facts as did the control group in a second grade (Kraus, 1981).
- Seventh graders learned from computer estimation games (Gordon, 1985). Feedback was helpful, but information on objectives did not increase estimation skill. Students learned to use the strategy of bisecting previous ranges of estimation, but transfer to more difficult tasks was not found.
- Some students in grades 4-6 had little difficulty applying probability concepts and explaining their strategies, while others could not relate the moves to probability (Schroeder, 1983).
- The roles of challenge, fantasy, and curiosity in games differed for elementary school boys and girls (Malone, 1980).

- No significant differences in scores were found between learning disabled and average pupils in grade 2 on a subtraction game in a destroy-the-invaders format (Nelson, 1984).

- Significant differences favored boys aged 13 on a spatial ability measure after playing computer games (Pepin et al., 1985).

- Achievement gain was higher for the sixth-grade class using a microcomputer for three geometry strategy games than for the class not using a computer (Morris, 1983).

Computer-Managed Instruction

No significant differences in student achievement characterize each of three studies of computer-managed instruction of mathematics. Bailey (1984) compared a computer-managed sequence with the instructional sequence suggested by a textbook series used in grades 4-6. Beck (1985) considered objectives across the K-12 curriculum, so that tests could be formed and data maintained for 300 students. Crenshaw (1983) gave some primary-grade children instruction managed by computers, while records were maintained systems to non-computer systems, but students had no preference and there was no significant difference in achievement. Each researcher concluded that computer-managed instruction was feasible, but could not be promoted on the basis of student achievement.

Attitudes Toward Computers

Seven studies that focused on attitudes varied in their approaches and thus in their findings.

- All students in fifth grade who used microcomputers for mathematics drill and practice and some programming daily for a semester were strongly positive toward microcomputer use (Kahn, 1985). The children believed that microcomputers will improve education, that all students should learn about them, and that both boys and girls, at all ability levels, are equally interested in them.

- Positive correlations were found for students in grades 11 and 12 betwen attitudes toward using a computer and attitudes toward mathematics (DeBlassio and Bell, 1981).

- Prior computer experience had the greatest impact on attitude toward computers in grades 7 and 8 (Fertsch, 1985).

- No significant sex differences were found in attitudes toward computers (Fertsch, 1985).

- Use of computer programs in either Logo or mathematics drill and practice in third grade did not increase intrinsic motivation or perceived competence (Forte, 1985).

- Students in the third national assessment of mathematics exhibited positive attitudes toward computers. About three-fourths of them at both ages 13 and 17 thought that computers were useful for teaching mathematics and made mathematics more interesting (Carpenter et al., 1983).

- Mathematics and science teachers had more favorable attitudes toward computers than did other teachers (Ruffin, 1985; Whitfield, 1984).

Review of Research

Computers in Mathematics Education, K-12

Summaries

Can computers help students learn?

Drill and practice	YES
Problem solving	YES
Tutorial instruction	YES
Management	YES
Games	YES
Simulations	YES
Information storage	YES
Programming	YES

- Jamison et al. (1974):

  CAI effective as a supplement.
- Edwards et al. (1975), Vinsonhaler and Boss (1972) agree.
- Begle (1979) adds:

  But - findings of no significant difference outnumber positive findings - though negative findings are rare.
- Burns (1982), Bracey (1982), Kulik (1983, 1984) all concurred.
- Clements (1984):

  Programming may affect cognitive style, more than ability.
- Bright (1983):

  CAI increases learning time.

Status

From the creation of the first computer in the '40s until 1977, about 1/2 million computers were sold.

In 1983 alone, 6.7 million microcomputers were sold.

- From Becker's 1983 survey (Becker, 1985a):
    - majority of schools now have micros
    - secondary schools more likely to have one
    - secondary schools with 5 or more micros doubled between June 1982 and January 1983
    - elementary schools in 1983 where secondary schools were in 1981
    - many elementary schools have only cassette-based micros
- From Becker's 1985 survey (Becker, 1985b):
    - majority of elementary schools had 5 or more computers
    - half of secondary schools had 15 or more
    - elementary schools in 1985 where secondary schools were in 1983
    - 7% of secondary and 15% of elementary schools lack computers

Percentages of Schools with Computers

	elementary	secondary
June 1980	3%	21%
June 1981	10%	38%
June 1982	22%	55%
January 1983	42%	76%
January 1985	85%	93%

Uses

- Mathematics has been predominant subject area using micros.
- 40% of computer-using teachers are mathematics or computer specialists -- yet more business teachers use them than mathematics teachers -- 30% vs. 20%.

NAEP: 1978 -- few students had computer experience

1981 -- number doubled: 1/4 of 13-year-olds, 1/2 of 17-year-olds

- Most commercial software is for arithmetic, with the emphasis on drill and practice
    - focus on recall of previously learned facts
    - no stress on higher-order skills
    - few programs teach concepts
    - rarely are graphics embedded in the instructional content
    - little user control
    - little remediation in feedback
- Most software targeted for elementary schools - yet secondary schools remain larger users -- teaching about computers and how to program them.
- However, this is changing -- rather quickly . . .

Type of Use in 1983 (Becker, 1985a)

Introduction to computers	64%	85%
Drill and practice	59	31
Programming instruction	47	76
Tutoring for special students	41	20
Programming to solve problems	27	29
Recreational games	24	19
Demonstrations, simulations	20	22
Administrative use	10	14
Teacher record-keeping	7	15
Teacher tests, worksheets	5	10
Student papers, word processing	3	7
Business education/vocational	-	29

- Major uses in 1985 (Becker, 1985b):

CAI	about 50%
programming	about 25%
problem solving/discovery	
word processing	

- As experience increases, use of computers for drill

  decreases

  and use for programming

  increases

  at both elementary and secondary levels.

Findings

Computer Literacy

- Drill-and-practice programs improved both cognitive and affective computer literacy.
- Mathematics interest and achievement were related to computer literacy outcomes.
- Boys and girls did not differ in achievement, although boys tended to sextype computer use as a male domain.

Programming

- No significant difference - 5 studies
- Achievement higher
    - with programming - 8 studies
    - without programming - 1 study
- Generally, no sex difference
- Same processes used in programming and problem solving
- Logo - no significant difference - 2 studies; specific success - 5 studies.
- Programming aids in developing logical reasoning skills.
- Programming or manipulating graphics aids in developing spatial skills.

Tutorial CAI

- Higher achievement - 25 studies
- No significant difference - 12 studies
- Lower achievement - 1 study
- Costs more than conventional instruction, but is more cost-effective.
- Useful for diagnosis

Summaries of studies with mainframe computers concluded:

- Instruction supplemented by CAI is more effective than regular instruction alone.
- When CAI is substituted for regular instruction, CAI was more or equally effective.
- No CAI mode (e.g., drill, tutorial, problem solving, simulation) was consistently more effective.
- CAI was equally effective compared with individual tutoring, programmed instruction, film.
- Students using CAI took less time to learn.
- Achievement gains were related to the amount of time spent on CAI.
- CAI seemed particularly helpful for low-ability students -- but micros aid all.
- Personality affected achievement on CAI -
    - Sensing types completed segments faster than Intuitive types
    - Extraverted Perceptive types tended to drop out of CAI courses
- Attitudes toward CAI and computers were generally positive.
- Students often considered the computer more "human" than humans.

Drill and practice

- Higher achievement - 4 studies
- No significant difference - 8 studies
- As with tutorial CAI, a number of studies considered program variables (e.g., feedback) or student variables: findings diverse.

Games

- reinforcing
- motivating
- little transfer
- roles of challenge, fantasy, and curiousity differed for boys and girls

CMI

- No significant difference - 3 studies
- Teachers preferred CMI to non-computer systems

Attitudes

- positive correlation between attitudes toward computers and attitudes toward mathematics
- generally, student attitudes toward computers positive
- generally, no sex difference

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Session 13E
Beginning Logo*

To begin using Logo follow these steps:

1. Insert the Logo disk into the disk drive.
2. Turn on the Apple microcomputer--the power switch is on the back left of the micro.
3. Turn on the monitor.
4. Press the RETURN key. Ignore the message regarding inserting your own disk. Wait until the screen is blank except for the message.

```
WELCOME TO LOGO
?
```

A white flashing box appears in the upper left corner next to the question mark. It is called the cursor.

5. If you make typing errors, or typos, just type the line over. If you discover an error before pressing RETURN, back up the cursor using the left arrow (find it on the right side of the keyboard) and type the line from the point of error.
6. Type in the word SHOWTURTLE (or ST for short) and press RETURN. You are now ready to create some turtle graphics. What in the world are turtle graphics? I'm glad you asked.

---

*Source: *Apple Logo Primer*, Reston Publishing Co., 1983.

In the center of the screen is a turtle (you may have thought it was just a triangle). The turtle knows how to move about the screen going forward and backward. The command FORWARD (or FD for short), followed by a space and a number, tells exactly how many "steps" you want the turtle to take in the direction in which it is pointing. Now it is pointing straight up. Type in this command (and always remember to press the RETURN key after you are finished typing a line):

```
FD 50            or            FORWARD 50
```

The turtle just marched 50 turtle steps forward. Make it go 25 more steps forward. Try having it go just one step and watch carefully as you press RETURN to see if you can determine any movement. One turtle step is very small.

The turtle can go backward too. FORWARD and BACK are the two most important commands in Logo. The command BACK (or BK for short) causes the turtle to move the opposite direction from which it is heading. Enter this line:

```
BK 155           or            BACK 155
```

Notice that the turtle went backward past the point it began by 79 steps. What happens if you instruct it to go back 200 steps? Were you right? It moves so quickly you can hardly tell that it is moving down the screen and coming back on the top--since it ran out of space at the bottom of the screen. This is called wrapping around the screen.

Try to get the turtle into the center of the screen again. Just backtrack the instructions you have already given. An easier way to do this is to type the word CLEARSCREEN (CS for short), which is quicker than backtracking since it clears the screen and places the turtle in the center of it.

This clever turtle can also make turns to the LEFT (LT for short) and RIGHT (RT for short). Whatever direction the turtle is pointing, it will turn the number of degrees you tell it, in the direction you indicate (to the left or right). Enter these lines and observe the turtle as you press RETURN each time:

```
CS
RT 90

LT 90

RT 180

LT 360
```

The last line causes the turtle to turn completely around. The movement is so fast that it can hardly be perceived.

Combining these four commmands, FORWARD, BACK, RIGHT and LEFT, the Logo user can design an unlimited number of graphic objects. If you wanted to draw a square, here are some lines that you might begin with:

```
CS
FD 100
RT 90
FD 100
RT 90
```

Finish this program by adding lines that instruct the turtle to complete the square that is begun. Once you have mastered the square, experiment with other objects. (Don't forget to use the CS command each time you begin a new design.)

Check out this command:

```
CS
REPEAT 4[FD 50 RT 90]
                          SHIFT  M
        SHIFT  N
```

The square brackets in this line are special Logo characters made by holding down the SHIFT key and pressing the N for the left bracket and the M for the right bracket. Parentheses cannot be substituted.

If you want to draw a square, the above program is one way to do it. It saves a lot of time compared to entering a series of FORWARD and RIGHT moves. The REPEAT 4 command tells the number of times you want the turtle to do whatever is inside the brackets. First the turtle goes forward 50 steps and then it turns 90 degrees; the second time around the turtle follows the instructions inside the brackets again, going another 50 steps forward and turning 90 degrees; and it does the same two more times and stops because it only was supposed to do it 4 times. What happens if you change the 4 to 8 and also change the numbers within the brackets? Enter this

```
CS
REPEAT 8[FD 25 RT 45]
```

You already learned SHOWTURTLE (ST is the abbreviated form). It is used to cause the turtle to appear when it is not on the screen. There is also the opposite command called HIDETURTLE (or HT for short). See what happens when you enter this command and press RETURN. The turtle is still in the same place, but you just can't see it. How can you prove this last statement? Simply type a FORWARD or BACKWARD instruction, indicate a number of steps and watch what happens. Now make the turtle reappear.

Another fun thing you can do with the turtle is to tell it when to draw and when not to draw. PENUP and PENDOWN are turtle commands that stay in effect until you give the next pen command. When you first load Logo, the pen is down, ready to draw. When the pen is up, the turtle will move, but no drawing will occur.

Refer to the following table:

Abbreviated Instructions	Nonabbreviated Instructions
CS	CLEARSCREEN
ST	SHOWTURTLE
PU	PENUP
RT 90	RIGHT 90
FD 75	FORWARD 75
PD	PENDOWN
BK 75	BACK 75
LT 100	LEFT 100

On the left is the short version and on the right is the long version. Both programs will produce the same thing. (Use the abbreviations whenever possible. There are quick references to abbreviations on the inside covers.)

PENERASE is another pen command which can also come in handy. Enter one of these four line programs:

```
CS                      CLEARSCREEN
FD 80                   FORWARD 80
PE                      PENERASE
BK 80                   BACK 80
```

If you draw a line you don't like, simply erase it by going back over it using the PENERASE command. Draw a square and then erase it. (It's not fair to use CLEARSCREEN!)

One additional pen command is PENREVERSE. The short form is simply PX. This command is tricky. It erases any line it goes over and draws on anything that is blank. In other words, it is actually reversing anything on the screen that it comes into contact with. As you enter each line below, watch the screen carefully:

```
PX
FD 80
BK 80  (Reverse line 2 since turtle has left a trail)
FD 80
RT 90
FD 60
LT 90
FD 15
LT 135
FD 150 (Erases the points on the lines already drawn where it crosses them--
                    otherwise it draws on the blank screen)
```

After using the PX command you must indicate your desire to change this pen mode by entering PD or PU or the PENREVERSE stays in effect. At this time type PD to eliminate the PX mode.

The cursor is an important symbol on the computer because it is always there to tell you your location. There are some helpful control keys that, when pressed after you hold down the key labeled CTRL located on the right side of the keyboard, produce special movements of the cursor. If you make a typing error and discover it before you press RETURN, you can move back or forward over the line to make the correction.

- CTRL B moves the cursor back one step (not the turtle).
- CTRL F moves the cursor forward one step.
- CTRL D deletes the character under the cursor.

For example, clear the screen and enter this line as you see it and press RETURN.

```
FF 100
```

The error message appears

```
I DON'T KNOW HOW TO FF
```

This is the way in which you are informed that you have made an error. (The error messages in Logo are considered to be quite polite compared to other languages!) Enter FF 100 again, but don't press RETURN. To correct the error, hold down the CTRL key and press the B. Each time you press the B, the cursor moves back one space over each character. Move it all the way back to the second F and not holding down the CTRL key, type a D. The line now reads

```
FDF 100
```

with the cursor on the second F. By holding the CTRL key and pressing D, the F will be deleted. You have now corrected the line. Now use CTRL F until the cursor is over the 1 and change it to a 2. Delete the 1 (CTRL D) and press RETURN. The instruction is now executed. Experiment with both the CTRL B and F.

There are also control commands that manipulate the screen. Remember that each time you type a control key, you must first hold down the key marked CTRL.

CTRL L (FULLSCREEN) allows the entire screen to be used for graphics. For example

```
CS
BK 100
CTRL L (Watch the screen as you press RETURN)
```

The turtle moves downward until its trail disappears due to the four lines at the bottom alotted to text. The CTRL L is the screen command that allows the use of the entire screen space for graphics, and you can see now where the line goes.

CTRL S (SPLITSCREEN) is the screen mode you have been using. In this mode, graphics are on the top lines with four lines for text at the bottom. Type

```
CTRL S
```

and the screen again shows four lines of text at the bottom, cutting off the turtle trail that was drawn.

CTRL T (TEXTSCREEN) clears the entire screen for text use only. Enter the following:

```
CS
CTRL T (Hold down the key marked CTRL and type a T)
```

Immediately the screen is filled with instructions that have been entered. As soon as one turtle instruction is entered, the screen reverts back to a splitscreen. Now enter

```
RT 180
FD 100
CTRL L (Fullscreen--pause here and notice the change)
CTRL S (Bottom four lines are back on the screen)
```

Enter the three screen control commands in their unabbreviated form: FULLSCREEN, TEXTSCREEN, and SPLITSCREEN. You will get the same results as when the CTRL key is used along with the appropriate letter: L, T, or S.

Note: When in the fullscreen mode, if you make a typo the screen reverts to a splitscreen in order to give the error message.

An additional screen command that comes in handy is HOME. This command places the turtle back in the center, but differs from CLEARSCREEN: it does not erase anything but leaves a trail. This might be used when you are writing a program that moves the turtle around the screen, and then you want to start a new design from the center point again without tampering with what is currently on the screen. Since it does leave a trail, if one is not desired use the PENUP command before HOME. Try this program:

```
CS
FD 45
RT 45
FD 45
PU
HOME
PD
FD 45
LT 45
FD 45
PU
HOME  (The turtle is now back in the center)
```

If you ever want the turtle to stop what it is doing, use CTRL G (This gives the message, "Stop, turtle!"). This comes in handy, especially when you use the REPEAT command. Enter these lines:

```
CS
LT 45
REPEAT 100[FD 50 LT 90]
```

The turtle is beating a mad path around the diamond. To stop this program from running, hold down the CTRL key and type a G. The following message appears:

```
STOPPED!
```

If you want to make the program pause, you should use CTRL W. When you are ready for the program to continue, simply type any character on the keyboard. Enter the above program again and use the CTRL W to make it wait and type any key to make it carry on. Stop the program using CTRL G and clear the screen.

Basically, the turtle can draw in six colors if a color monitor is being used. (If a black and white monitor is being used, the background will be black with white lines. Skip to the next section.)

Think of the turtle as carrying a pen. To change the color of the lines or objects being drawn, the command SETPC (for set pen color) is used. Using the program above, add this as the first line:

```
SETPC 5  (This sets the pen color at number 5)
```

The program is drawn in color number 5, which is blue. There are six pen colors in Apple Logo:

0 = black
1 = white
2 = green
3 = violet
4 = orange
5 = blue

Change the pen color in the above program to check out each different shade available. If a color doesn't look like you think it should, you might want to adjust the color controls on the monitor. One caution--you must specify the pen color before any drawing is done.

You can also use these colors for the background or screen. Black is the automatic screen background color.

Clear the screen and type in these lines:

```
SETBG 1
SETBG 2
SETBG 3
SETBG 4
SETBG 5
```

The following program uses both pen color and background color:

```
CS
SETBG 4
SETPC 1
RT 45
FD 80
SETBG 1
SETPC 5
FD 40
```

Try the other colors in this program until you find pen colors and background colors that go well together. Some colors just don't mix, and the quality may depend on your monitor. Use different combinations of pen colors and background colors.

```
CS
SETPC 1  (White on white)
FD 70
LT 100
HOME
SETBG 5
BK 100
SETBG 2
SETPC 1
RT 90
FD 100  (Produces a white line)
SETBG 3 (Reverses again)
SETPC 4
FD 100  (Double line)
SETBG 4
LT 90
SETPC 5
FD 100
```

As with color monitors, the screen is automatically black with a white pen. Although the magic of color is not possible, there are some interesting effects that can be achieved on a black and white monitor. Enter these lines:

```
CS
FD 50
RT 90
FD 75
HOME
```

Watch what happens when you enter this line:

```
SETBG 1
```

This reverses the screen and pen colors. Experiment with this concept using your own designs.

So far you have written short programs and run them immediately. There is a way to put your programs into memory - that is, a way to write the program so that after it is run once it is there to run again and again.

Note: Unless you just loaded Logo, enter

```
SETBG 0
SETPC 1
```

As you enter the following lines, the graphics will not be on the screen. In the procedure mode, only text lines appear. Because of this, you should have already perfected the graphic program before you write a procedure:

```
TO TRIANGLE
RT 30
FD 50
RT 120
FD 50
RT 120
FD 50
END
```

The keyword in this program is TO, which indicates that you want the procedure name and the instruction lines placed into memory. Enter the lines above; press RETURN; and type TRIANGLE and the procedure will be produced.

Note: If you make an error or there is a bug in a procedure, it will not be indicated until you run it. For example, in a procedure named TRI, if you have a line that reads FF 10, the following message will appear when you run it:

```
I DON'T KNOW HOW TO FF IN TRI:
```

Soon you will learn how to make alterations in your procedures, called editing, to allow for additions, deletions, or corrections.

After you enter END the screen will say TRIANGLE DEFINED. END signals the completion of the procedure and is a necessary part of the process.

Type TRIANGLE over and over, and the turtle keeps drawing triangles. Why aren't they all in the same place? It is due to the direction which the turtle is heading at the end of the program.

You may not use the name TRIANGLE for any other procedure or you will be in big trouble. Check the section on saving programs in this chapter for clarification of the naming process. Here is the entire program, including your input and the screen output:

```
TO TRIANGLE   (Names the procedure)
RT 60      (The program instructions follow)
FD 50
RT 120
FD 50
RT 120
FD 50
END
TRIANGLE DEFINED  (Message on screen)
?
TRIANGLE (Type this in to run the program)
```

The triangle appears! The screen does not automatically clear without the explicit instructions, CS. It is a good idea to include it in procedures if appropriate.

Here is a more sophisticated use of procedures:

```
TO PRESENT
FD 40
RT 90
FD 70
RT 90
FD 40
RT 90
FD 70
BK 35
RT 90
FD 40
BK 20
RT 90
FD 35
BK 70
END
```

Type in the procedure name PRESENT to see the graphic design before entering the following procedure:

```
TO PRESENTS
PU        (Lifts the pen)
LT 90
FD 100
PD        (Lowers the pen)
PRESENT     (Runs the PRESENT procedure above)
PRESENTS    (Sends the turtle back to the beginning of PRESENTS starting with
          the PENUP command)
END
```

The first procedure PRESENT produces a picture of a present on the screen. The second procedure, named PRESENTS, has the turtle travel somewhere on the screen, draw a present, and then go somewhere else and draw another present, and on and on. The practice of including the procedure within itself is called recursive, which means that it will go on and on. It is like an infinite loop, if you are familiar with BASIC.

Use CTRL G to stop the procedure, and the screen output is

```
STOPPED! IN PRESENT:
RT 90   (Or whatever line it was executing when CTRL G was entered)
```

Everyone has a right to change their mind, and the turtle agrees. Therefore, if you want to change a procedure in any way after you have written it, here is what to do:

1. Type ED "PRESENT or EDIT "PRESENT (ED is short for EDIT). It must be followed by a space, a quotation mark, and the name of the procedure.

2. Your program instructions will now be shown on the screen. At the bottom of the screen it will say LOGO EDITOR. Use the following control characters to manipulate the cursor by holding down the key marked CTRL and the letter specified.

CTRL A	Cursor moves to beginning of line
CTRL B	Cursor moves one space back
CTRL C	Puts your changes in memory and gives the message "PRESENT DEFINED"
CTRL D	Deletes the character under the cursor
CTRL E	Cursor moves to the end of the line
CTRL F	Cursor moves one space forward
CTRL N	Cursor moves to the next line
CTRL O	Line is opened up above the line with the cursor. Be sure cursor is at the beginning of the line.
CTRL P	Cursor moves to the previous line

Try to associate each control character with a meaningful word so that you do not have to refer to the manual frequently.

Change some of the lines in the PRESENT procedure so they match the following program. Use the editor and control characters above.

```
TO PRESENT
FD 20
RT 45
FD 35
RT 45
FD 20
RT 45
FD 35
BK 17
RT 45
FD 20
BK 10
RT 45
FD 17
BK 35
END
```

Follow these instructions to initialize or prepare a blank disk to save your programs:

1. Turn off the computer (the switch is on the back left) and insert the 3.3 DOS System Master disk into the drive. Turn on the computer.
2. When the light on the drive goes off, remove the disk.
3. Insert your blank disk into the drive. Type INIT HELLO and press RETURN. The drive will run for nearly a minute while it prepares your disk.
4. Type CATALOG and press RETURN. You will see that HELLO is now present on your disk. If not, repeat the procedure from step 1 above.
5. Remove your disk and insert the Logo disk. Press RETURN.
6. Even though the message on the screen tells you to insert your disk at this time, do not do it the first time you are using the new disk. Press RETURN again.
7. When you see the message WELCOME TO LOGO now is the time to insert your new disk. After doing so, type SAVE "STARTUP "AIDS and press RETURN. This is necessary the first time you use your disk because it places a library of prewritten procedures on your disk.
8. Again you receive the friendly greeting

   ```
   WELCOME TO LOGO
   ?
   ```

   and you are ready to write a procedure.

If your new disk is not in the drive at this time, follow these steps:

1. Turn off the power, insert the Logo disk. Turn on the power.
2. When the light on the drive goes off, insert your initialized disk and press RETURN.
3. Wait until you receive the message

```
WELCOME TO LOGO
?
```

Now you are in business.

Here are the steps to follow in order to save a program. Follow these steps only if this is the first procedure you have entered after loading Logo into your computer.

1. Write a procedure:

```
TO STAIRS
CS
REPEAT 4[FD 20 RT 90 FD 20 LT 90]
END
```

2. Look for the message

```
STAIRS DEFINED
```

3. Type

```
SAVE "STAIRS
```

4. STAIRS is now saved. You will see the message

```
1 PROCEDURES SAVED
```

If this was not the first procedure written after loading Logo, more than one procedure will be indicated as saved.

Checkpoint: Turn off the power and reload Logo into the computer. Insert your disk and press RETURN. To see if your program was really saved, type

```
LOAD "STAIRS
```

5. After the drive light goes off, type STAIRS and your procedure should be drawn on the screen.

6. Type the command CATALOG and you will see all the procedures you have saved. Since this was your first one, you should only have HELLO and STAIRS. It is not necessary to turn off the computer each time as in the CHECKPOINT above. Only the CATALOG operation is needed.

If you are defining several procedures at one sitting, do not save each one separately! When it comes to saving more than the first procedure, Logo does something weird. To explain this statement, consider this hypothetical situation: Logo is loaded, and the following procedure is entered:

```
TO LINE1
FD 100
END
```

If you typed SAVE "LINE1, you would receive the message:

```
1 PROCEDURES SAVED
```

If you then entered a second procedure:

```
TO LINE2
BK 100
END
```

and typed SAVE "LINE2, you would receive the message

```
2 PROCEDURES SAVED
```

Each time you write and save a new procedure, every other procedure you have written up to that point will also be saved under the file name you place after the word SAVE. It is not efficient to have every procedure saved over and over again. However, this will continue to occur with each subsequent procedure you enter and save, but there are two ways to get around it.

One way to erase all procedures in the memory before saving a new one so that there is nothing in memory that you don't want saved in that file. In other words, since the computer saves every single procedure in memory as the result of one SAVE command, then just erase what you don't want saved. Be sure that if you wanted other procedures saved, you would already have done so before erasing the procedures in memory. Here is a situation where you want to clear the memory, but save specific procedures:

1. You have written procedures named CHAIR, TABLE, and LAMP.

2. You saved them with the command

```
SAVE "ROOM       (CHAIR, TABLE, and LAMP are in this file now)
3 PROCEDURES SAVED (Screen output)
```

3. Now you write procedures named FLOWER, VASE, and BEE.

Before you attempt to save them, you must omit the other three procedures, under the name ROOM. Use ERASE (or ER for short). Enter the command

```
ERASE [CHAIR TABLE LAMP]
```

4. If there is more than one procedure to erase, the names must be listed inside the brackets, separated by a space. If only one procedure is to be omitted, it can be in this form:

```
ER "CHAIR
```

5. The three procedures named in the ERASE command (CHAIR, TABLE, and LAMP) are now gone from the current memory but, of course, are still saved on the disk under the file named ROOM.

6. Now the SAVE command can be given:

```
SAVE "PHOTO          (User input)
3 PROCEDURES SAVED      (Screen output)
```

The second alternative to avoid saving procedures on top of procedures is to use a command called PACKAGE. It is like placing certain procedures in a special package with a special name. For example, if you had written procedures named TABLE, CHAIR, and LAMP, you could enter these lines:

```
PACKAGE "ROOM[TABLE CHAIR LAMP]
```

Inside the package named ROOM, the procedures TABLE, CHAIR, and LAMP would be stored and no others. After packaging these procedures, you still must save them, but a special format is necessary:

File name	Package name
SAVE "ROOM	"ROOM

It is OK to use the same two names for the file to be saved and the package. The command above saves in the file named ROOM whatever is in the package named ROOM. The file name in which you wish everything to be saved comes first, preceded by a quotation mark, and the package name comes second, also preceded by a quotation mark. When using packages to store your procedures, it is not necessary to erase any procedures or to worry about piling up all your programs.

Note: It is possible to save the same procedure in more than one package. For example, let's say that you want TABLE to be saved in the file named ROOM as well as in a file named PHOTO. Just name TABLE in each package you want to include it in. Then you decide that you want to change the TABLE procedure in the ROOM file but leave it the same in the PHOTO package. This is how it is done:

1. Erase all procedures from the workspace (memory)--assuming ROOM and PHOTO have been saved. The command to use is ERALL.

2. Load ROOM and call TABLE into the Logo editor:

```
LOAD "ROOM
ED "TABLE
```

3. Make the changes you wish, type CTRL C to exit the editor, and enter

```
ERASEFILE "ROOM
 SAVE "ROOM
```

The ERASEFILE command eliminates the old file ROOM and then saves it once again with all the same procedures plus the edited TABLE. TABLE was not edited as part of the PHOTO file, so it will remain in its original state. If you desire to have both TABLES edited in the same way, you would have to follow the above three steps for each TABLE you want changed in each package it appears.

If you ever want to check to see exactly what procedures you stored under a certain file name, use these commands:

```
ERALL          (To erase all procedures in memory)
LOAD "ROOM       (Loads just the procedures in ROOM)
POTS
```

The PRINT OUT THE TITLES command (POTS for short) will cause a listing of each procedure in memory--in this case "ROOM. If the ROOM file holds TABLE, CHAIR, and LAMP, the screen output resulting from the above line would be:

```
TO TABLE
TO CHAIR
TO LAMP
```

Another command, PRINT OUT (PO), will list each procedure named, if it is in memory, along with the name of the program instruction lines. It is necessary to specify certain files:

```
PO "TABLE        (TABLE procedure will be output)
PO [TABLE CHAIR]  (TABLE and CHAIR procedures will be output)
```

When you want to see the procedure instructions for procedures in a certain package, the following command is used. Here ROOM is a package name.

```
POPS "ROOM                    or                    POPS [TABLE CHAIR]
```

The prewritten programs mentioned earlier are in the STARTUP file. They are on the Logo disk and also on your initialized disk if you followed the instructions on page 126. They are really just shortcuts that are available and include seven different files. Six will be presented here. Only two of them have abbreviated forms.

```
ARCLEFT                or                ARCL (Short form)
ARCRIGHT               or                ARCR (Short form)
CIRCLEL
```

```
CIRCLER
ARCL 1
ARCR 1
```

All of the procedures deal with producing arcs and circles. They require inputs--numbers that help define what kind of circle or arc to draw. The following procedure means to draw an arc to the left with a radius of 30 turtle steps and 180 degrees around.

```
CS
ARCL 30 1
```

This will produce a half-circle (since 180 degrees is half of 360 degrees, a complete circle; the circle is 60 turtle steps across and the radius is half of the diameter).

```
PU
HOME
PD
ARCR 30 180
```

This is a mirror image of the first line, with the arc going to the right, using identical radius and degrees. Why are PU, HOME, and PD used here? Perhaps you will discover it as we go along.

The CIRCLEL command needs only one input number, the radius, since it will automatically be 360 degrees or one revolution.

```
PU
SETPC 5  (Add some color for fun)
HOME
PD
CIRCLEL 20
```

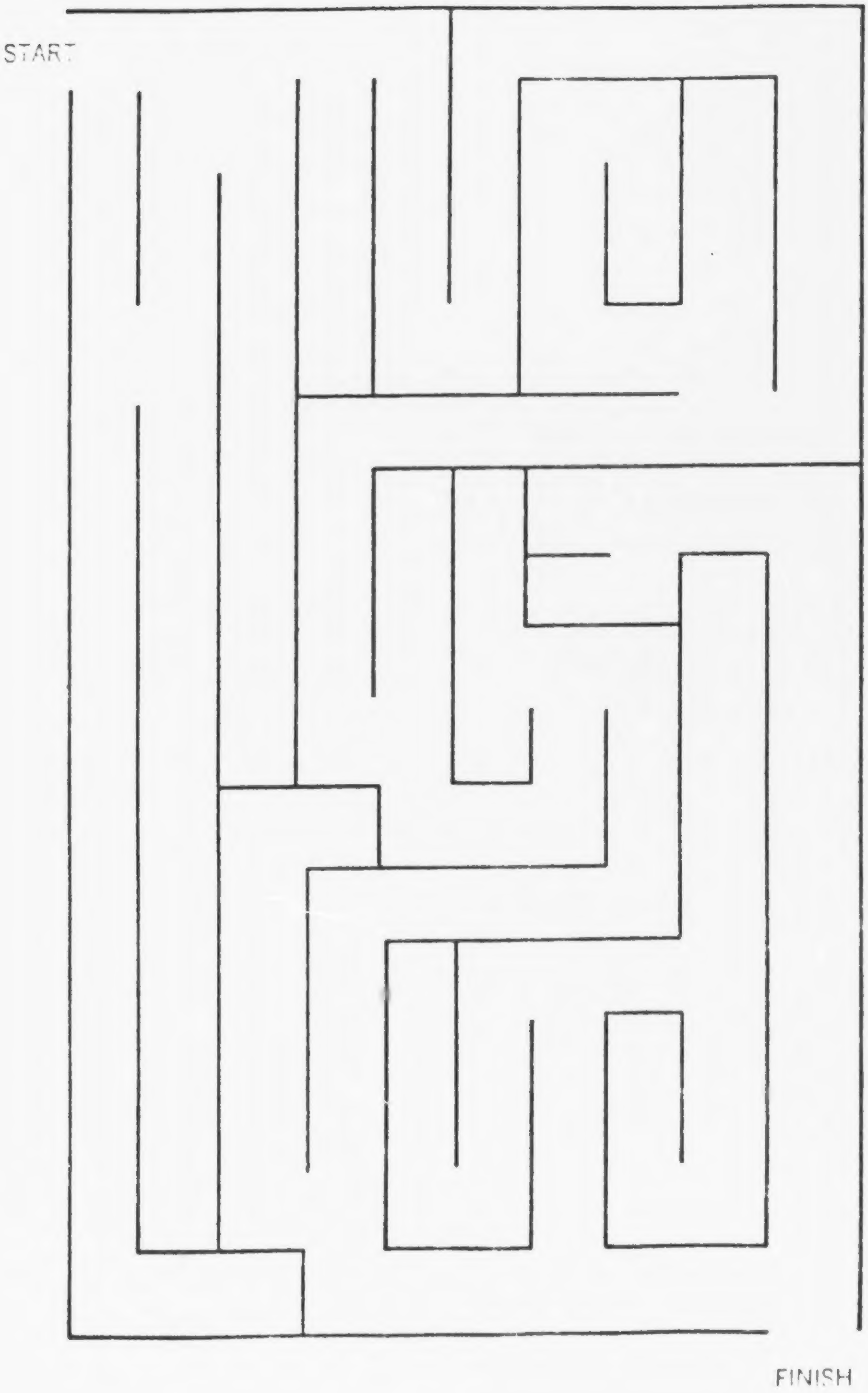
Add the mirror image of this simply by entering these lines to the previous ones:

```
PU
HOME  PD
CIRCLER 20
```

The other two commands, ARCR1 and ARCL1, require still different inputs.

```
PU
SETPC 1
HOME
PD
ARCL1 4 36
ARCR1 4 36
HT
```

The first number (4) represents turtle steps, and the second number (36) refers to the number of times you want the turtle to go 4 steps. The turtle automatically turns 10 degrees before each new step size is executed. Since 10 degrees times 36 is 360 degrees, we can expect a circle again. Predict what would happen with the input numbers of 10 and 18. Try it.


START
FINISH

Session 13M Functions

Slope, Intercept, and Function Solutions

Objective

To be able to find the zero of a linear function by approximation in a t-table and to determine the relationship between the slope, y-intercept and the x-coordinate of the zero of the function.

Description

This spreadsheet template (in Appleworks) allows the student to put in any linear equation of the form y=mx+b. The student changes m and b and then changes what value s/he wants to start the t-table with. The student can also change the increment value which the x-coordinate increases by. Ten entries are displayed in the table with their x-values and corresponding y-values. The student can use the program to "close in" on the function zero.

Procedure

Load the spreadsheet template "Lines". In cell B1 put in the value for m. In cell B2 put in the value for b. Put in 2 for m and 3 for b. Set the start value to -5 and the increment to 1. Now watch the table display.

The purpose of this session is to demonstrate how to solve linear and quadratic functions on the computer.

Observe that the y-values go from negative to positive when the x-value goes from -2 to -1. What should we reset the start value and increment to. (A good choice would be to set the start value to -2 and the increment to

.1) Read zero of the function. Put this information into the accompanying table.

Repeat the experiment for another function. Try m as 4 and b as 7. Put the results into the table.

Repeat the experiment for any linear function where m and b are both between 1 and 10. Does a pattern exist? Is it possible to determine the x-value of the zero directly using the slope and y-intercept?

Without doing the experiment, predict the x-value of the function zero for the following linear function:

$y=5x+12$

Now try it out. Were you right? Enter the data into the table.

Test your prediction with negative values for m and b. Does your prediction still hold? How about fractional values of m and b? Does it hold here as well?

Use what you know to generate a linear function which passes through the point (4,0) (4 is the zero of the function). How many lines pass through that point? Is there only one? What do these functions all have in common?

Linear function:

Roots of a Quadratic Polynomial

Objective

The student will use the program to determine the roots of a quadratic polynomial of the form:

$ax^2 + bx + c$

The student will determine what types of quadratics have real roots and which do not.

Description

This spreadsheet template allows the student to put in the values for a, b, and c. The solutions, if they exist are displayed along with the polynomial in factored form. If a root does not exist (discriminant < 0) then ERROR is printed where the roots are displayed.

Procedure

Load the spreadsheet template "Roots". Put in the following polynomial:

$a=1$ $b=-2$ and $c=-8$

Find the solutions for this polynomial and write them down.

If you multiply the coefficients and constant by a constant term, how does the solution set change?

If you divide by a constant how does the solution set change?

Can you add or subtract by a constant and still have the same solution set?

Put in the following polynomial:

$a=1$ $b=1$ and $c=-2$

Does it have integer roots?

Change the value of c keeping a and b the same. Can you change c so that the new polynomial also has integer roots?

(Hint: try c=-6)

Keep trying different values of c to generate polynomials with integer roots where a and b are both equal to 1. Can you see a pattern?

Predict what the next value for c will generate integer roots. Check your answer.

Repeat the experiment with a = 1 b = -5 and c =6. Give at least three values of c which will generate integer roots. What is the pattern?

Repeat the experiment with any solvable quadratic of your choice. Try to determine if a pattern exists and what that pattern is.

Session 13S.1
Sketching, Graphs and Solving Problems

ACTIVITY

LAB

OBJECTIVE

To present the graphing of the absolute value function in order to have the student derive general rules that apply to the sketching of functions once the shape of the "core" function is known.

DESCRIPTION

An algebraic method of graphing absolute value functions is presented. The process will be further discussed in the upcoming classroom setting. The computer will be used to produce many graphs in order that the student can draw generalizations about sketching absolute value functions - and thus apply it to functions in general.

PROCEDURE

This session provides the initial development of a generalized method of sketching the graphs of functions. In addition, several small programs are listed for participants to use to solve problems and illustrate concepts.

1. The algebraic method of approaching the graphing of absolute value functions is developed :

GRAPH $Y = |X|$

Recall the definition of $|X|$

$|X| = X$ when $X >= 0$

$|X| = -X$ when $X < 0$

Clearly either $X < 0$ OR $X >= 0$

If $X < 0$, $|X| = -X$	If $X >= 0$, $|X| = X$

Now, substitute in the original problem $Y = |X|$, the values found in each case above.

$Y = |X|$	$Y = |X|$
substituting $Y = -X$	$Y = X$
BUT only where $X < 0$	BUT only where $X >= 0$

"OR" means UNION

NOW GRAPH IN THESE LIMITED PORTIONS OF THE COORDINATE PLANE.

2. Use the program RELATION GRAPHER from "Chalkboard Graphics Tool Box I" by Scharf Systems choosing option #6, "Any Function" --- setting the domain between -5 and 5, the range between -5 and 5 with increment of 2 in both cases and a plot speed of "Fast/Low".

- Graph the set of functions in each group below on the same set of coordinate axes by selecting option #1, "Add Another Relation" after each graph is drawn.
- After EACH function is graphed, observe the attributes of the graph and how the previous function(s) graphed in the particular group is(are) related to the function just graphed.
- Further, take note of how the new function's graph might have been predicted by studying the form of the equation of the function.

a. On one set of axes, graph:

$Y = |X|$

Remember to enter as Y = ABS(X).

$Y = 2|X|$

$Y = -|X|$

$Y = .5|X|$

Discuss conclusions.

b. On one set of axes, graph:

$Y = |X| + 2$

Remember to enter as Y = ABS(X)+2.

$Y = |X + 2|$

$Y = |X| - 3$

$Y = |X - 4|$

Discuss conclusions.

c. Without the computer, sketch the graph of each of the following and check your results with the computer.

1.) $Y = |X - 2| + 3$

2.) $Y = -|X + 3| + 4$

3.) $Y = 2|X - 1| - 3$

d. Now let's graph each of the following and search for analogies to the graphs of the absolute value functions. Graph all of the following on one set of axes:

1.) $Y = X^2$

2.) $Y = (X - 2)^2$

3.) $Y = (X - 2)^2 + 3$

4.) $Y = 2X^2$ -- don't forget to enter as Y = 2*X^2

5.) $Y = -X^2$

e. Use one set of axes on which to plot each of the following. It might be wise to try to do the graphs first, then check with the computer.

1.) Y = |X|

2.) Y = |X - 2|

3.) Y = |X - 2| + 3

4.) Y = 2|X| -- enter as Y = 2*ABS(X)

5.) Y = - |X|

f. For the following functions, use one set of axes. First graph the "core" function Y = sin(X). Before graphing each of the variations, try to predict what will happen.

Then use the computer to check whether or not you were correct.

It is suggested that the domain be selected from -7 to 7 with increment of 2, the range from -3 to 3 with increment of 1 and the plot speed be "Fast/Low".

1.) Y = sin(X)

2.) Y = sin(X - 2)

3.) Y = sin(X - 2) + 2

4.) Y = 2sin(X) -- enter as Y = 2*sin(X)

5.) Y = -sin(X)

g. Graph the following two functions on one set of axes. When complete, decide whether or not you could have predicted the result you obtained for Y = sin(X + PI/2):

1.) Y = sin(x)

2.) Y = sin(X + PI/2) **NOTE** You must use a numerical approximation for PI. For our purposes, let us agree to approximate PI as 3,14.

ACTIVITY

LAB

OBJECTIVE

To work with several short programs that teach various mathematical topics.

DESCRIPTION

The programs are on disks in DOS 3.3 and are entitled :

a. Signed Number Practice

b. Cupcake Problem

c. Quarter Problem

d. Percents Illustrated

e. Triangle Classification

f. Decimals Illustrated

g. Freq. Dist. Bar Graph 80 Col.

The program "Signed Number Practice" was written to permit students to practice adding, subtracting and multiplying signed numbers while allowing the teacher to monitor their progress by sound. To be certain the student is working:

- "one beep" from the computer is heard each time a student provides a correct response
- "five beeps" from the computer indicates that the student has responded incorrectly
- after the student has made a total of eleven errors, a series of "fifteen beeps" is heard -- this alerts the teacher -- also the student receives a message to see the teacher for help.

The documentation for all of the other programs in this series are on the pages which follow. They are taken from Monograph #2 of the Association of Mathematics Teachers of New Jersey (AMTNJ). Additional copies of the monograph can be obtained for $5.00 each from AMTNJ by writing to:

AMTNJ Monograph
c/o Fran Masat
Glassboro State College
Glassboro, New Jersey 08028

Make checks payable to "AMTNJ"

One further note - the program "Freq. Dist. Bar Graph 80 Col." should be run in 80-column format in order for the bar graph to appear in proper form.

Enjoy the problems on the pages that follow during this lab period and whenever you get a chance to work on them again!

General Topic : Computing/Programming

Specific Topic : Computer Generated Solutions (Cupcakes and Quarters)

Objectives : Students will use a computer program to consider the following combinatorial problem:

A customer ordered 15 cupcakes. Cupcakes are placed in packages of 4,3, or 1. In how many ways can you fill the order?

The student (programmer) will need to make several decisions about the task and the resulting program:

1. What is the fundamental strategy for generating a solution list?
2. How efficient is their "search" process?
3. Is it enough to print the list or should the program also summarize by giving a "final answer?"

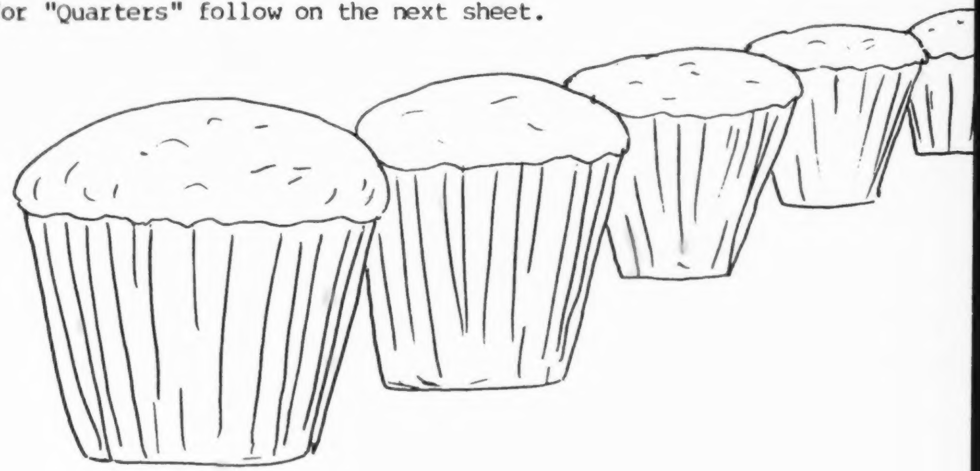
The program and output for "Cupcakes" follow on a separate sheet.

Uses and Extensions : Approaches to this problem will vary with the age and experience of the learner. For example, manipulative materials (to represent the cupcakes) could be used with primary children. Students will be able to see that this problem is typical of the kind of problem where an organized list is required. Hence, this problem is process oriented and not the kind of problem which can be solved by a number sentence and some quick arithmetic.

The objective is clearly not to have elementary and middle school youngsters writing programs of this magnitude. However, it is essential for these learners to explore programs which illustrate the connection between computer use and problem solving. For elementary and middle school students (and also for senior high students), CUPCAKE is an excellent example.

An excellent extension is exemplified by the following: A clerk wants to make change for a quarter and has only dimes, nickels and pennies. In how many ways can the clerk make change for the quarter?

Program and output for "Quarters" follow on the next sheet.



```
]LIST

120  PRINT  TAB( 15);"CUPCAKE PROBLEM": PRINT
150  PRINT "4/PKG","3/PKG"," 1/PKG"
160  PRINT "-----","-----"," -----"
180  FOR J = 0 TO 3: FOR K = 0 TO 5: FOR I = 0 TO 15
210  LET A = J * 4 + K * 3 + I: IF A <  > 15 THEN  GOTO 250
230  LET C = C + 1: PRINT J,"  ";K,"   ";I
250  NEXT I: NEXT K: NEXT J
280  PRINT "THE TOTAL NUMBER OF WAYS IS ";C
```

```
]RUN
                 CUPCAKE PROBLEM

4/PKG            3/PKG            1/PKG
-----            -----            -----
0                0                15
0                1                12
0                2                9
0                3                6
0                4                3
0                5                0
1                0                11
1                1                8
1                2                5
1                3                2
2                0                7
2                1                4
2                2                1
3                0                3
3                1                0
THE TOTAL NUMBER OF WAYS IS 15
```

```
RUN
               QUARTER PROBLEM

  HOW MANY WAYS DO YOU THINK THERE ARE ?
            THERE ARE TWELVE WAYS
  DIMES          NICKELS          PENNIES
  0              0                25
  0              1                20
  0              2                15
  0              3                10
  0              4                5
  0              5                0
  1              0                15
  1              1                10
  1              2                5
  1              3                0
  2              0                5
  2              1                0
  THE TOTAL NUMBER OF WAYS IS 12
```

```
]LIST

1  REM    CHANGE OF A QUARTER
10  INPUT "HOW MANY WAYS DO YOU THINK THERE ARE TO MAKE CHANGE FOR A QUAR
    TER? ";A
15  IF A <  > 12 THEN  PRINT "SORRY, THERE ARE TWELVE WAYS-LETS SEE THEM!
    !": FOR X = 1 TO 1000: NEXT X: GOTO 25
20  IF A = 12 THEN  PRINT "VERY GOOD!!! LET'S SEE THEM": FOR X = 1 TO 100
    0: NEXT X
25  PRINT "DIMES         NICKELS          PENNIES"
30  FOR D = 0 TO 2: FOR N = 0 TO 5: FOR P = 0 TO 25
40  LET X = (D * 10) + (N * 5) + P
50  IF X <  > 25 THEN  GOTO 100
60 C = C + 1
70  PRINT D,N,P
100  NEXT P: NEXT N: NEXT D
130  PRINT "THE TOTAL NUMBER OF WAYS IS ";C
```

General Topic : Arithmetic and Measurement

Specific Topic : Percentages

Objective : Student will gain practice in calculating percentages.

Description : The activity allows students to use a computer to calculate percentages. If the answer is an integer, a pictorial display will be given along with the answer. Many examples are given, after which the program can be changed to allow entering any number and percent.

Uses and Extensions : Use the program as it is, recording the results in a chart similar to the one below:

Problem	Percent (P)	Number (N)	Answer (A)
1)			
2)			
3)			
⋮	⋮	⋮	⋮

Alter the program by typing these lines:

```
50 INPUT "ENTER THE PERCENT TO USE ";P
60 INPUT "ENTER THE NUMBER ";N
```

Enter your own numbers and percents. Record the results in the chart.

What do you notice about the size of the percents in problem numbers 6,8,13,14? What do you notice about the answers in problems 6,8,13,14?

```
]LIST

10  HOME : PRINT "THIS PROGRAM WILL DETERMINE THE PERCENT OF A NUMBER AND
      DISPLAY THE PERCENT, "
20  PRINT "NUMBER AND ANSWER"
60  READ P,N: IF P = 0 THEN  END
70 A = N * P / 100
80  PRINT "PERCENT","NUMBER","ANSWER"
81  PRINT P,N,A: IF A =  INT (A) THEN 200
85  PRINT : PRINT : PRINT
90  GOTO 60
200  PRINT : PRINT
210  PRINT "A DISPLAY FOR THE NUMBER (N) AND THE PERCENTAGE (A) ANSWER."
220  PRINT : PRINT : PRINT
230  FOR R = 1 TO N: PRINT "N";: NEXT R
260  PRINT : PRINT
270  FOR S = 1 TO A: PRINT "A";: NEXT S
300  PRINT : PRINT : PRINT
305  FOR X = 1 TO 300: NEXT X: RESTORE : GOTO 60
315  DATA  7,200,40,80,16,300,2,150,10,70,22,400,20,90,9,400,4,350,5,60,.
     5,200,.8,500,2.5,400,.5,400,.5,800,0,0
```

General Topic : Geometry

Specific Topic : The Pythagorean Theorem

Objective : Students will be able to apply the Pythagorean Theorem, its converse, and related theorems regarding obtuse and acute triangles.

Description : The activity allows students to use a computer program that accepts the lengths of the three sides of a triangle, and then determines and displays information as to whether the triangle formed is acute, right, or obtuse. The program also prints out information when no triangle is formed.

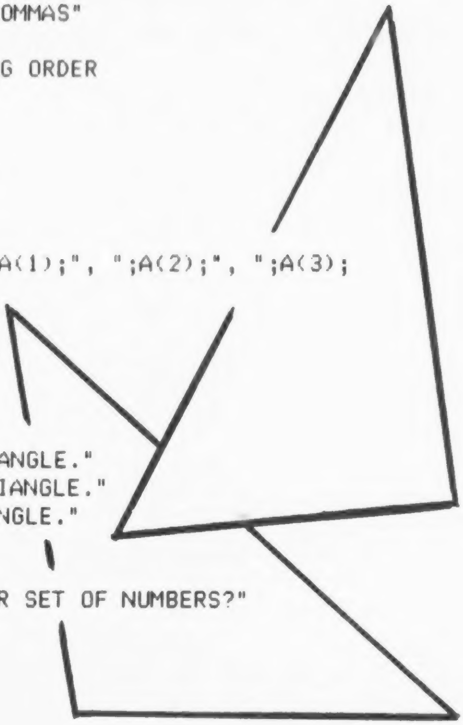
Uses and Extensions : Please be aware that difficulties arise under certain conditions when using exponential notation such as A(1) ^ A(2) = A(3)^2. Due to the fact that the computer uses logs to compute exponents, decimals may become involved when squaring an integer. Therefore, it was necessary to use A(1) * A(1) instead of A(1)^2 (see lines 500 and 510 of the program).

Since the program is lengthy, you may find it easier to store the program on a disk.

An interesting extension is to have those 10th and 11th grade geometry students with programming experience write and use their own versions of an interactive program that uses the Pythagorean Theorem and its related theorems to determine if a given triangle is obtuse, acute or right.

```
]LIST

150  PRINT "TYPE YOUR THREE SIDES SEPARATED BY COMMAS"
170  INPUT A(1),A(2),A(3)
185  REM  BUBBLE SORT: ARRANGE SIDES IN ASCENDING ORDER
230  FOR N = 1 TO 2
240  FOR I = 1 TO 2
250  IF A(I) < A(I + 1) THEN 290
260 TEMP = A(I):A(I) = A(I + 1):A(I + 1) = TEMP
290  NEXT I
300  NEXT N
400  PRINT "IF THE SIDES OF YOUR TRIANGLE ARE ";A(1);", ";A(2);", ";A(3);
     " THEN ";
410  IF A(1) + A(2) <  = A(3) THEN 550
420  IF A(2) + A(3) <  = A(1) THEN 550
430  IF A(1) + A(3) <  = A(2) THEN 550
500  LET L = A(1) * A(1) + A(2) * A(2)
510  LET M = A(3) * A(3)
520  IF L > M THEN  PRINT "YOU HAVE AN ACUTE TRIANGLE."
530  IF L < M THEN  PRINT "YOU HAVE AN OBTUSE TRIANGLE."
540  IF L = M THEN  PRINT "YOU HAVE A RIGHT TRIANGLE."
545  GOTO 600
550  PRINT "NO TRIANGLE IS FORMED."
600  PRINT : PRINT "WOULD YOU LIKE TO TRY ANOTHER SET OF NUMBERS?"
610  INPUT "TYPE YES OR NO. ";A$
620  IF A$ = "YES" THEN 150
630  END
```



SAMPLE WORKSHEET

1. Enter the attached BASIC program (or load it from a disk).

2. Enter RUN, and respond to the questions asked. That is, you will have to enter three sides (lengths) of a triangle.

3. Can you decide whether the triangle formed with the given sides is acute, obtuse or right? Consider the possibility that NO TRIANGLE will be formed. See if you can guess the correct answer before entering your data.

Examples:

a) 12,5,13
b) 8,18,15
c) 9,12,14
d) 23,9,13
e) 0.7,0.7,1
f) 1.1,6.0,6.1
g) 0.6,0.8,1
h) 2*(SQR(15)),6,5
i) SQR(7),SQR(15),SQR(8)
j)2,4,3
k) 7,6,5

4. State the Pythagorean Theorem.

5. Under what conditions will your triangle be a right angle?

6. Can you discover when a triangle will be obtuse? Acute?

7. Why are there situations when no triangle is formed?

General Topic : Number Theory

Specific Topic : The Division Algorithm

Objective : Students will be able to investigate digit patterns in division problems.

Description : This program uses the division algorithm to perform long division to any desired number of places. The resulting data is applicable in number theory and statistics for such investigations as digit patterns in quotients, cycles of repetition and distribution of digits. An interesting sequence of digits for students to examine is the result of 355/113 as an approximation for pi.

Uses and Extensions : Use to emphasize place values, to search for number patterns, to generate data for statistics, or to approximate mathematical constants such as pi and e.

Note that the program can be used for the study of error generation, particularly when dividing by a decimal.

```
]LIST

5  INPUT "NUMERATOR =";N
10  INPUT "DENOMINATOR =";D
15  INPUT "NUMBER OF DIGITS IN QUOTIENT =";N1
17  PRINT
20  FOR D1 = 1 TO N1
30 Q =  INT (N / D)
40  PRINT Q;
45  IF D1 = 1 THEN  PRINT ".";
50 N = 10 * (N - Q * D)
60  NEXT D1
70  END
```

```
]RUN
NUMERATOR =355
DENOMINATOR =113
NUMBER OF DIGITS IN QUOTIENT =10

3.141592920
```

```
]RUN
NUMERATOR =1237
DENOMINATOR =524
NUMBER OF DIGITS IN QUOTIENT =22

2.3606870229007633587 78
```

```
]RUN
NUMERATOR =58763
DENOMINATOR =675.2
NUMBER OF DIGITS IN QUOTIENT =50

87.030509472526751954936539615157765757059537923233 7
```

```
]RUN
NUMERATOR =3124
DENOMINATOR =513
NUMBER OF DIGITS IN QUOTIENT =79

6.089668615984405458089668615984405458089668615984405458089668615984405458089
```

General Topic : Graphing

Specific Topic : Computerized Horizontal Bar Graphs

Objective : Students will be able to produce bar graphs using the computer as a tool.

Description : Students will follow directions to enter the number of items in the graph, the label part of the data, the number part of the data, the factor used for division so that the graph fits on the screen, and the symbol choice for the actual display. The computer will produce the graph on the screen.

Uses and Extensions : Carefully type in a copy of the attached program, or use one that has already been saved on a disk. The teacher may want the program entered prior to the day of use due to its length. Note that an Apple IIe should have an 80 column card for this activity.

Sample Items for Student Worksheet

Enter RUN and respond by entering your data or the data provided by your teacher. Ideas for the graph:

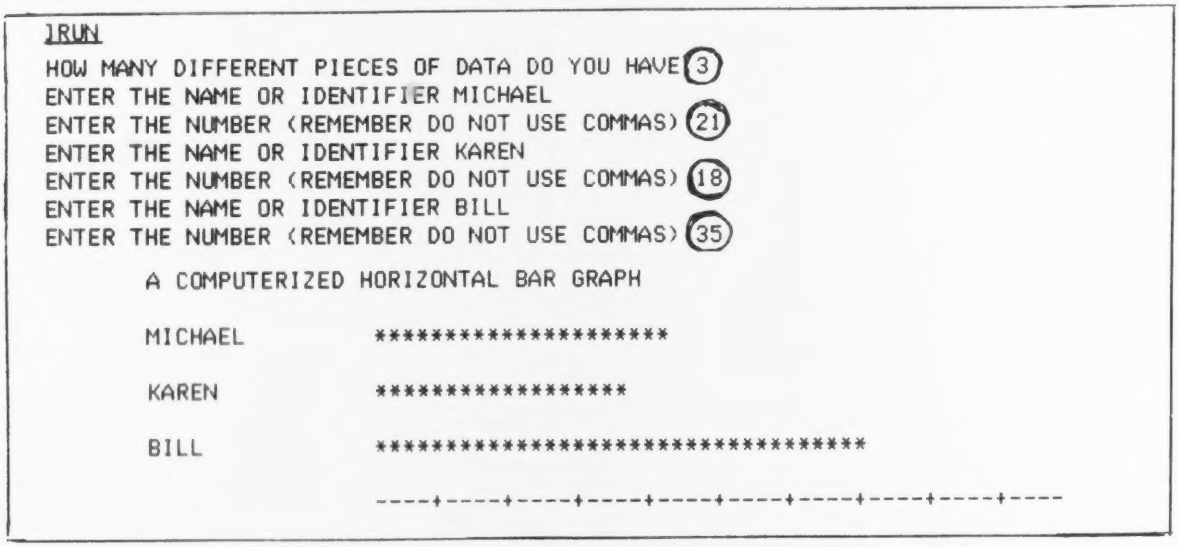
Make a graph of the names and ages of everyone in your house. Your factor for doing the division can be 1 unless family members are over 60.

Make a graph listing several members of your class and their scores on an assignment.

Make a graph comparing the area or the population sizes of the continents (use a copy of the World Almanac for this information).

Make a graph comparing the records of some of the teams that play your favorite sport. (The World Almanac is a good reference).

Listing : Please see next sheet



```
]RUN
HOW MANY DIFFERENT PIECES OF DATA DO YOU HAVE 3
ENTER THE NAME OR IDENTIFIER MICHAEL
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 21
ENTER THE NAME OR IDENTIFIER KAREN
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 18
ENTER THE NAME OR IDENTIFIER BILL
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 35
       A COMPUTERIZED HORIZONTAL BAR GRAPH

       MICHAEL         *********************

       KAREN           ******************

       BILL            ***********************************

                       ----+----+----+----+----+----+----+----+----+----
```

```
]LIST

3  REM  THIS WILL RUN ON APPLE IIe WITH AN EXTENDED 80 COLUMN CARD
10  REM  THIS PROGRAM WILL PRINT OUT A HORIZONTAL GRAPH FOR A STUDENT
15  REM  THE MAXIMUM NUMBER OF PIECES OF DATA IS 10
20  REM  TEACHING SUGGESTION, HAVE A STUDENT WORK OUT THE GRAPH ON A PIEC
     E
25  REM  OF GRAPH PAPER AND THEN CHECK HIS OR HER WORK WITH THE COMPUTER
30  REM  N$(X) HOLDS THE NAME PART OF THE DATA
35  REM  S(X) HOLDS THE NUMBER PART OF THE DATA (NO COMMAS)
40  REM  DB IS THE DIVIDING NUMBER
45  REM  T IS THE NUMBER OF SYMBOLS TO BE PRINTED PER LINE
80  DIM N$(10),S(10)
88  HOME
90  PRINT "THIS PROGRAM WILL HELP YOU PRINT A HORIZONTAL BAR GRAPH ON THE
      SCREEN.": PRINT
92  PRINT "YOU WILL NEED TO KNOW THE NAME OR IDENTIFIER AND THE NUMBER TH
     AT"
94  PRINT " GOES WITH THAT INFORMATION. REMEMBER YOU MUST TYPE THE NUMBER
     "
96  PRINT "INTO THE COMPUTER WITHOUT USING ANY COMMAS!!!!!"
100  INPUT "HOW MANY DIFFERENT PIECES OF DATA DO YOU HAVE ";N
110  FOR X = 1 TO N
120  INPUT "ENTER THE NAME OR IDENTIFIER ";N$(X)
130  INPUT "ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) ";S(X)
140  NEXT X
150  PRINT "THE GRAPH MUST FIT ON THE SCREEN. 60 IS THE MAXIMUM NUMBER"
160  PRINT "OF CHARACTERS THAT CAN BE PRINTED ACROSS THE SCREEN."
170  PRINT "IF THE NUMBERS ARE LARGER THAN 50, THEN YOU HAVE TO DIVIDE AL
     L"
180  PRINT " OF THEM BY THE SAME NUMBER."
190  INPUT "WHAT SYMBOL DO YOU WANT TO USE FOR THE GRAPH ?";SY$
205  HOME
206  PRINT "IF YOUR NUMBERS ARE NOT ABOVE 50, THEN ENTER 1 OTHERWISE ENTE
     R THE NUMBER YOU WISH TO DIVIDE THE NUMBERS BY."
208  INPUT DB
209  PR# 1
210  PRINT "A COMPUTERIZED HORIZONTAL BAR GRAPH": PRINT
220  FOR X = 1 TO N
230  PRINT N$(X),
240  LET T =  INT (S(X) / DB + .5)
250  FOR Y = 1 TO T
260  PRINT SY$;
270  NEXT Y
280  PRINT : PRINT
290  NEXT X
300  PRINT ,"----+----+----+----+----+----+----+----+----+----+----+---"
310  PRINT ,," IN ";DB" UNITS"
315  PR# 0
320  END
```

Session 13S.2
Advanced Topics

Teacher Notes for Worksheet on Trigonometric Identities

OBJECTIVE

To provide the student with a visual means for verifying trigonometric identities.

DESCRIPTION

Any graphics package which handles trigonometric functions is appropriate for using with this worksheet. Teachers need to give instructions for use of the particular software that they choose. This worksheet is appropriate for individual or small group use or as a whole class activity when only one computer is available.

SPECIAL NOTES

This worksheet takes a full class period to complete.

It must be explained that each side of the identity equation worked with will be set equal to y and graphed. If the graphs of both sides of the equation are the same, the function is an identity.

Most graphics packages do not label the x axis in terms of π. The x axis will be labeled ... -3, -2, -1, 0, 1, 2, 3 ... Instead of $\pi/2$, the value will be found at 1.57. Prepare the students for this.

For any graphics package, setting the x values from -6 to 6 and the y values in the same range usually works well.

Some graphics packages have flaws in their plotting routines which cause them to connect points where no y value exists, for instance, at $\pi/2$ on the tangent graph. Point this out to the students if your software is like this.

Be sure to emphasize the restrictions to the domain of particular graphs and point out the corresponding display on the computer screen.

Teacher Notes for Worksheet on Shrink/Stretch Trigonometric Functions

OBJECTIVE

To provide the student with experience in graphing trigonometric functions in the form y=AsinBx. The student will change values of A and B to derive general rules that apply to the amplitude and period of trigonometric functions.

DESCRIPTION

Any graphics package which handles trigonometric functions is appropriate for use with this worksheet. The teacher will need to give instructions for use of the particular software chosen. This worksheet is appropriate for individual or small group use or as a whole group activity when only one computer is available.

SPECIAL NOTES

This worksheet takes a whole class period to complete.

Students may want to label individual graphs with the problem number rather than the equation due to space limitations.

Setting the x axis coordinates from 0 to 7 and the y coordinates from -5 to 5 provides the best results for this worksheet.

Most graphics packages will label the x axis in terms of decimal radian values rather than in multiples of π. You may want to discuss this with the students beforehand.

This worksheet is an excellent introduction to the ideas of amplitude and period or it can be used as a follow-up activity. In either case, the students still need experience in actually plotting some of these functions by hand themselves.

Because graphics packages plot sine or cosine functions as continuous wave forms, phase shift is more difficult to show. For instance, students cannot determine the direction of phase shift when studying graphs such as $y = \sin(x + \pi)$ and $y = \sin(x - \pi)$. Both appear as the same graph as far as the student can tell. Finding zeros of the function is also more difficult since the x axis is not labeled in terms of π.

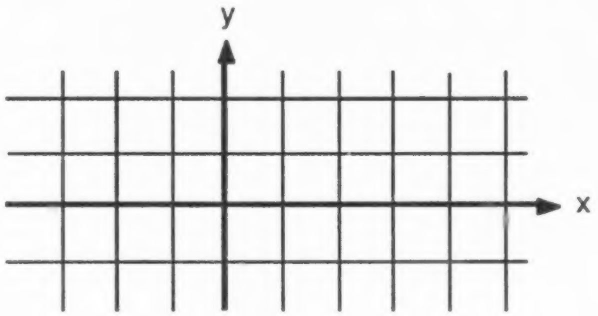
Student Worksheet Stretch/Shrink Trigonometric Functions

Trigonometric equations are usually in the form $y = A * \sin(Bx + C)$, where $|A|$ is the amplitude of the function, $2\pi/|B|$ is the period of the function, and $-C/B$ describes the phase shift of the function.

I. In the following set of exercises, you will be examining trigonometric equations of the form $y = A * \sin(x)$. For these equations, $B = 0$ and $C = 0$.

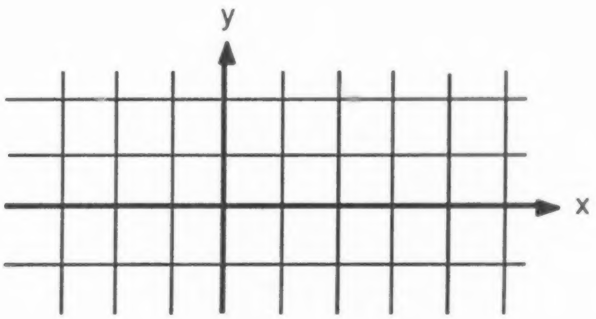
A. Type in each equation, observe its graph, and sketch and label it before typing in the next equation.

1) $y = \sin(x)$
2) $y = 2 * \sin(x)$
3) $y = .5 * \sin(x)$
4) $y = 3 * \sin(x)$
5) $y = .2 * \sin(x)$



B. Erase all graphs from the previous section. Type in the following equations and observe them one at a time. Sketch and label them below.

1) $y = \sin(x)$
2) $y = -\sin(x)$
3) $y = -3 * \sin(x)$
4) $y = -.5 * \sin(x)$



C. Describe the changes in the graph of y = A * sin(x) when | A | > 1 increases.

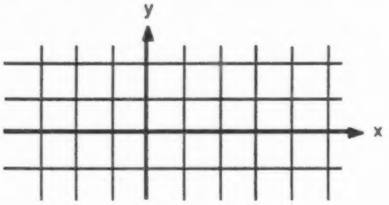
When | A | approaches 0, what happens to the graph of y = A * sin(x)?

What is the effect on the graph of y = A * sin(x) when A < 0?

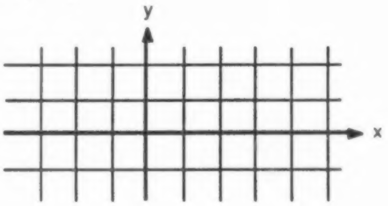
In general, we can say that | A | determines the amplitude of the graph y = A * sin(x).

II. Erase all graphs. The following equations will be of the form y = sin(B*x). That is, A = 1 and C = 0 for all of these equations. Type in each of the following functions and sketch them on separate axes below. Erase each graph before continuing on to the next.

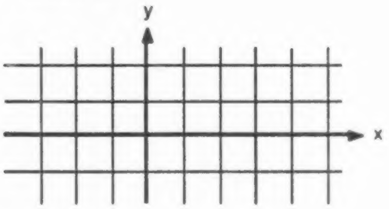
a) y = sin(x)



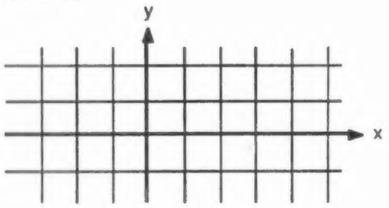
b) y = sin(2*x)



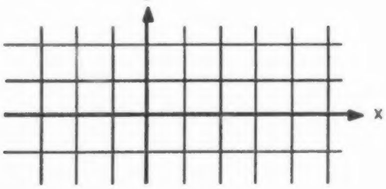
c) y = sin(.5*x)



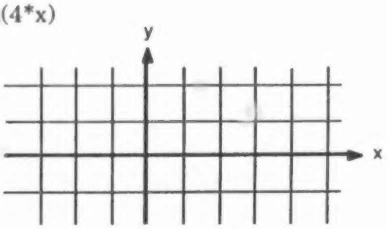
d) y = sin(.25*x)



e) y = sin(3*x)



f) y = sin(4*x)



The value of B tends to stretch or shrink the sine curve along the x axis. This is a change in the period, the interval necessary for one repetition of the function.

The period of $y = \sin(x)$ is 2π units. What is the period of $y = \sin(2*x)$?

What is the period of $y = \sin(3*x)$?

In general when the value of $B > 1$, how is the period affected?

What is the period of $y = \sin(.5*x)$?

What is the period of $y = \sin(.25*x)$?

In general when the value of $B > 0$ and $B < 1$, how is the period affected?

Can you find a formula that relates B to the period of a function?

III. Erase all graphs. Change several equations from parts I. and II. above from the form $y = \sin(x)$ to $y = \cos(x)$. What are your observations?

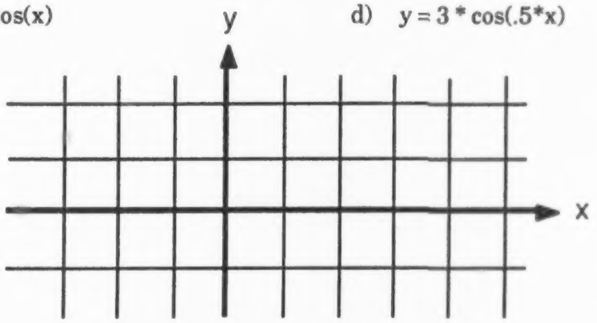
IV. Sketch the following equations on the axes provided below without using the computer. Check your answers afterwards by typing them into the computer.

a) $y = 2 * \sin(2*x)$

b) $y = -.5 * \sin(3*x)$

c) $y = -2 * \cos(x)$

d) $y = 3 * \cos(.5*x)$



Teacher Notes for Worksheet on Trigonometric Identities

OBJECTIVE

To provide the student with a visual means for verifying trigonometric identities.

DESCRIPTION

Any graphics package which handles trigonometric functions is appropriate for using with this worksheet. Teachers need to give instructions for use of the particular software that they choose. This worksheet is appropriate for individual or small group use or as a whole group activity when only one computer is available.

SPECIAL NOTES

This worksheet takes a full class period to complete.

It must be explained that each side of the identity equation worked with will be set equal to y and graphed. If the graphs of both sides of the equation are the same, the function is an identity.

Most graphics packages do not label the x axis in terms of π. The x axis will be labeled ... -3, -2, -1, 0, 1, 2, 3 ... Instead of $\pi/2$, the value will be found at 1.57. Prepare the students for this.

For any graphics package, setting the x values from -6 to 6 and the y values in the same range usually works well.

Some graphics packages have flaws in their plotting routines which cause them to connect point where no y value exists, for instance, at $\pi/2$ on the tangent graph. Point this out to the students if your software is like this.

Be sure to emphasize the restrictions to the domain of particular graphs and point out the corresponding display on the computer screen.

Student Worksheet
Trigonometric Identities

I. This worksheet will explore identities, those equations that are true for all values of the variables for which both sides of the equation are defined. Those values for which the function is not defined are referred to as the restrictions for the domain of the function.

Recall the following relationships between the trigonometric functions. You may use these for reference later.

Definitions-

$\tan x = \sin x / \cos x$

$\cot x = \cos x / \sin x$

Reciprocal Relationships-

$\csc x = 1 / \sin x$ and $\sin x = 1 / \csc x$

$\sec x = 1 / \cos x$ and $\cos x = 1 / \sec x$

$\cot x = 1 / \tan x$ and $\tan x = 1 / \cot x$

Pythagorean Relationships-

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

II. Proving an identity such as

$$1 - 2\cos^2 x = 2\sin^2 x - 1$$

involves using the trigonometric relationships above to rewrite one side of the equation so it has the same form as the other side, as follows:

$$\begin{aligned} 1 - 2\cos^2 x &= 2\sin^2 x - 1 \\ &= 2(1 - \cos^2 x) - 1 \\ &= 2 - 2\cos^2 x - 1 \\ &= 1 - 2\cos^2 x \end{aligned}$$

IF THE RIGHT AND LEFT SIDES OF AN EQUATION ARE EQUAL, THEIR GRAPHS SHOULD BE THE SAME.

Using the equation above, if you enter

$$y = 1 - 2\cos^2 x \quad \text{and} \quad y = 2\sin^2 x - 1$$

and look at the resulting graphs that are sketched, they should be equivalent. Type them in and make that comparison. Notice that there are no asymptotic lines in the graphs and that there are no restrictions for the domain of either function.

III. Erase the equations from part II and type in the following:

1) $y = \tan x + \cot x$

2) $y = \sec x \csc x$

Notice that vertical asymptotes appear in the graphs of these functions. What are the restrictions on the domains of each?

Is $\tan x + \cot x = \sec x \csc x$ an identity? If so, prove it by rewriting it below.

IV. Erase the graphs from part III and repeat the process for:

1) $y = (1 - \sin x) / \cos x$

2) $y = \cos x / (1 + \sin x)$

If the graphs were the same, prove the identity below.
State what restrictions apply to the domains.

V. If $\cos^4 x - \sin^4 x + 2\sin^2 x = 1$

describe the graph of the function

$$y = \cos^4 x - \sin^4 x + 2\sin^2 x$$

without actually graphing it.

VI. Using the graphing methods described in parts III and IV, determine whether the following are trigonometric identities. Circle those which are and prove them in the space provided. State the restrictions on their domains.

A) $(\sin A / \csc A) + (\cos A / \sec A) = \sin A \csc A$

B) $\tan^2 A - \sin^2 A = \tan^2 A \sin A$

C) $\sec A + \csc A = (1 + \tan A) / \sin A$

D) $\cos A + \sec A + 2 = 0$

E) $(\cos A - \sin A) / \cos A = 1 - \tan A$

Teacher Notes for Worksheet on Absolute Values

OBJECTIVES

To familiarize the student with the shape of the absolute value function and its relationship to a linear graph. To show how changing the value of A, B, and C in an equation of the form $y = A\,|x + B| + C$ will change the graph.

DESCRIPTION

Any graphics package which handles absolute value functions is appropriate for use with this worksheet. Students may work alone, in small groups, or in a teacher led discussion with one computer at the front of the classroom.

SPECIAL NOTES

This is a rather lengthy worksheet. The best strategy may be to let students do sections I-IV on their own in class. The next day the teacher could review some of the ideas from the previous day and the group could answer sections V-VII together.

A good follow-up worksheet would be to consider equations of the form $y = |A*x + B| + C$, again varying A, B, and C.

Student Worksheet
Absolute Values

Recall the definition of $|X|$

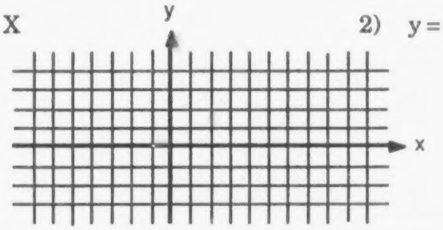
$$|X| = X \text{ when } X >= 0$$

$$|X| = -X \text{ when } X < 0$$

I. Type the following equations into the computer and sketch their graphs on the axes provided below. Erase each set before going on to the next.

1) $y = X$

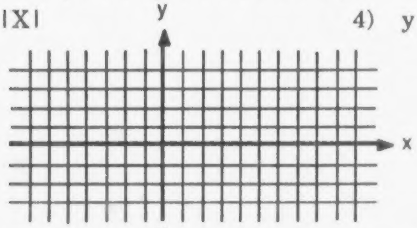
2) $y = |X|$



3) $y = |X|$

4) $y = 2X$

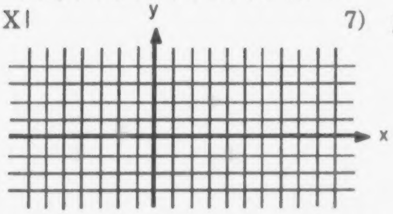
5) $y = 2|X|$



6) $y = |X|$

7) $y = .5X$

8) $y = .5|X|$



Describe how the pairs of graphs above (1 & 2, 4 & 5, 7 & 8) are related.

How do you account for this?

II. Erase the screen and type in the following equations. Sketch their graphs below.

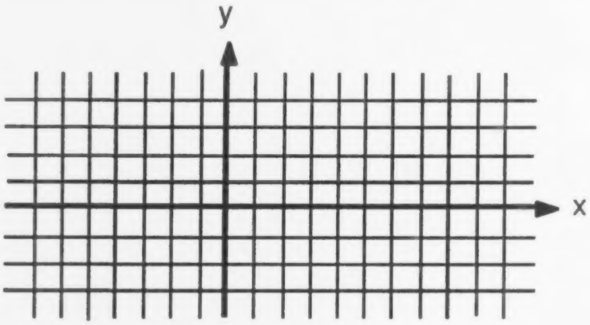
1) $y = |X|$

2) $y = 3\,|X|$

3) $y = -\,|X|$

4) $y = -2\,|X|$

5) $y = -.5\,|X|$



What relation does the value of the multiplier of the absolute value have on its graph?

How does this compare with the slope, m, of a line on a graph of $y = mx + b$?

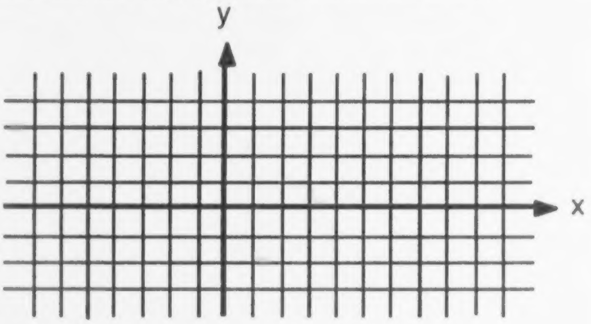
III. Erase the previous graphs. Type in the following equations and sketch below.

1) $y = |X|$

2) $y = |X+2|$

3) $y = |X-3|$

4) $y = |X-1|$



In equations of the form $y = |X - h|$, what effect does h have on the graph?

Describe what the graph of $y = |X + 5|$ will look like.

IV. Erase all graphs. Type in the following equations and sketch below.

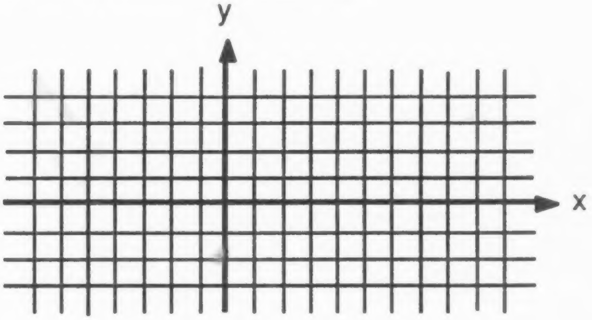
1) $y = |x|$

2) $y = |X| + 2$

3) $y = |X| - 3$

4) $y = |X| - 4.5$

5) $y = |X| + .5$



In equations of the form $y = |X| + k$, what effect does k have on the graph?

Describe what the graph of $y = |X| + 6$ should look like.

V. Sketch the following equations on the graph below. Afterwards, check your answers by entering the equations into the computer.

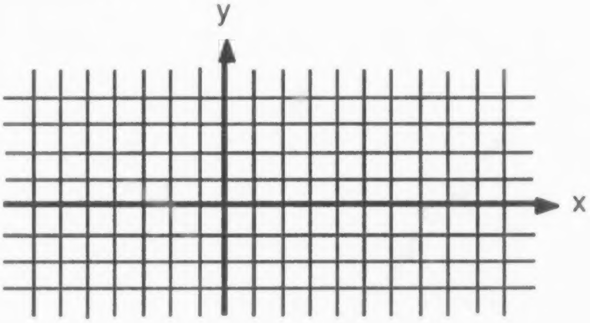
1) $y = |X - 2| + 3$

2) $y = - |X + 3| + 4$

3) $y = 2 |X - 1| - 3$

4) $y = -3 |X + 2|$

5) $y = |X - 1| - 1$



VI. Answer the following questions based on the information above.

1) In an equation of the form $y = |X - h|$, if $h > 0$ the absolute value graph is shifted h units __________.

2) In the same equation, if $h < 0$ the absolute value graph is shifted h units __________.

3) In the equation $y = |X| + k$, if $k > 0$ the absolute value graph is shifted k units __________.

4) In the same equation, if $k < 0$ the absolute value graph is shifted k units __________.

Teacher Notes for Worksheet on Exponential Graphs

OBJECTIVES

To provide the student with experience in graphing equations of the form $y = bx$.

DESCRIPTION

Any graphics package which handles exponential function graphs will work with this worksheet. Students may work alone, in small groups, or in a teacher-directed lesson where one computer is located in front of the classroom.

SPECIAL NOTES

This is a lengthy worksheet which may take more than one class period.

Students should be able to handle sections A-E on their own. It may be desirable to use section F for full class discussion or in some cases, to omit it.

Some of the ideas in this worksheet are not usually covered in an Algebra II class, but are certainly not beyond their understanding.

Student Worksheet
Exponential Graphs

This worksheet explores equations in the form $y = b^x$. You will observe the effects of varying x with a number of different bases b.

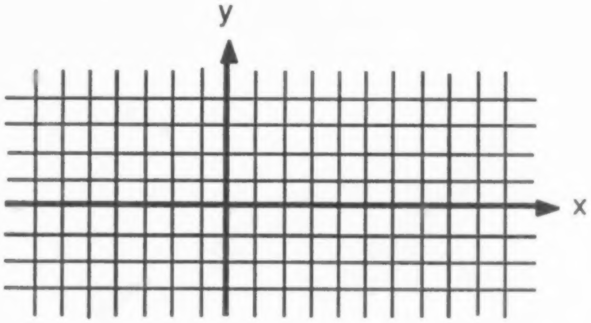
A. In this first set of exercises the value of b in $y = b^x$ will be greater than one. Type the following equations into the computer. Sketch and label their graphs on the axes below.

1) $y = 2^x$

2) $y = 3^x$

3) $y = 5^x$

4) $y = 12^x$



Notice that the graphs of the above equation extend left and right along the x axis without any gaps or holes. We say that the domain of this function, the allowable x values, is the set of real numbers. Observe that the y values of the graph are always in the first or second quadrants. We say that the resulting y values, the range, are real numbers greater than zero.

Did you notice that there was a point common to the graphs of equations 1-4 above? What was that point?

Explain why this point is present on each of the curves.

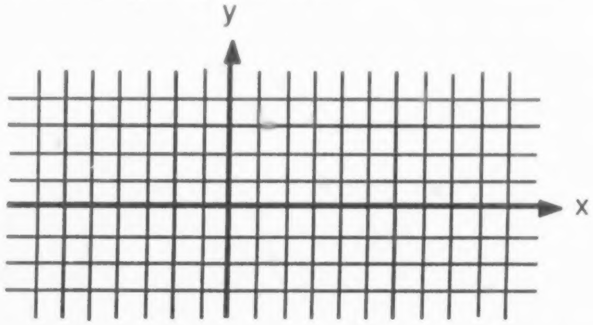
B. This section will explore graphs of $y = b^x$ where $0 < b < 1$. Erase the graphs from above and type in the following. Sketch and label your graphs below.

1) $y = (3/4)^x$

2) $y = (1/2)^x$

3) $y = (1/3)^x$

4) $y = (1/5)^x$



What is the domain for the graphs above?

What is the range for the graphs above?

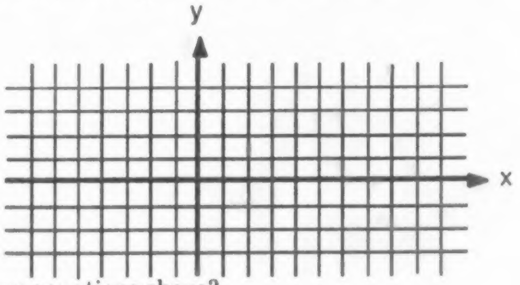
What point do these four graphs have in common?

Why is that a common point to all four curves?

C. Erase the graphs from above and type in the following. Sketch and label their graphs below.

1) $y = (1/2)^x$

2) $y = 2^{-x}$

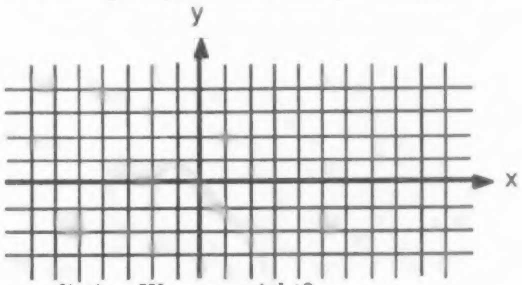


What do you notice about the graphs of the two equations above?

Why?

Rewrite the equations for 1, 3, and 4 from part B using negative exponents.

D. Erase the graphs from above. Sketch below what your prediction that $y = 1^x$ should look like.

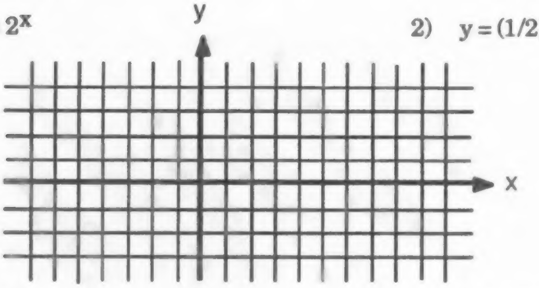


Now type in the equation above to check your prediction. Were you right?
What is another way to write that equation?

E. Erase your previous graphs. Type in the following pairs of equations, erasing the screen in between pairs. Sketch and label them on the axes below.

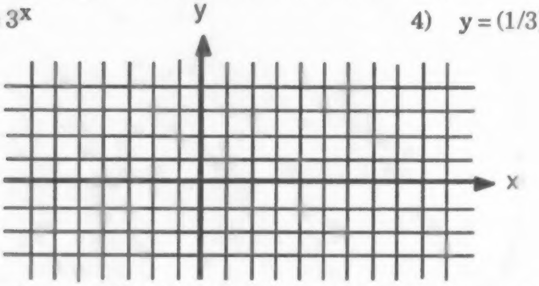
1) $y = 2^x$

2) $y = (1/2)^x$

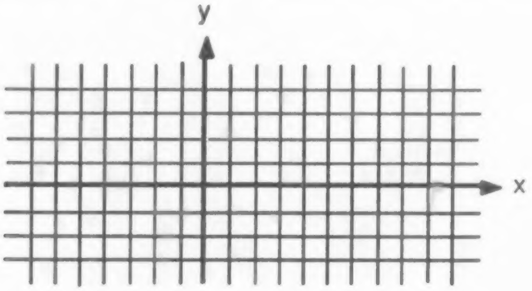


3) $y = 3^x$

4) $y = (1/3)^x$



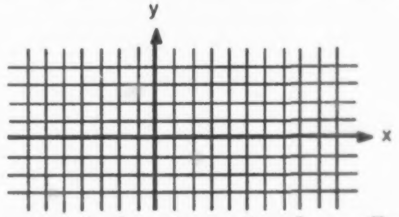
Predict what $y = 4^x$ and $y = (1/4)^x$ should look like by sketching their graphs on the axes below.



Erase any previous graphs and type these two equations in to check your predictions.

F. Can you predict what the graph of $y = b^x$ will look like if $b < 0$? Erase all graphs from before and type in the following. Sketch and label it on the axes below.

1) $y = (-2)^x$



Notice that the computer accepted the function and made an effort to graph the points that were calculated. However, nothing showed up on the screen. Consider a table of values for the function above.

x	$y = (-2)^x$
-2	1/4
-1	-1/2
0	1
1	-2
2	4

Did you notice that the values of y varied from negative to positive depending on whether the exponent, x, was odd or even? For all integer values of x, real values of y will result.

Now consider x to be a fractional exponent, such as 1/2 What is the value of $(-2)^{(1/2)}$?

Whenever the numerator of the fraction is odd, the negative base raised to that power will remain negative. If the denominator is also odd, the answer will be a real number. For instance, $(-27)^{(1/3)}$ can be rewritten as $\sqrt[3]{-27}$ and can be simplified to give an answer of -3. However, whenever the numerator isodd and the denominator is even, the resulting answer will be an imaginary number, an even root of a negative number. An example would be $(-25)^{(1/2)}$ which can be written as $\sqrt{-25}$. The simplified answer here is +/- 5i.

The computer only plots real values. Over a given number of x values, a large number of imaginary range values will occur. In fact, depending on the programmer''s algorithm for dividing up the interval, few points may turn out to have real number range values. As a result, the computer does not plot these functions where $b < 0$.

Session 14S
From Algebra to Calculus

ACTIVITY ONE

OBJECTIVE

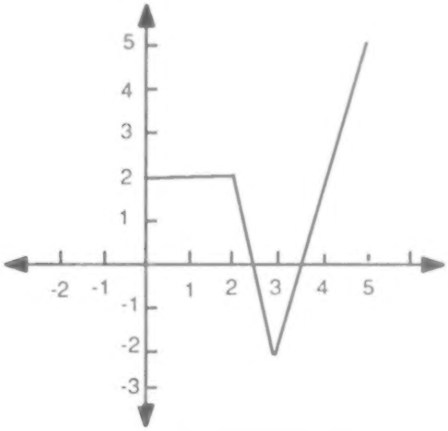
To apply the relationships discovered among the coefficients and constants to the sketching of a graph of a basic "core" function as it passes through a variety of parameter changes.

DESCRIPTION

The graph of an abstract "core" function is given. Students are to apply the observations of the previous lab to this situation.

PROCEDURE

On a sheet of graph paper, given the graph of "f" below, sketch the graph of:



a. f(X - 2)

b. f(X - 2) + 3

c. 2*f(X)

d. -f(X)

The material developed in this session is appropriate for many levels of instruction in the secondary school. Graphics packages and spreadsheets are utilized.

ACTIVITY TWO

OBJECTIVE

To discuss a comprehensive algebraic method for solving algebraic equations and inequalities containing variables within absolute value signs as well as graphing absolute value functions of all kinds.

DESCRIPTION

An algebraic method, similar to that illustrated in the material in section 13S.1, pp.1-4 will be presented.

PROCEDURE

Solve:

1. $|X - 3| = 5$
2. $|X + 4| > 7$
3. $|X - 5| \leq 3$
4. $|X + 2| + |X - 6| < 10$

Graph:

$$Y = |X - 3| + |X + 2|$$

ACTIVITY THREE

OBJECTIVE

To present students with a clear picture of the speed with which a function value increases when the abscissa value approaches a point at which a vertical asymptote occurs. Also to look at several functions with non-vertical asymptotes and to become better able to sketch the graphs of the functions from observation of critical aspects of the equation.

DESCRIPTION

The program ASYMPTOTE, found on your disk, has been written to illustrate the very fast rate at which the value of the function $F(X) = 1/X$ increases or decreases as the value of X approaches zero. The program RELATION GRAPHER from "Chalkboard Graphics Tool Box I" will be used to extend these ideas.

PROCEDURE

1. Before proceeding with this material, the student should be made thoroughly familiar with the term asymptote and the graph of $F(X) = 1/X$.

2. Be certain that students are familiar with the way the computer expresses scientific notation and the fact that the computer rounds off certain answers as it computes in base two.

3. Run the program ASYMPTOTE.

4. In response to the questions:

    a. Begin at the X-value of -2.

    b. End at -.1.

    c. Increment by .1.

5. Run the program again. This time:

    a. Begin at the X-value of -.1.

    b. End at -.001.

    c. Increment by .001.

6. Finally:

   a. Begin at the X-value of -.001.

   b. End at -.00001.

   c. Increment by .00001.

7. Stress that the abscissa value is being incremented at equal intervals and observe how the value of the function, F(X), behaves. Discuss the implications this has for the graph of the function near zero.

8. It is instructive to at this point launch a discussion of functions like those which follow in order to present the notion of both vertical and non-vertical asymptotes.

   a. For each numbered problem below, it is suggested that the graph be plotted on a "fresh" set of coordinate axes. Use a graphics program such as Scharf Systems' "Chalkboard Graphics Tool Box I", RELATION GRAPHER with domain and range set at -10 to 10 with increment of 2, plot speed of "Fast/Low" and graph:

      1.) $Y = (1/X) + X$ - Analyze results.

      You might wish to graph $Y = X$ on the same set of axes. Before clearing the graph, sketch the results displayed on the computer screen on a sheet of paper.

      2.) $Y = X^2 + (1/X)$ - Analyze results.

      You might wish to graph $Y = X^2$ on the same set of axes. Again, before clearing the graph from the computer screen, sketch the display on a sheet of paper.

      Now graph:

      3.) $Y = (1/X^2)$ - Observe and analyze the results.

      Based on the previous results, predict what will happen by sketching, on paper, the graph of:

$$Y = X^2 + (1/X^2)$$

      Then graph using "Chalkboard Graphics ..." RELATION GRAPHER to check your prediction. Now, on the same set of axes, graph $Y = X^2$. Compare graphs and analyze results.

b. In each of the following cases, first sketch the graph on paper, then check it using the computer and RELATION GRAPHER. The suggested bounds for the domain and range are from -5 to 5 with increment of 1 and plot speed of "Fast/Low".

1.) $Y = X/(X^2 - X)$

2.) $Y = 1/(X^2 + X - 6)$

After checking the graphs with the computer program, be sure to discuss the reasons for the results as well as any errors you might have made in your initial sketches.

ACTIVITY FOUR

OBJECTIVE

To use the power of the computer to gain insight into the concept of limits using numerical examples including the application to the delta process and the definition of the derivative.

DESCRIPTION

Several programs, most of which are merely extensions of a basic "table of values" program, will be used to find the limits of sequences and expressions. A simple adjustment in the program will allow us to explore the limit of the sum of the terms of an infinite sequence. The definition of the derivative will be employed to find an approximation to the derivative of a function at various points using the program "DELTA.PROCESS" , found on your disk. In addition, a discussion of right and left hand limits will be undertaken and continuity will be mentioned.

PROCEDURE :

FIRST

- We will consider the definition of a sequence as that of a function whose domain is the set of positive integers.
- We will look at the sequence generated by $F(X) = (1 + 1/X)^X$. The question is what number, if any, do the terms of the sequence approach as the value of X takes on larger and larger positive integral values?
- At this point, it is interesting to make a conjecture before actually running the program "LIMIT.OF.(1+1/X)^X". The program is on your disk.
- After forming your conjecture, try running the program from 1 to 100 in single unit increments...then try going from 100 to 10000 in hundred unit increments...finally 1000 to 100000 in increments of 1000.

SECOND

- Consider the function F(X) = sin(X)/X as X becomes unboundedly large. Remember, X is in radians. Don't forget to make a conjecture first as to the value the individual terms of the sequence approach as X becomes very large. Then run the program LIMIT.OF.SIN(X)/X (found on your disk) and test your "guess".
- Now think of the same function as X becomes infinitesimally close to zero. Make your conjecture, then run the program to test it.

THIRD

- The sequence 1/2, 1/4, 1/8, 1/16, ... is generated by the function $F(X) = 1/2^X$ as X assumes the values of all the positive integers. Make a conjecture regarding the limit of the individual terms of the sequence as X becomes unboundedly large. From your disk, use the program "LIMIT.OF.1/2^X" to see if you're correct.
- Suppose we took the terms of this sequence and added them up, in order, forever. Does that sum have a limit? Take a guess! Then, with a slight modification of the previously used program, we can check the conjecture. Use the program LIMIT.OF.SUM.OF.1/2^X, found on your disk, and add up the first 30 terms. Add on more to convince yourself.

FOURTH

- Suppose we want to talk about the slope of a curve at a point. Let's consider the function

  $$F(X) = X^2$$

  and determine its slope at the point (3,9).
- It is assumed that you are already convinced that the slope of a curve at a point is the value of the slope of a line tangent to the curve at that point.
- Normally we choose another point on the curve, different from (3,9), and look at the slope of the secant line through (3,9) and this other point.
- The "other" point is then moved closer and closer to (3,9). As this happens we try to determine the limit that the slopes of the secant lines are approaching as the point moves ever closer to (3,9) along the curve.
- It is important to remember that we must look at slopes of secant lines as the "moving" point becomes closer to (3,9) from either side of (3,9) along the curve.
- Use the program "DELTA.PROCESS". Run it for the function $F(X) = X^2$ starting at 3 and choosing delta X as 2 with 10 iterations. In the process of using this program, it should be realized that delta X is being halved (any fraction between 0 and 1 would serve the purpose) each time to bring the "moving" point closer and closer to (3,9).

- Then try again (this time starting at 3 with a delta X of -2) to get on the other side of (3,9) once more using 10 iterations.
- By observing the two results, what appears to be the limit of the slope of the secant line as the "moving" point approaches (3,9)?
- That limit is called the value of the "derivative" at that point.

NOW, TO FURTHER INVESTIGATE

- Replace the function $F(X) = X^2$ with $F(X) = |X|$ and find the slope of the function at (0,0).
- Start at 0 with a delta X of 2 and 5 iterations. Can you explain the results?
- Now, to get to the other side of (0,0), start at 0 with a delta X of -2 and 5 iterations.
- What appears to be the limit of the slope of the secant line as the "moving" point approaches (0,0) from either side?
- Explain how you determined your answer.
- Finally, what is the value of the derivative of the function $F(X) = |X|$ at (0,0)? Again, be sure you can explain your answer.

ACTIVITY FIVE

OBJECTIVE

A numerical property relating the solution of linear systems of equations and a variety of sequences will be discovered (inductively) and proved (deductively).

DESCRIPTION

The program entitled "SOLUTION OF LINEAR SYSTEMS" (Hirsch, Christian R. "Families of Lines." MATHEMATICS TEACHER 76 (November 1983) :590-92, 597-98) found on your disk will be used to discover a property of the solution set of a linear system when the coefficients are taken from certain familiar sequences.

PROCEDURE

- Run the program "SOLUTION OF LINEAR SYSTEMS".
- Enter the coefficients 1, 2, 3, 4, 5 and 6 for A, B, C, D, E and F respectively. Observe the solution.
- Run the program again and enter the coefficients 3, 4, 5, 6, 7 and 8. Again note the solution.
- Next try 87, 88, 89, 90, 91 and 92 - then 54, 56, 58, 60, 62 and 64 - then 101, 105, 109, 113, 117 and 121. Try again using 83, 88, 93, 2, 7 and 12 - finally try 1,2,3,123,130 and 137.
- At this point a conjecture should be possible. Obviously it appears as if whenever the coefficients of a system of linear equations are in arithmetic sequence, even if the coefficients of one equation are from one arithmetic sequence and the coefficients of the other equation are from a different arithmetic sequence, the solution will always be (-1, 2). This conclusion is a generalization based upon observations and illustrates the use of inductive reasoning.
- A deductive proof using a method like Cramer's Rule should now be attempted.
- Similar results can be obtained using coefficients that are in geometric sequence. Fibonacci coefficients should also be tried.

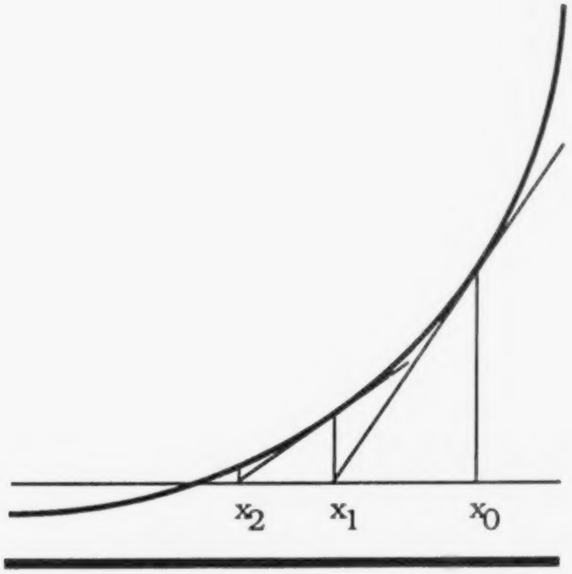
ACTIVITY SIX

OBJECTIVE

To demonstrate the power of Newton's Method in obtaining the real roots (real zeros) of a polynomial equation (function) correct to many decimal places.

DESCRIPTION

The program called FUNCTION ANALYZER by Scharf Systems will be used to review the method of finding the real roots of a polynomial equation to the nearest hundredth by the "location principal" and employing the "table of values" option. The program "NEWTONS", found on your disk, will then be employed to demonstrate the rapidity with which Newton's Method can obtain the same results correct to many more decimal places. We will then use a spread sheet to demonstrate the calculations involved in employing Newton's Method. CAUTION - users should be aware of the "risks" of employing Newton's Method. Further, a user of the "NEWTONS" program as well as the spreadsheet application of Newton's Method must be familiar with a method of finding the derivative of a function.



Newton's Method

PROCEDURE

1. Use the program FUNCTION ANALYZER to obtain a sketch of the function $Y = 2X^3 - 9X^2 + 2X + 1$
   Be sure to enter it as:

   F(X) = 2*X^3 - 9*X^2 + 2*X + 1.

   The domain and range are left for you to determine. This may take a period of trial and error.

2. From your "sketch" of the function, determine the region in which the largest real zero of the function occurs. Use the "table of values" option in the program to find this largest zero to the nearest tenth.

3. Use the "sketch" of the graph obtained in number one above to illustrate how Newton's Method operates. Try to discover some of the "risks" in using Newton's Method that will make it fail to produce the results you desire.

4. Use the program "NEWTONS" to find the largest real zero as well as the other two real zeros of the function (and thus the real roots of the corresponding equation) correct to the nearest hundred-thousandth.

5. The spreadsheet component of AppleWorks will now be used to analyze the function:

   $$Y = X^3 - 5X^2 + 7X - 2$$

   You are reminded that when using the spread sheet in AppleWorks, you must also enter the function as F(X) = X^3 - 5*X^2 + 7*X - 2.

- Look at the first page of the AppleWorks spreadsheet on the pages which follow. Rows 1 through 3 merely contain labels and blanks.
- The initial value or "guess" for Newton's Method is placed in cell C4.
- Rows 5 through 7 are again reserved for labels and blanks.
- Cell A8 contains the initial value that you place in cell C4.
- Cell C8 contains the function evaluated at the initial value from cell A8.
- Cell E8 contains the value of the derivative of the function at the initial value.
- Cell A9 contains the new "guess" or "next estimate" for the function's zero based upon Newton's formula:

  $$X_2 = X_1 - F(X_1)/F'(X_1)$$

- Cell C9 contains the function evaluated at the "next estimate" from cell A9.
- Cell E9 contains the value of the derivative of the function at the "next estimate" from cell A9.
- Cell A10 contains the "next estimate" for the function's zero based upon Newton's formula shown above.
- Each cell thereafter repeats the process using the next set of values obtained from the previous row.
- IMPORTANT : Be sure you set the spreadsheet to calculate in rows, NOT COLUMNS!! If you happen not to do this, you will get very strange results.
- On the second spreadsheet page that follows, you'll note how Newton's Method quickly converges on the zero 2.618034 from an initial "guess" of 500.
- For the sake of illustration, this function was chosen containing zeros that are relatively close together. Therefore on the third spreadsheet page that follows, it can be seen that an initial "guess" of 2.2 converges to a zero of 2; on the fourth spreadsheet page, the initial "guess" of 2.3 converges to .381966; and the fifth spreadsheet page illustrates the initial "guess" of 2.4 converging to the same zero as 500 did earlier - namely 2.618034.
- The sixth of the spreadsheet pages that follow illustrate how the initial "guess" of 1 results in an "ERROR". Why did that happen?

```
File:  NEWTONS2                          Page 1

       NEWTON'S              METHOD
       F(X) = X^3 - 5*X^2 + 7*X - 2

   INIT X =  -500

    X                          F(X)                                   F'(X)

+C4                  (A8^3)-((5)*(A8^2))+(7*A8)-2         (3*(A8^2))-(10*A8)+7
+A8-(C8/E8)          (A9^3)-((5)*(A9^2))+(7*A9)-2         (3*(A9^2))-(10*A9)+7
+A9-(C9/E9)          (A10^3)-((5)*(A10^2))+(7*A10)-2      (3*(A10^2))-(10*A10)+7
+A10-(C10/E10)       (A11^3)-((5)*(A11^2))+(7*A11)-2      (3*(A11^2))-(10*A11)+7
+A11-(C11/E11)       (A12^3)-((5)*(A12^2))+(7*A12)-2      (3*(A12^2))-(10*A12)+7
+A12-(C12/E12)       (A13^3)-((5)*(A13^2))+(7*A13)-2      (3*(A13^2))-(10*A13)+7
+A13-(C13/E13)       (A14^3)-((5)*(A14^2))+(7*A14)-2      (3*(A14^2))-(10*A14)+7
+A14-(C14/E14)       (A15^3)-((5)*(A15^2))+(7*A15)-2      (3*(A15^2))-(10*A15)+7
+A15-(C15/E15)       (A16^3)-((5)*(A16^2))+(7*A16)-2      (3*(A16^2))-(10*A16)+7
+A16-(C16/E16)       (A17^3)-((5)*(A17^2))+(7*A17)-2      (3*(A17^2))-(10*A17)+7
+A17-(C17/E17)       (A18^3)-((5)*(A18^2))+(7*A18)-2      (3*(A18^2))-(10*A18)+7
+A18-(C18/E18)       (A19^3)-((5)*(A19^2))+(7*A19)-2      (3*(A19^2))-(10*A19)+7
+A19-(C19/E19)       (A20^3)-((5)*(A20^2))+(7*A20)-2      (3*(A20^2))-(10*A20)+7
+A20-(C20/E20)       (A21^3)-((5)*(A21^2))+(7*A21)-2      (3*(A21^2))-(10*A21)+7
+A21-(C21/E21)       (A22^3)-((5)*(A22^2))+(7*A22)-2      (3*(A22^2))-(10*A22)+7
+A22-(C22/E22)       (A23^3)-((5)*(A23^2))+(7*A23)-2      (3*(A23^2))-(10*A23)+7
+A23-(C23/E23)       (A24^3)-((5)*(A24^2))+(7*A24)-2      (3*(A24^2))-(10*A24)+7
+A24-(C24/E24)       (A25^3)-((5)*(A25^2))+(7*A25)-2      (3*(A25^2))-(10*A25)+7
+A25-(C25/E25)       (A26^3)-((5)*(A26^2))+(7*A26)-2      (3*(A26^2))-(10*A26)+7
+A26-(C26/E26)       (A27^3)-((5)*(A27^2))+(7*A27)-2      (3*(A27^2))-(10*A27)+7
+A27-(C27/E27)       (A28^3)-((5)*(A28^2))+(7*A28)-2      (3*(A28^2))-(10*A28)+7
+A28-(C28/E28)       (A29^3)-((5)*(A29^2))+(7*A29)-2      (3*(A29^2))-(10*A29)+7
```

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File:  NEWTONS2                    Page 2

     NEWTON'S          METHOD
     F(X) = X^3 - 5*X^2 + 7*X - 2

   INIT X =              500

          X                       F(X)                       F'(X)

      500                 123753498.                 745007.

     333.8894829           36667653.9985399          331114.6655798
     223.1494348           10864457.3670029          147162.5164075
     149.3231805            3219076.7096954           65406.0048557
     100.106343              953786.0285412           29069.7762931
      67.2961136             282593.6504509           12920.339582
      45.4241149              83725.0750065            5742.8094968
      30.8450008              24803.2124991            2552.7922137
      21.1288898               7346.3222425            1135.0010541
      14.6563652               2174.8691019             504.8634704
      10.3485291                643.2246649             224.7908705
       7.4870932                189.82755               100.2987607
       5.5944721                 55.7671617              44.9496327
       4.3538132                 16.2277758              20.3289359
       3.5555532                  4.6282359               9.3703438
       3.0616294                  1.2619444               4.5044297
       2.7814731                   .3064729               2.3950468
       2.653512                    .0526664               1.5882576
       2.6203521                   .0032189               1.3952143
       2.618045                    .0000152               1.3820289
       2.618034                    .0000000               1.381966
       2.618034                    .0000000               1.381966
       2.618034
```

```
File:  NEWTONS2                        Page 3
       NEWTON'S              METHOD
       F(X) = X^3 - 5*X^2 + 7*X - 2

   INIT X =                        2.2

     X                             F(X)                      F'(X)

          2.2                      -.152                      -.48
   1.8833333                   .1286898                    -1.1925
   1.9912493                   .0088266                 -1.0172716
    1.999926                    .000074                 -1.0001479
            2                   .0000000                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
            2                          0                        -1
```

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File:  NEWTONS2                    Page 4

      NEWTON'S            METHOD
      F(X) = X^3 - 5*X^2 + 7*X - 2

   INIT X =              2.3

     X                           F(X)                   F'(X)

      2.3                       -.183                   -.13
       .8923077                  .9755558                .4655621
     -1.2031285               -19.4010419              23.3738403
      -.3730963                -5.3596133              11.1485652
       .1076484                -1.3031546               5.9582805
       .3263616                 -.2132668               4.0559196
       .3789432                 -.0109718               3.6413616
       .3819563                 -.000035                3.6181087
       .381966                  -.0000000               3.618034
       .381966                   .0000000               3.618034
       .381966                   .0000000               3.618034
       .381966                   .0000000               3.618034
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       .381966                   .0000000               3.618034
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File:  NEWTONS2                    Page 5

      NEWTON'S              METHOD
      F(X) = X^3 - 5*X^2 + 7*X - 2

   INIT X =                         2.4

X)          F'(X)

      2.4                       -.176                .28
      3.0285714                 1.1175743            4.2310204
      2.7644332                  .2666276            2.2819406
      2.6475907                  .0433655            1.5533025
      2.6196724                  .0022719            1.3913266
      2.6180395                  .0000076            1.3819975
      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
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      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
      2.618034                   .0000000            1.381966
```

```
File:  NEWTONS2                    Page 6

       NEWTON'S            METHOD
       F(X) = X^3 - 5*X^2 + 7*X - 2

    INIT X =             1

          X                        F(X)                       F'(X)

          1                        1                          0
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
          ERROR                    ERROR                      ERROR
```

ACTIVITY SEVEN

OBJECTIVE

To use the spreadsheet to provide some insight into the concept of compound interest.

DESCRIPTION

The spreadsheet component of AppleWorks will be used to develop the concept of compound interest beginning with simple interest for the first period and then building to the idea of interest on principal plus interest over a number of interest periods. Finally, the concept will be extended to evaluate the limit of (1 + 1/h)h as h gets unboundedly large. It is assumed that the person using this activity is familiar with the operation of a spreadsheet.

PROCEDURE

- The class must be aware of the formula for simple interest -- I = p.r.t, where p is the principal, r is the rate of interest for the interest period under consideration and it is the amount of time over which the investment is made.

- It must be noted that if "r", the rate of interest, is expressed in terms of a yearly interest rate, and there are "n" interest periods per year, the rate per interest period is "r/n".

- Let's look at the initial setup of our spreadsheet and formulas:

Cells

	"A"	"B"	"C"	"D"	"E"
1				COMPOUND	INTEREST
2					
3	PRINCIPAL				=100
4	YEARLY RATE (in decimal form)				= .04
5	NUMBER OF PERIODS PER YEAR				= 4
6	PERIODS				AMOUNT IN ACCOUNT
7	1				+E3+(E3*(E4/E5))
8	+A7+1				+E7+(E7*(E4/E5))
9	+A8+1				+E8+(E8*(E4/E5))
10	+A9+1				+E9+(E9*(E4/E5))

- Note that rows 1 and 2 are occupied only by labels and blanks.

- Cell E3 contains the value of the principal; E4 the yearly rate in decimal form; and E5, the number of interest periods per year.
- The formulas begin in row 7.
- Column 1 from row 7 down merely contains the interest periods beginning at 1 in cell A7, then adding 1 in each successive row.
- Cell E7 contains the formula for the total amount in the account after one interest period. It is principal (E3) plus principal (E3) multiplied by the interest rate for the period (E4/E5).
- Note that the interest rate for the period is the yearly rate (E4) divided by the number of interest periods per year (E5).
- Remember the total amount in the account immediately before the second interest period can be found in cell E7.
- Cell E8 contains the total amount in the account at the end of the second interest period. Therefore it contains the principal at the end of the first interest period (E7) plus this principal (E7) multiplied by the rate of interest per period which will always be the yearly rate (E4) divided by the number of periods of interest per year (E5).
- At this point, a great spreadsheet capability will be employed --- that of copying columns and being able to choose which variables remain constant and which vary according to the cell in which they're located.
- The result, for twenty interest periods, is as follows on the next page.

```
          COMPOUND    INTEREST

PRINCIPAL                              =        1000
YEARLY RATE  (in decimal form)         =         .04
NUMBER OF PERIODS PER YEAR             =           4

     PERIODS           AMOUNT IN ACCOUNT
        1                   1010
        2                  1020.1
        3                  1030.301
        4                  1040.60401
        5                  1051.0100501
        6                  1061.5201506
        7                  1072.1353521
        8                  1082.8567056
        9                  1093.6852727
       10                  1104.6221254
       11                  1115.6683467
       12                  1126.8250301
       13                  1138.0932804
       14                  1149.4742132
       15                  1160.9689554
       16                  1172.5786449
       17                  1184.3044314
       18                  1196.1474757
       19                  1208.1089504
       20                  1220.1900399
```

- Solve the following problems:

1. Compare the total amount in the account after depositing \$1000 at 4% yearly interest compounded quarterly for five years with the same amount deposited for the same period of time -- still compounding quarterly -- at a rate of 5%, 6%, 7%, 8%. See the pages which follow.

2. Extend the spreadsheet on the previous page to account for 360 interest periods. Compare the amount in the account after one year from depositing \$1000 at 4% compounded quarterly with the amount in the account at 4% compounding daily (a bank uses a 360 day year.)

3. If one permits the principal to be "1", the rate to be "1" and the number of interest periods per year to be "many", then extends the spreadsheet so that a full year's balance can be seen, the total amount in the account approaches the constant "e". Try it for the spreadsheet with 360 entries. Let the principal be "1". the rate be "1" and the number of interest periods per year be "360". The beginning and end of this spreadsheet can be found on one of the following pages. Obviously, better approximations of "e" can be obtained by extending the spreadsheet further.

\$1000 at 5% interest compounded quarterly:

PERIODS	AMOUNT IN ACCOUNT
1	1012.5
2	1025.15625
3	1037.9707031
4	1050.9453369
5	1064.0821536
6	1077.3831805
7	1090.8504703
8	1104.4861012
9	1118.2921774
10	1132.2708297
11	1146.424215
12	1160.7545177
13	1175.2639492
14	1189.9547486
15	1204.8291829
16	1219.8895477
17	1235.138167
18	1250.5773941
19	1266.2096116
20	1282.0372317

$1000 at 6% interest compounded quarterly:

PERIODS	AMOUNT IN ACCOUNT
1	1015
2	1030.225
3	1045.678375
4	1061.3635506
5	1077.2840039
6	1093.4432639
7	1109.8449129
8	1126.4925866
9	1143.3899754
10	1160.540825
11	1177.9489374
12	1195.6181715
13	1213.552444
14	1231.7557307
15	1250.2320667
16	1268.9855477
17	1288.0203309
18	1307.3406358
19	1326.9507454
20	1346.855007

$1000 at 7% interest compounded quarterly:

PERIODS	AMOUNT IN ACCOUNT
1	1017.5
2	1035.30625
3	1053.4241094
4	1071.8590313
5	1090.6165643
6	1109.7023542
7	1129.1221454
8	1148.881783
9	1168.9872142
10	1189.4444904
11	1210.259769
12	1231.4393149
13	1252.989503
14	1274.9168193
15	1297.2278636
16	1319.9293512
17	1343.0281149
18	1366.5311069
19	1390.4454012
20	1414.7781958

```
$1000 at 8% interest compounded quarterly:
  PERIODS                    AMOUNT IN ACCOUNT
     1                            1020
     2                             1040.4
     3                             1061.208
     4                             1082.43216
     5                             1104.0808032
     6                             1126.1624193
     7                             1148.6856676
     8                             1171.659381
     9                             1195.0925686
    10                             1218.99442
    11                             1243.3743084
    12                             1268.2417946
    13                             1293.6066305
    14                             1319.4787631
    15                             1345.8683383
    16                             1372.7857051
    17                             1400.2414192
    18                             1428.2462476
    19                             1456.8111725
    20                             1485.947396
```

```
        COMPOUND     INTEREST
PRINCIPAL                              =        1000
YEARLY RATE (in decimal form)          =         .04
NUMBER OF PERIODS PER YEAR             =         360

    PERIODS                AMOUNT IN ACCOUNT
       1                      1000.1111111
       2                      1000.2222346
       3                      1000.3333704
       4                      1000.4445185
       5                      1000.555679
       6                      1000.6668519
       7                      1000.7780371
       8                      1000.8892346
       9                      1001.0004446
      10                      1001.1116668
      11                      1001.2229015
      12                      1001.3341485
      13                      1001.4454078
      14                      1001.5566795
      15                      1001.6679636
      16                      1001.77926
      17                      1001.8905688
      18                      1002.00189
      19                      1002.1132236
      20                      1002.2245695
=======================================================
=======================================================
     340                      1038.49825
     341                      1038.6136387
     342                      1038.7290402
     343                      1038.8444545
     344                      1038.9598817
     345                      1039.0753217
     346                      1039.1907745
     347                      1039.3062401
     348                      1039.4217186
     349                      1039.5372099
     350                      1039.652714
     351                      1039.768231
     352                      1039.8837608
     353                      1039.9993034
     354                      1040.1148589
     355                      1040.2304272
     356                      1040.3460084
     357                      1040.4616024
     358                      1040.5772092
     359                      1040.6928289
     360                      1040.8084615
```

```
          COMPOUND    INTEREST

PRINCIPAL                          =          1
YEARLY RATE  (in decimal form)     =          1
NUMBER OF PERIODS PER YEAR         =        360
    PERIODS                      AMOUNT IN ACCOUNT
         1                           1.0027778
         2                           1.0055633
         3                           1.0083565
         4                           1.0111575
         5                           1.0139663
         6                           1.0167828
         7                           1.0196072
         8                           1.0224395
         9                           1.0252796
        10                           1.0281276
        11                           1.0309835
        12                           1.0338473
        13                           1.0367191
        14                           1.0395989
        15                           1.0424867
        16                           1.0453825
        17                           1.0482863
        18                           1.0511982
        19                           1.0541182
        20                           1.0570463
========================================================
========================================================
       341                           2.5751533
       342                           2.5823065
       343                           2.5894796
       344                           2.5966726
       345                           2.6038856
       346                           2.6111186
       347                           2.6183717
       348                           2.6256449
       349                           2.6329384
       350                           2.6402521
       351                           2.6475861
       352                           2.6549405
       353                           2.6623154
       354                           2.6697107
       355                           2.6771266
       356                           2.684563
       357                           2.6920202
       358                           2.699498
       359                           2.7069966
       360                           2.714516
```

ACTIVITY EIGHT

OBJECTIVE

The spreadsheet component of AppleWorks will be employed to solve many problems from the variety that occur in high school algebra courses to maximum-minimum problems that are traditionally found in calculus courses, but can now be done by students whose background is no higher than first year algebra.

DESCRIPTION

The problems will be stated. A portion of the AppleWorks spreadsheet including the formulas will be displayed. Finally the spreadsheet (either partial or complete - depending on length) will be presented to show the solution. In some instances, follow-up questions will be provided to extend the ideas presented.

PROBLEMS

COOKIE MIXTURE PROBLEM

A storekeeper has two kinds of cookies - one worth seventy-five cents per pound and the other worth fifty cents per pound. How many pounds of each should he use to make a mixture of sixty pounds worth fifty-five cents per pound?

SPREADSHEET WITH FORMULAS

	cookie 1	cookie 2
# of pounds	=0	60-C9
price per pound	=.75	.5
		TOTAL COST
		|||||||||||
		(C9*C11)+(E9*E11)

The general idea here is to first read and understand the problem - determining that if you must wind up with sixty pounds of cookies worth 55 cents per pound, the total cost of the cookie mixture is $33.00. Once that is determined, the expressions relating the quantities in the problem are entered into the spreadsheet. This can be accomplished either by the student or the teacher depending upon the sophistication of the student. Finally, various numbers of pounds of "cookie #1" are entered into spreadsheet cell C9 to see when the total cost will become the $33.00 previously determined to be the value which will produce the solution.

The table which follows shows that if 0 pounds of "cookie #1" are used, the entire 60 pounds will be made up of "cookie #2" and will cost a total of $30.00, not $33.00!

SPREADSHEET WITH ONE SET OF VALUES

		cookie 1	cookie 2
# of pounds	=	0	60
price per pound	=	.75	.5

TOTAL COST

|||||||||||

30

In the following table, after several trials using the spreadsheet program, we see that 12 pounds of "cookie #1" and thus 48 pounds of "cookie #2" will produce a total cost of $33.00 - our desired result!!

SPREADSHEET WITH CORRECT SET OF VALUES

		cookie 1	cookie 2
# of pounds	=	12	48
price per pound	=	.75	.5

TOTAL COST

|||||||||||

33

It is interesting as well as instructive, to enter values outside the domain of the problem to permit students to observe the results. For example, try selecting 70 pounds of "cookie #1" to see what happens. Use the results to begin a discussion of the values that can be chosen for the number of pounds of "cookie #1" that will make sense in the problem.

SHOW TICKETS

Tickets to the school show are priced at $3.00 for students and $5.00 for adults. The auditorium can seat 150 people. List all possible combinations of total revenue intake from one performance if every seat is sold.

While this problem is very similar to that found in most algebra texts, the call here is for ALL combinations of seats in order to permit the answering of several interesting questions usually missed in traditional texts.

The typical textbook question (after the second sentence in the statement of the problem above) might be "How many adult tickets and how many student tickets were sold to produce a total revenue intake of $622.00?" The solution can easily be read from the table.

Students should be taken through the development of the "set-up" of the problem and, subsequently, the spreadsheet.

Let x be the number of student tickets sold.

Let 150 - x be the number of adult tickets sold.

Total revenue intake = $3x + 5(150-x)$.

BEGINNING OF SPREADSHEET WITH FORMULAS

# of students	# of adults	Total Amount
0	150-A15	(3*A15)+(5*C15)
+A15+1	150-A16	(3*A16)+(5*C16)
+A16+1	150-A17	(3*A17)+(5*C17)
+A17+1	150-A18	(3*A18)+(5*C18)
+A18+1	150-A19	(3*A19)+(5*C19)
+A19+1	150-A20	(3*A20)+(5*C20)
+A20+1	150-A21	(3*A21)+(5*C21)
+A21+1	150-A22	(3*A22)+(5*C22)

EVALUATED SPREAD SHEET FOR ALL ENTRIES

# of students	# of adults	Total Amount
0	150	750
1	149	748
2	148	746
3	147	744
4	146	742
5	145	740
6	144	738
7	143	736
8	142	734
9	141	732
10	140	730
11	139	728
12	138	726

13	137	724
14	136	722
15	135	720
16	134	718
17	133	716
18	132	714
19	131	712
20	130	710
21	129	708
22	128	706
23	127	704
24	126	702
25	125	700
26	124	698
27	123	696
28	122	694
29	121	692
30	120	690
31	119	688
32	118	686
33	117	684
34	116	682
35	115	680
36	114	678
37	113	676
38	112	674
39	111	672
40	110	670
41	109	668
42	108	666
43	107	664
44	106	662
45	105	660
46	104	658
47	103	656
48	102	654
49	101	652
50	100	650
51	99	648
52	98	646
53	97	644
54	96	642
55	95	640
56	94	638
57	93	636
58	92	634
59	91	632
60	90	630
61	89	628
62	88	626

63	87	624
64	86	622
65	85	620
66	84	618
67	83	616
68	82	614
69	81	612
70	80	610
71	79	608
72	78	606
73	77	604
74	76	602
75	75	600
76	74	598
77	73	596
78	72	594
79	71	592
80	70	590
81	69	588
82	68	586
83	67	584
84	66	582
85	65	580
86	64	578
87	63	576
88	62	574
89	61	572
90	60	570
91	59	568
92	58	566
93	57	564
94	56	562
95	55	560
96	54	558
97	53	556
98	52	554
99	51	552
100	50	550
101	49	548
102	48	546
103	47	544
104	46	542
105	45	540
106	44	538
107	43	536
108	42	534
109	41	532
110	40	530
111	39	528
112	38	526
113	37	524

114	36	522
115	35	520
116	34	518
117	33	516
118	32	514
119	31	512
120	30	510
121	29	508
122	28	506
123	27	504
124	26	502
125	25	500
126	24	498
127	23	496
128	22	494
129	21	492
130	20	490
131	19	488
132	18	486
133	17	484
134	16	482
135	15	480
136	14	478
137	13	476
138	12	474
139	11	472
140	10	470
141	9	468
142	8	466
143	7	464
144	6	462
145	5	460
146	4	458
147	3	456
148	2	454
149	1	452
150	0	450

Now for a few non-traditional, yet interesting questions.

1. What audience population will produce the greatest intake of revenue? the least? Could you have predicted this without looking at the spreadsheet? Explain your response.

2. If a total of $586 was collected, how many adults and how many students were in the audience?

3. If 12% of the audience was adults, what was the total revenue collected?

4. How many adults and how many students attended if the total revenue collected was $563.00?

VOLUME OF A RECTANGULAR SOLID

If congruent squares are cut from each of the four corners of a square piece of cardboard 12 inches on each side, the four remaining flaps can be folded up to obtain a tray with no top. What size squares should be cut in order to maximize the volume of the tray?

This is a problem traditionally reserved for the beginning student in calculus. The difficulty with many problems of this kind is that students have never set up or worked with such problems until they encounter them in the calculus course. Now, using the spreadsheet, they need not wait until the calculus course to tackle problems of this nature.

Again, it is expected that the students be involved in the set-up of the problems and the spreadsheet. Proceed as follows :

Let x be a side of one of the four congruent squares cut from the corners of the large cardboard square.

The volume of the resulting cardboard tray is :

$$V(x) = x\,(12 - 2x)(12 - 2x)$$

SPREADSHEET WITH FORMULAS

Enter the value of x = 1 inc.=.1

The corresponding volume =+D14*(12-(2*D14))*(12-(2*D14))

x	VOLUME
+D14+F14	+A19*(12-(2*A19))*(12-(2*A19))
+A19+F14	+A20*(12-(2*A20))*(12-(2*A20))
+A20+F14	+A21*(12-(2*A21))*(12-(2*A21))
+A21+F14	+A22*(12-(2*A22))*(12-(2*A22))
+A22+F14	+A23*(12-(2*A23))*(12-(2*A23))
+A23+F14	+A24*(12-(2*A24))*(12-(2*A24))
+A24+F14	+A25*(12-(2*A25))*(12-(2*A25))
+A25+F14	+A26*(12-(2*A26))*(12-(2*A26))
+A26+F14	+A27*(12-(2*A27))*(12-(2*A27))
+A27+F14	+A28*(12-(2*A28))*(12-(2*A28))
+A28+F14	+A29*(12-(2*A29))*(12-(2*A29))
+A29+F14	+A30*(12-(2*A30))*(12-(2*A30))
+A30+F14	+A31*(12-(2*A31))*(12-(2*A31))
+A31+F14	+A32*(12-(2*A32))*(12-(2*A32))
+A32+F14	+A33*(12-(2*A33))*(12-(2*A33))
+A33+F14	+A34*(12-(2*A34))*(12-(2*A34))
+A34+F14	+A35*(12-(2*A35))*(12-(2*A35))
+A35+F14	+A36*(12-(2*A36))*(12-(2*A36))
+A36+F14	+A37*(12-(2*A37))*(12-(2*A37))
+A37+F14	+A38*(12-(2*A38))*(12-(2*A38))

EVALUATED SPREADSHEET

```
Enter the value of x    = 1      inc.=     .1

The corresponding volume =              100

        x                               VOLUME

        1.1                             105.644
        1.2                             110.592
        1.3                             114.868
        1.4                             118.496
        1.5                             121.5
        1.6                             123.904
        1.7                             125.732
        1.8                             127.008
        1.9                             127.756
      128
        2.1                             127.764
        2.2                             127.072
        2.3                             125.948
        2.4                             124.416
        2.5                             122.5
        2.6                             120.224
        2.7                             117.612
        2.8                             114.688
        2.9                             111.476
      108
```

It is obvious from examination of the results above that the maximum volume of the tray occurs when the squares cut from the corners of the original square have two inch sides and the maximum volume is 128 cubic inches.

Again, choosing values outside the meaningful domain of this problem will produce "interesting" results that should stimulate discussion. For example - let x vary from 5 to 8 with an increment of .2.

TRAIN PROBLEM

A railroad company agrees to run a train from New York to Washington for a group of people if at least 200 people will go. The fare is to be $8.00 per person if 200 go, and will decrease by one cent for everybody for each person over 200 that goes - thus if 250 people go, the fare per person for each of the 250 people will be $7.50. Find the number of passengers that will produce the greatest gross income for the railroad.

Another problem traditionally left for the calculus. Now we can introduce problems of this kind to any students with a background in elementary algebra.

Don't forget to have the students play an integral part in the set-up of the problem as well as the spreadsheet.

Let x be the number of people over 200 that take the trip.

Total Income = # of people X price per person

# of people = x + 200

price per person = 8.00 - .01 X x

$F(x) = (x + 200)(8.00 - .01x)$

BEGINNING OF SPREAD SHEET WITH FORMULAS

Enter value of x (over 200)	=1
Total income for the railroad	=(E16+200)*((8-(.01*E16)))
+E16+1	(A19+200)*((8-(.01*A19)))
+A19+1	(A20+200)*((8-(.01*A20)))
+A20+1	(A21+200)*((8-(.01*A21)))
+A21+1	(A22+200)*((8-(.01*A22)))
+A22+1	(A23+200)*((8-(.01*A23)))
+A23+1	(A24+200)*((8-(.01*A24)))
+A24+1	(A25+200)*((8-(.01*A25)))
+A25+1	(A26+200)*((8-(.01*A26)))
+A26+1	(A27+200)*((8-(.01*A27)))

EVALUATED SPREADSHEET

Enter value of x (over 200)	= 1
Total income for the railroad	= 1605.99
2	1611.96
3	1617.91
4	1623.84
5	1629.75
6	1635.64
7	1641.51
8	1647.36
9	1653.19
10	1659
11	1664.79
12	1670.56

==

293	2499.51
294	2499.64
295	2499.75
296	2499.84
297	2499.91
298	2499.96
299	2499.99
300	2500

**** Here it is - the maximum income is $2500 when a total of 500 passengers ride the train.

301	2499.99
302	2499.96
303	2499.91
304	2499.84
305	2499.75
306	2499.64
307	2499.51
308	2499.36
309	2499.19
310	2499
311	2498.79
312	2498.56
313	2498.31
314	2498.04
315	2497.75
316	2497.44
317	2497.11
318	2496.76
319	2496.39
320	2496
321	2495.59
322	2495.16
323	2494.71
324	2494.24

It would be interesting in this spreadsheet to find out what would happen if the number of passengers was 800 or even 1000. A discussion of the results observed should prove logical and fascinating (--- and possibly expected.)

COOKIE MIXTURE PROBLEM

A storekeeper has two kinds of cookies - one worth seventy-five cents per pound and the other worth fifty cents per pound. How many pounds of each should he use to make a mixture of sixty pounds worth fifty-five cents per pound?

SPREADSHEET WITH FORMULAS

	cookie 1	cookie 2
# of pounds	=0	60-C9
price per pound	=.75	.5

TOTAL COST

|||||||||||

(C9*C11)+(E9*E11)

The table which follows shows that if 0 pounds of "cookie #1" are used, the entire 60 pounds will be made up of "cookie #2" and will cost a total of $30.00, not $33.00!

SPREADSHEET WITH ONE SET OF VALUES

		cookie 1	cookie 2
# of pounds	=	0	60
price per pound	=	.75	.5

TOTAL COST

|||||||||||

30

In the following table, a second trial for the problem is attempted using the spreadsheet program, we see that 5 pounds of "cookie #1" and thus 55 pounds of "cookie #2" will produce a total cost of $31.25.

SPREADSHEET WITH SECOND TRIAL SET OF VALUES

		cookie 1	cookie 2
# of pounds	=	5	55
price per pound	=	.75	.5

TOTAL COST

| | | | | | | | | | |

31.25

In this table, after several trials using the spreadsheet program, we see that 12 pounds of "cookie #1" and thus 48 pounds of "cookie #2" will produce a total cost of $33.00 - our desired result!!

SPREADSHEET WITH CORRECT SET OF VALUES

		cookie 1	cookie 2
# of pounds	=	12	48
price per pound	=	.75	.5

TOTAL COST

||||||||||||

33

Session 15E Beginning Basic

PRINT AND END

```
NEW
10 PRINT "THIS COMMAND IS ";
20 PRINT "USED FOR OUTPUT"
30 END
RUN
```

```
NEW
100 PRINT "YOUR NAME"
200 PRINT "TODAY'S DATE"
300 PRINT "CITY YOU WERE BORN IN"
400 END
RUN
```

```
NEW
50 PRINT "COMPUTERS ARE ";
60 PRINT "NOT SMART"
70 PRINT "COUNTDOWN 5,4,3:2;ONE"
80 END
RUN
```

```
NEW
10 PRINT 2*3, 2+3
20 END
RUN
```

```
NEW
100 PRINT 5*4, 5+4, 5-3, 5+3, 5^3
120 END
RUN
```

```
NEW
50 PRINT "THE ANSWER OF 5+3="; 5+3
60 PRINT "FIVE TIMES THREE EQUAL"; 5*3
70 PRINT "12 DIVIDED BY 3 IS", 12/3
80 PRINT "FIVE TO THE THIRD POWER="; 5^3
90 END
```

COMPUTER PRINTING A LETTER

```
NEW
100 PRINT " E E E E E E E "
110 PRINT " E E E "
120 PRINT " E E E "
130 PRINT " E E E "
140 PRINT " E E E E E "
150 PRINT " E E E E E "
160 PRINT " E E E "
170 PRINT " E E E "
180 PRINT " E E E "
190 PRINT " E E E E E E E "
200 END
RUN
```

INPUT WITH STRINGS

```
NEW
10 PRINT "WHAT IS YOUR NAME"
20 INPUT N$
30 PRINT "YOUR NAME IS ";N$
40 END
RUN
```

```
NEW
100 PRINT "YOUR AGE"
110 INPUT A
120 PRINT "YOUR NAME";
130 INPUT N$
140 PRINT N$; " YOU ARE "; A; " YEARS OLD."
150 END
RUN
```

REMARK

```
NEW
100 REM PROGRAM TO DESCRIBE REM STATEMENT
110 PRINT "THE REMARK IS FOR THE  ";
120 PRINT "PROGRAMMER'S NOTES!"
130 PRINT "REM AND REMARK ARE THE SAME"
140 REMARK PRINT BY ITSELF WILL SKIP A LINE
150 PRINT
160 PRINT
170 PRINT
180 PRINT "THIS PROGRAM JUST SKIPPED";
190 PRINT "THREE LINES"
200 END
RUN
```

LET

```
NEW
10 LET R = 5
20 PRINT R
30 END
RUN
```

```
NEW
100 LET B = 10
110 LET C3 = 20
130 PRINT B, C3, B+C3, B-C3, C3-B, C3/B
140 END
RUN
```

LET STRINGS

```
NEW
130 LET K$ = "E.T. "
140 LET L$ = " PHONE HOME "
150 PRINT K$; L$
160 END
RUN
```

MEMORY

```
NEW
1000 LET N = 1
1010 LET P = 2
1020 LET X = 3
1030 LET X = 4
1040 LET P = 8
1050 PRINT "P =";
1060 PRINT P; " N="; N; " X="; X;
1070 PRINT " N X P = "; N*P
1080 END
RUN
```

COUNTER

```
NEW
10 LET B = 0
20 LET B = B+1
30 PRINT B
40 GO TO 20
50 END
RUN
```

```
NEW
100 LET L = 0
110 LET L = L-1
120 PRINT L
130 GO TO 110
140 END
RUN
```

ACCUMULATOR

```
NEW
30 LET K = 0
40 PRINT "INPUT A NUMBER"
50 INPUT M
60 LET K = K+M
70 PRINT K
80 GO TO 40
90 END
RUN
```

IF-THEN

```
NEW
110 PRINT "ENTER ANY TWO NUMBERS"
120 INPUT A,B
125 PRINT
130 IF A>B THEN PRINT A; "IS THE LARGER NUMBER"
140 IF A<B THEN PRINT B; "IS THE LARGER NUMBER"
150 IF A=B THEN PRINT "THE TWO NUMBERS ARE EQUAL"
999 END
RUN
```

OTHER FUNCTIONS

```
NEW
100 LET X = 0
110 LET X = X + 2
120 LET Z = SQR(X)
130 PRINT X,Z
999 END
RUN
```

FOR/NEXT

```
NEW
110 FOR LOOP = 1 TO 10
120 PRINT LOOP
130 NEXT LOOP
999 END
RUN
```

THE FIBONACCI SEQUENCE

```
NEW
10 A = 1
20 B = 1
30 PRINT A
40 PRINT B
50 C = A + B
60 PRINT C
70 A = B
80 B = D
90 IF C 100 THEN 50
200 END
RUN
```

GRAPHING

```
NEW
10 FOR X = -3 TO 3 STEP .125
20 LET Y = X-INT(X)
30 PRINT X, Y
40 NEXT X
50 END
RUN


-3             0
-2.875        0.125
-2.75         0.25
-2.625        0.375
-2.5          0.5
-2.375        0.625
-2.25         0.75
-2.125        0.875
-2             0
-1.875        0.125
-1.75         0.25
-1.625        0.375
-1.5          0.5
-1.375        0.625
```

```
-1.25          0.75
-1.125         0.875
-1             0.
-0.875         0.125
-0.75          0.25
-0.625         0.375
-0.5           0.5
-0.375         0.625
-0.25          0.75
-0.125         0.875
 0              0
 0.125         0.125
 0.25          0.25
 0.375         0.375
 0.5           0.5
 0.625         0.625
 0.75          0.75
 0.875         0.875
 1              0
 1.125         0.125
 1.25          0.25
 1.375         0.375
 1.5           0.5
 1.625         0.625
 1.75          0.75
 1.875         0.875
 2              0
 2.125         0.125
 2.25          0.25
 2.375         0.375
 2.5           0.5
 2.625         0.625
 2.75          0.75
 2.875         0.875
 3              0
```

CLASSIFYING TRIANGLES

```
10 PRINT "GIVE THE LENGTHS, SMALLEST TO LARGEST"
20 INPUT "WHAT ARE THE LENGTHS"; A, B, C
30 IF A + B < C THEN PRINT "NOT A TRIANGLE": GOTO80
40 IF A = B AND B = C THEN PRINT "EQUILATERAL": GOTO80
50 IF A^2 + B^2 = C^2 THEN PRINT "RIGHT":
60 IF A = B OR B = C THEN PRINT "ISOSCELES": GOTO 80
70 PRINT "SCALENE"
80 END
```

Session 18

The Many Faces of Classroom Management

When an educator hears classroom management and computers mentioned at the same time, several different scenes may come to mind. One view might be of a teacher typing test grades into a student's data base record and later printing out a list which details the entire class performance. Another image might reveal a student working at a computer using software which keeps track of his performance and channels his progress according to those records. Yet another scene might include a teacher accessing the math resource data base to find out what extra audio visual, printed, or other supplementary materials are available for a particular topic. This same teacher may also search for interesting problems associated with the topic which have been filed by the department in the past. Looking in on one last teacher, a different scene might come into view. Picture this: a room with one teacher, one computer, and thirty students who want to use that one computer. In each classroom up and down the hall, the same scene is repeated: One teacher, one computer, thirty students each waiting impatiently for his or her turn. What can these people do to best manage their mathematics classroom computer?

Managing with One Computer

Until this time, many schools have made do with one computer and thirty students. Sometimes that computer was placed in a classroom on a rotating basis. For many of the teachers concerned, the computer proved to be an intrusion, one they felt they would rather do without. Under these conditions, the computer could not act as a tool for learning, it was a novelty item. There was a real difficulty in trying to infuse its use into the curriculum, almost a disincentive in doing so. The time was frustrating and disappointing for teacher and students alike. The given curriculum was already so full. Teachers had a hard time justifying spending the many necessary hours to use a computer when its availability was so limited and transient.

Later, when one teacher showed particular interest or talent in using a computer, that roving computer might get parked in his or her room (sometimes to the great relief of others). Maybe through some funding process, from bake sales to district support, lone computers might be found stationed in several different classrooms. Now that a computer was available to more individuals, important changes could start to take place: Software could be used; a curriculum could be designed; time could be scheduled. But did this work?

In a number of different studies, teachers have been asked just that question. Did it work? Their answers have revealed a number of anticipated and unanticipated problems. Problems of access, equity, and scheduling seem to lead the lists. Most teachers agree that one computer is better than none, but most hope that a better solution can be developed.

Those teachers who use one computer successfully with a class full of children tend to offer similar advice in several areas. -First of all, these teachers design a computer use schedule with their students and post it. They teach the students to use the schedule, and they withdraw computer time from students who do not comply. Some teachers are successful in allowing computer time for those not involved in small group instruction; others limit computer time to recesses and noon hour. Some teachers form student computer pairs, while in other classes, students work alone. Many teachers find that if the student using the computer shows the next student on the schedule what to do, all parties benefit.

-Often the computer is isolated from the rest of the room by a computer center screen or other cloaking device. (See figure 1). If these computers were kept on moveable carts, teachers could pull them in from other classes to form a small lab when the opportunity arises. (See figure 2).

-Teachers should teach all students how to turn the computer on and off and how to load software. Teachers should also spend an equal amount of time showing care for computer equipment and proper computer etiquette.

-Some educators have found that assigning computer time by who finishes assignments first or who behaves the best leads to very inequitable usage. They recommend that time be allowed as per schedule.

-Many computer using teachers find that examining the software well in advance to learn its capabilities is the wisest method. Appropriate software is then chosen for student use in a particular week. Some software is used by all students; some software is more individually chosen. If a computer coordinator is available, this job of preview and selection is much quicker and easier. However, most teachers find that they still must do the selection and screening themselves.

-An appropriate software preview, evaluation, and cataloging system is necessary. Obviously, not everyone wants to be involved directly in the process of selecting and ordering of software. However, everyone should have input as to his or her needs. Standards in housing and checking out software are two other processes that need to be set (understood) to facilitate communication between computer users. Having a check-out procedure and an accessible storehouse for software can head off potential problems.

-Today there are many types of software that lend themselves very well to use in front of a classroom. Sometimes teacher led discussions are necessary for full understanding of concepts being taught. Often inquiry types of software can produce classroom discussions not possible before due to the quick calculations and comparisons made by a computer. Graphics packages turn a computer into an electronic chalkboard. Most of these software packages have wide application across the mathematics curriculum, but these packages also require teacher written materials to be used with them. Spreadsheets allow a class to explore "what if...?" situations that may involve calculations too numerous, time consuming, or unpleasant to perform with pencil and paper. Often they allow younger students an experience with higher-level problems that would otherwise be too difficult for them. When using a package such as "Interpreting Graphs," by Conduit, a good part of the lesson involves not only arriving at the correct answer, but also examining the other answers to determine why they are incorrect. This is not part of the questioning strategy provided in the software package and so it is up to the teacher to provide that experience for the students. This type of software use requires that the teacher be knowledgeable about the topic and the software being used. All of this takes time, but the rewards of seeing "aha!" in the eyes of many students will be well worth the time invested.

-At the junior high or secondary level, using a single computer may be appropriate to explore mathematical concepts through programming in languages like Logo, BASIC, or Pascal. Often times requiring a student to develop short programs or to modify existing programs can go a long way toward cementing the mathematical concept being explored.

Managing a Computer Lab

Another alternative to managing with one computer, of course, is to pull in the single computers and form a computer lab. More teachers seem to be willing to use this type of an arrangement since scheduling individual use can be such a problem. Entire groups have the capablility to all learn the same concepts at the same time. Students get more actual computer use time. Computers are used more hours of the day.

-Since someone in a school must be responsible for organizing computer labs, a school computer coordinator to perform this task is the perfect solution. Not all schools can or will provide that position, though. A second choice to help in planning lab time would be a paraprofessional computer aide who has proper training. If neither of those options is possible, the school's computer committee should appoint one member to be responsible. If someone isn't responsible for organizing the computer facilities, decisions are not made and regular upkeep is not done.

-When many people are sharing the same facility, the room must be accessible to all. For this reason, a computer lab situated in a room used for other classes is not a good idea. Sometimes hallways, large closets, and temporary buildings have been used as temporary housing for a computer lab. These make-shift facilities are usually not good choices, however, because of noise, poor security, and inadequate electrical systems. Remember, using old, discarded furniture may save money, but that fifty dollar table might hold thousands of dollars of equipment. Is it worth the risk? If a school system is serious about providing a computer lab for student use, then they should seriously consider the setting and furnishing of it. Again, there must be some software checkout system available in a computer lab. If software is housed in the library and checked out through library procedures, the lab should be close to the library.

-Scheduling computer lab time is a necessity. It is an awful waste of money to have fifteen to thirty computers sitting, unused, for three or more hours each day. If the opportunity presents itself, individual student use could be scheduled for those times that no regular classes are scheduled. Many students could use the opportunity for programming, word processing, or using software related to course work. Some computers could be left on moveable carts so they could be checked out for use in a classroom when possible.

-Most software used in a computer lab will probably be copyrighted material. Make sure that all teachers know the laws dealing with computer software copyrights. It is illegal to load a single copy of a program into RAM on more than one computer unless the district has a site license agreement with the manufacturer/author. The availability of reduced price laboratory packaging, site licensing, and licensed copy permission is much more prevalent today. Teaching students honesty by example is the best way. Each school district should have a written policy on software and computer use.

-Using a computer is always a time consuming task. When teaching a class in a computer lab, the teacher must remember the amount of time it takes to move to and from the classroom to the lab. There will also be time spent loading software and explaining its use. Planning a lesson which will fit in the remaining time is important to successful use of the computers.

-The physical arrangement of the computer lab can also have significant effects on best use. Each student needs enough room to be able to place necessary materials. If two students are to work at each lab station, then enough chairs should be available and there should be room enough to handle them. This seems obvious, but sometimes the obvious is overlooked. One popular arrangement for computer labs is shown in figure 3. This set up lines all computers stations along the walls in an 'L' or a 'U' shape. Teachers have more room to circulate and help students in this kind of an arrangement. Electrical outlets and countertops are easier to locate along walls. However, students do not face any common point in the room. Drawing their attention to a particular focus in the room, perhaps a larger monitor, may be difficult due to shifting position and attention. Tables set up in the center of this arrangement, figure 3, could be used when only half the class is at the computers or when all need to be grouped for full class discussion.

Another room set up, figure 4, places computers on rows of tables in a more standard classroom order. This allows for easier transition from individual to whole group efforts; however, this arrangement makes it more difficult for the teacher to circulate to help individuals. Furthermore, this arrangement may create problems in safely wiring the room.

-In mathematics, computer labs can provide students with excellent opportunities to discover new relationships for themselves. Some drill and practice is necessary in learning new concepts. It seems that the computer can help students to learn these at a faster rate. There are many ideas that are simply presented in class, or even skipped over, that students could have the experience in developing on a computer. Often this will require that the teacher examine software and write appropriate materials for learning these concepts, more like a science lab, perhaps. Activities of this nature are being presented at this conference. Teachers should share their good ideas along these lines with others in their departments, districts, and localities.

Managing computers in the mathematics classroom is not different from managing computers in any other classroom. In fact, the school can realize a real benefit when teachers from many different areas or grade levels begin better communication with computers as their initial commonality. Using a computer is somewhat like having a new food processor. At first you forget you have it and you grate the carrots by hand. Later you realize that carrots, cheese, and chocolate for fudge all grate nicely with the food processor. Soon, with confidence building each time you enter the kitchen, you start making notations on recipe cards that the processor with a particular blade works great. Next you start to trade recipes; then you are a pro and don't need recipes. You might even star in your own PBS cooking show! The process can be the same with using computers in mathematics. Try using a computer with a simple activity, to build your confidence. Write down what techniques and software work-right in your text book so you don't forget-and also note what doesn't work. Keep going and next you will be using the computer on a more regular basis to explore a variety of mathematical ideas, some that you thought far too complex for a certain level of student. Finally, with skill and confidence, you will be presenting your own workshop on using the computer in a mathematics classroom.

CLASSROOM MANAGEMENT SESSION QUESTIONS

1. I think I'd like to use Logo with my geometry students. Must I spend the two weeks that appear necessary to introduce them to the language? If I do spend the two weeks on introduction, what can I eliminate from my geometry course? Is Logo that valuable a tool for teaching geometry to warrant such an expenditure of time preparing for its use?

2. Our school (elementary, junior high - middle school, high school) has 900 students and one microcomputer. What can be done to ensure equitable use of the computer for each student?

3. If I teach mathematical concepts on the computer, should the computer be used in the testing (student evaluation) process? Explain your response in detail.

4. It seems as if each time I take my students to the computer lab, I spend so much time setting up, dealing with hardware problems and student difficulties in running the software that little time remains to get to the day's mathematical objective. Any ideas or suggestions??

5. How can all students have equitable access to computers if not all teachers will use computers?

6. How can I find time to use the computer in an already crowded curriculum?

7. I have one computer in an elementary school classroom. My supervisor wants to see the computer being used throughout the day. Each student is permitted to work on the computer for thirty minutes on a rotating basis. How can I be certain that students are getting all of the information being covered in class during the time they are working on the computer?

8. Our business department has all of the computers in our high school and claim that their courses require constant computer use, therefore they cannot share them with other departments in the school. What avenue(s) can mathematics teachers suggest in order to get some computer time for their students?

9. I want to use one computer to facilitate instruction of the entire class at one time. Please suggest ideas as to how the computer's output can be made visible to everyone in the class.

10. Our school has a computer coordinator who works out of the media center. Teachers send students to the coordinator for computer literacy instruction, but fail to come for such instruction themselves. Any suggestions?

11. I'm scheduled to use our elementary school's one computer with my class one day each month. I'd like to use it for mathematics instruction. What type of activities would be most beneficial under these circumstances?

12. Our high school has a computer lab in which computer languages are taught during each of the eight periods in the day - a different teacher for each period. While we have little trouble with vandalism, the machines do get dirty. No\ONE\teacher feels that it is her/his responsibility to see that the machines are kept clean since each is in the lab one eighth of the school day. Custodians are instructed not to touch the equipment due to their lack of knowledge regarding the devices. What suggestions can you provide to ensure that the machines and lab are kept tidy?

13. Our school recently acquired a computer lab, but spent all of the money on hardware - leaving no money budgeted for software. Our principal wants us to begin using the computer with our students, but most of our teachers have had little, if any, experience with computers and most feel that it's not their job to develop curriculum around computers. I feel the computer can be a valuable asset to the instructional process. Any ideas on how I can proceed?

14. In order to have each child in grades 4 through 8 have time on the computers, our school district moves one classroom set of computers from room to room for a two week period in the school year. We are to spend our mathematics class time for the two week period on a set of mathematics activities written by a group of teachers in a summer workshop. The computers arrive at a pre-determined time of the year at which time we must stop the current instruction and use the activities provided to instruct our students. Often the activities do not relate at all to the current instruction. Please provide me with suggestions regarding better management of this equipment to serve the needs of the student population designated.

15. Our district has five elementary schools. The superintendent has decided that each school will get three microcomputers. We have no computer coordinator in the district, yet each school is expected to use the computers in the instructional process. Some schools have teachers who are interested in computers and must be doing wonderful things with the equipment - from what I've heard. The answer obviously is that we must share ideas, but the teachers' contract does not permit extra days to be added to the teachers' year for inservice training unless teachers are paid. Our Board of Education claims they have no funds to cover such training! What can we do?

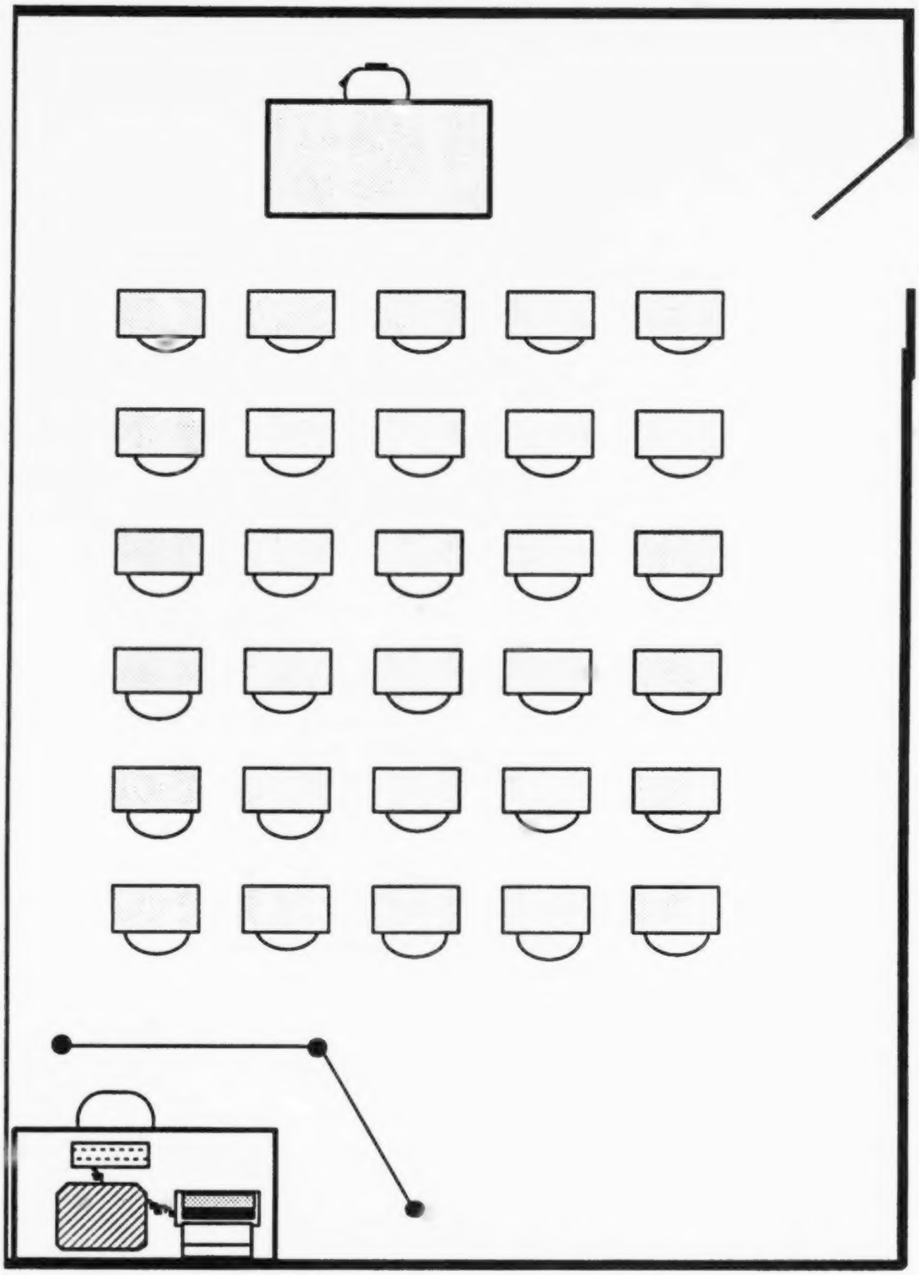


Figure 1.

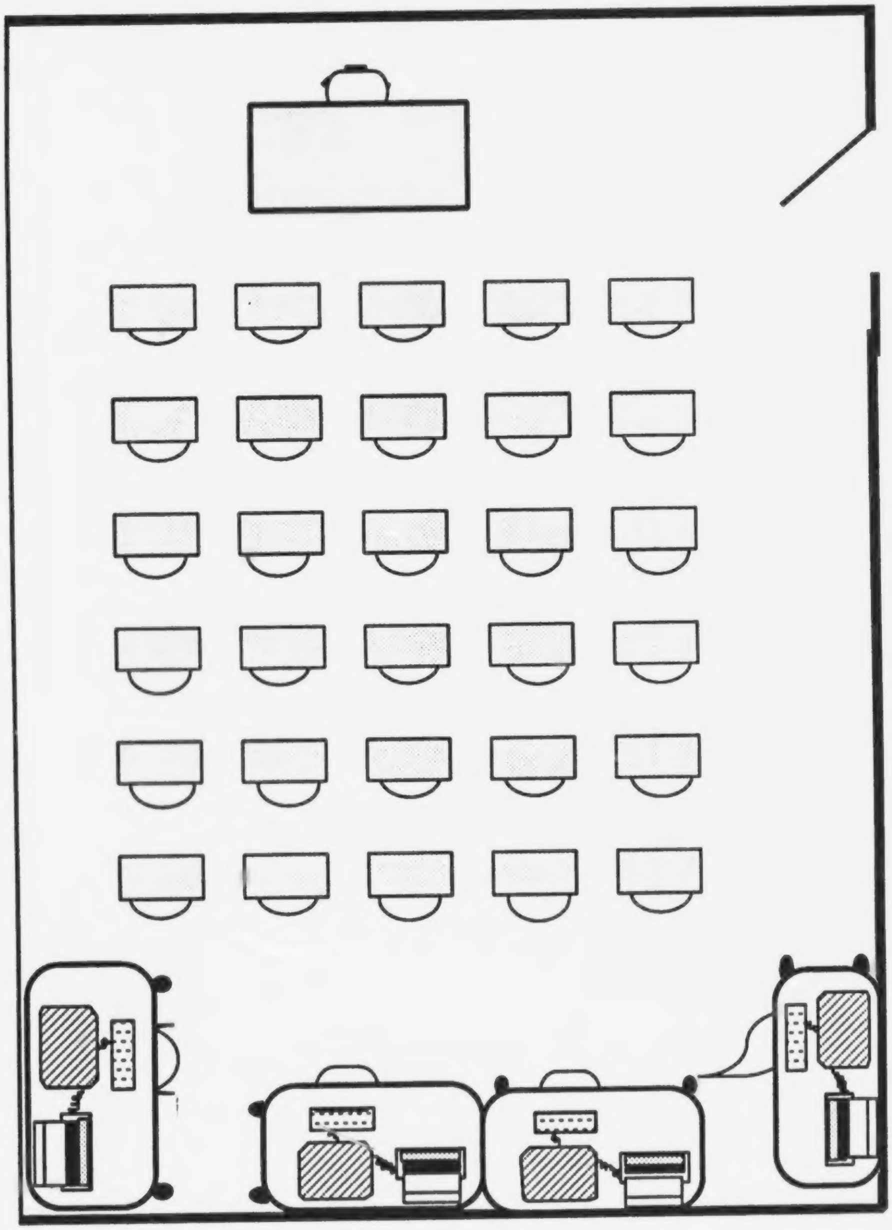


Figure 2.

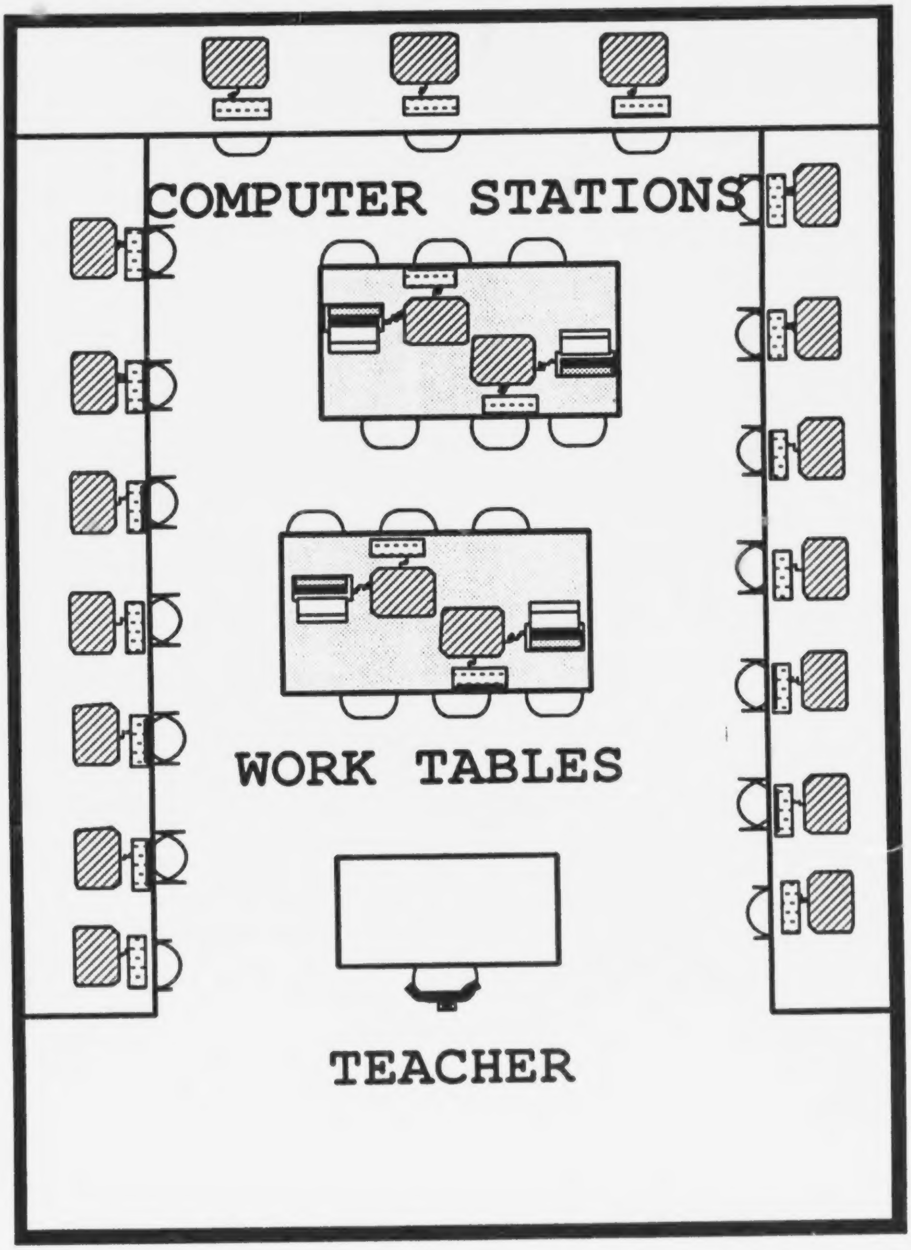

COMPUTER STATIONS
WORK TABLES
TEACHER

Figure 3.

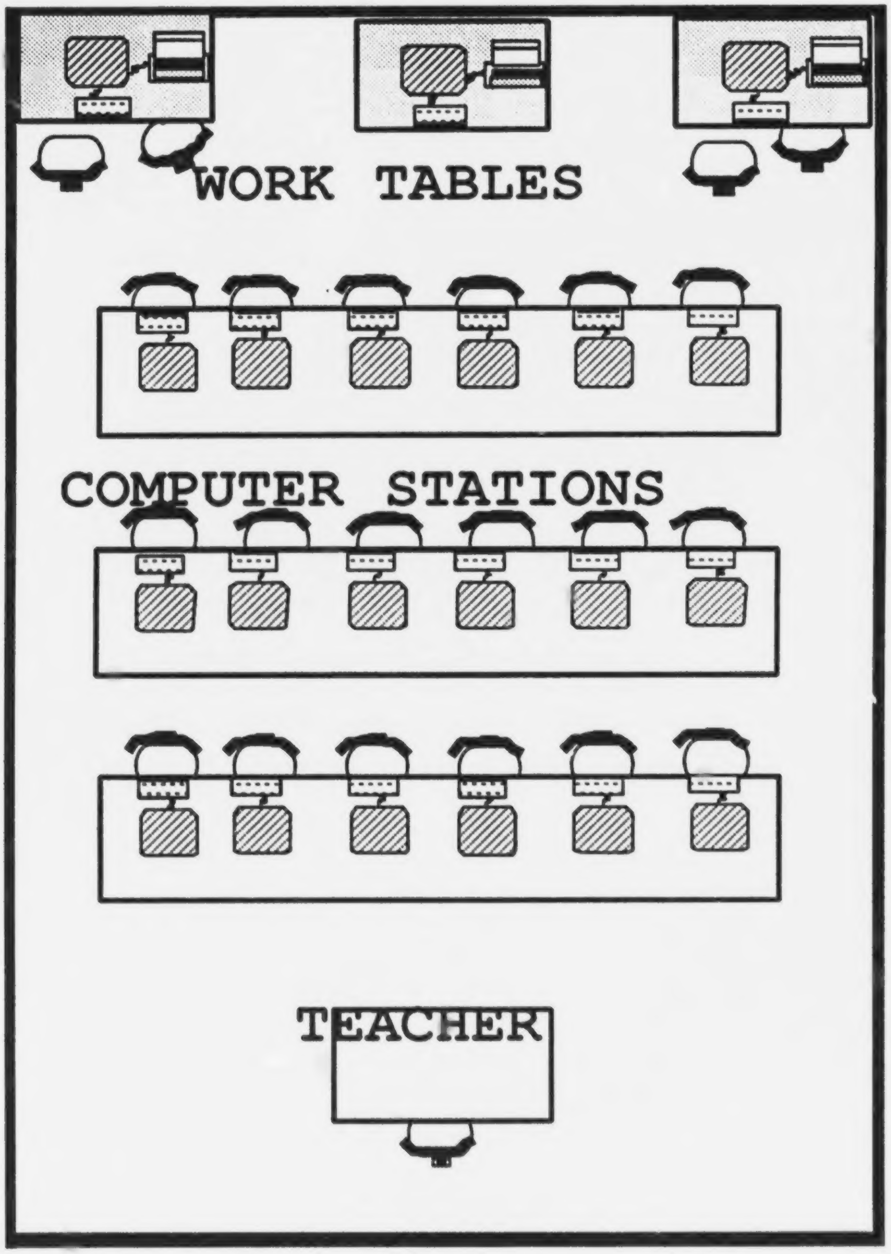

WORK TABLES
COMPUTER STATIONS
TEACHER

Figure 4.

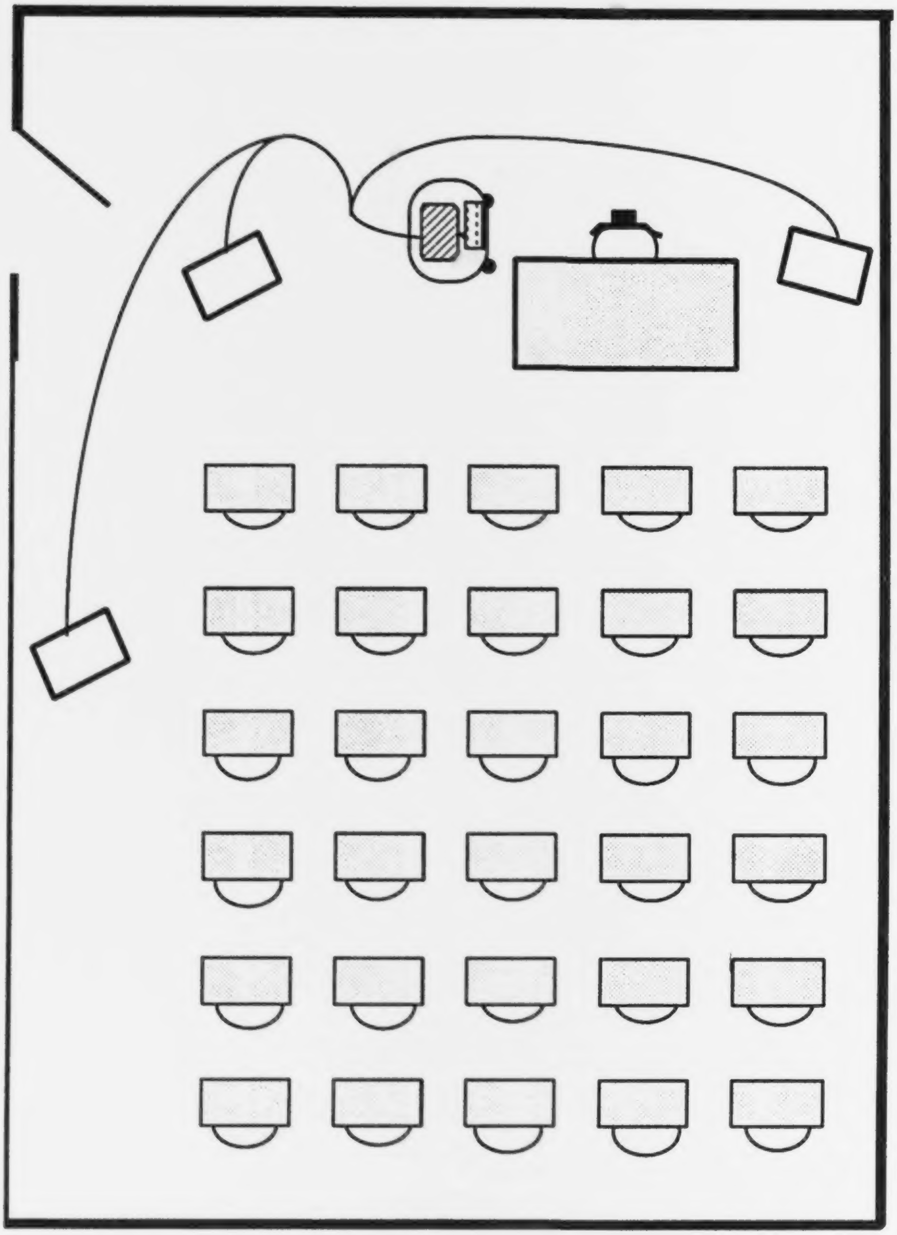


Figure 5.

Session 19 Authoring Systems

Issues to Consider

With all of the commercial software available on the market today one may wonder why some teachers want to develop their own. There are various reasons why it may be particularly appropriate for teachers to expend considerable effort to develop educational computer programs:

1. Certainly there are many titles of software products on the market today, however, many of these pieces are simply reproductions of the same thing. Software publishing companies are in the market to sell products, therefore the titles they carry generally fall into the more common curricular categories. Many curriculum areas have no software available because these areas would just not be big sellers.

2. Sometimes a teacher may have an unusual way of teaching a particular topic. This approach may not be duplicated with any available software even though a significant number of titles exist in this curriculum area. If the teacher wants the software to complement her teaching, she may have to develop the software to do it.

3. The cost of multiple copies of some software can be very high even if the per copy cost is low. If a school needs multiple copies of many different subject areas, the software budget could become depleted quickly. Unlike businesses which function on four or five software packages, schools are voracious consumers of software. A single grade level in mathematics alone could require over 30 pieces of software. Multiply this by the need for many copies, the different grade levels in a school, and the different subject areas besides

mathematics which might want software and it is easy to see why existing software budgets cannot meet the need.

4. Teachers can supplement their earnings by producing software. Some teachers have managed to add to their income by establishing consulting contracts to either develop or program educational software. Furthermore, teachers may create programs for the school system they work in and receive some increase in prestige within their own district.

5. If the teacher has a logical mind, can problem solve effectively, has patience to work with details, and has some creative flair, she may find programming fun. Most teachers who take introductory programming courses may program one or two applications for their own students, but cease to do so after the course has been completed. This is because many teachers who take these courses only begin to find out what programming is like once they are in the course. After they program for a while, they decide it may not really be for them. But for some, it can be fun and an interesting diversion from other teaching tasks.

DEVELOPMENT SYSTEMS

BASIC. If teachers wanted to program their own materials even as late as 1984, they had to learn a programming language like BASIC or possibly Pascal. BASIC is popular because it is resident in most microcomputers, everybody has it, and there are other people knowledgeable about the language if one encounters difficulty. BASIC, however, has deficiencies which make it less satisfactory as a development tool than some other products on the market today.

First, even though BASIC is widely available, BASIC dialects vary from one type of machine to another. Some of the commands are significantly different, or even non-existent, as one migrates software from machine to machine. For example, most commands which deal with display on the screen, such as graphics or text layout, are different. While some of these commands are just one-for-one translations of each other (e.g. HOME on the Apple is like CLS on the MS-DOS machines) many of the other commands have no counterpart in another dialect. An example is the ELSE command in MS-Basic does not exist in Applesoft. BASIC has additional problems, however. Most versions can only utilize a minimal amount of memory even though a computer may have more. BASIC on the IBM can only take advantage of the first 64K of memory despite the fact that your computer may have up to 640K in it. As a generic language, BASIC has no way of easily handling spelling errors which student software users may make. While commands exist in the language to parse test in order to check for spelling or case (alphabetic upper or lower) errors, the programmer must write the subroutines which do this. These routines are not built into the language.

However, BASIC does have some advantages as an authoring language. Since BASIC is so predominate one does not have tc supply a copy of the language when distributing software; there are many people around who can answer programming questions which the teacher may have; it has good computational ability and can handle a student answer within a numeric range; it can handle student generated text in response to questions although the routines need to be supplied by the programmer; most versions have reasonably strong capabilities in the graphics area; and many versions can be compiled to protect source code and increase execution speed considerably.

PASCAL. As a generic authoring system, Pascal has many advocates. Many of the capabilities of the language are well known and will not be repeated here. Pascal does have some advantages over BASIC. It generally executes faster; often is easier to debug because it forces modular thinking; many versions allow the programmer to take advantage of more memory available in the computer system (however, the Pascal Language must occupy memory itself somewhere and this takes up RAM space. Frequently BASIC is either ROM resident or almost entirely ROM resident and therefore uses little or no additional RAM memory); and there are a growing number of Pascal programmers who can provide assistance in coding. On the disadvantage side, some versions of Pascal are difficult to use. One must become familiar with a compiler and an editor, all things which are relatively transparent for the BASIC programmer. There are, however, some new versions of Pascal which reduce the difficulty of learning a new system. Instant Pascal on the Apple, for example, acts like an interpreted language. Since BASIC is still more widely available, unless one needs the speed or memory size, Pascal does not offer a significant advantage over BASIC in the development of CAI materials.

PILOT. Perhaps the language with the greatest opportunity for providing a strong authoring environment for computer assisted instruction is Pilot. Presently, there are versions available for most of the popular microcomputers. Although Pilot has been in existence for many years, it is only relatively recently that it has gained favor as a development language. This is attributable to the enhanced versions that are now available which can control videodisc players and interface to sophisticated graphics tools. Pilot's success may also be due to the availability of relatively cheap memory which allows large CAI programs to co-reside in RAM with the authoring language itself. Among the advantages of Pilot: the ability to call subroutines by name; ease in handling spelling mistakes by the student in any way the programmer wants; the ability to keep records of responses made by the student user; the elimination of the need to check for upper or lower case on student respones; the removal of unnecessary blanks from a students answer before error judging; the ease of use in student-answer matching capabilities; and the existence of fairly powerful graphics capabilities (and the existence of fairly powerful graphics packages to develop graphics for Pilot programs). Because of the larger amounts of disk space and computer memory, the IBM compatible versions of Pilot can do more than the Apple versions. Nevertheless, the effectiveness of Pilot on the IBM can be generalized to other machines as well. Pilot is reasonably easy to learn and has a much smaller command set than BASIC or Pascal. However, one still must know the concepts of programming to develop software in Pilot. Pilot does not eliminate the need to understand step-by-step logic, looping structures, counters, and flags. The programming and debugging processes are much the same regardless of the language one is coding in.

AUTHORING SYSTEMS. Unlike computer languages which require a knowledge of computer programming, authoring systems which eliminate the need to understand programming at all are available. Examples are AVA from Bell and Howell available for both the Apple and IBM/compatible computers and Private Tutor from IBM for the IBM PC and compatibles. Authoring systems generally have the courseware author enter the text, graphics, questions, etc. in a pre-formatted screen template which is then preserved for presentation later. The author enters a screen at a time and has control over how that screen should look. When the student uses the program he "jumps" from screen to screen. In the authoring system the template prompts the author to place text on the screen and state where that text should be placed. It asks the author what question the student is to be asked, queries about what constitutes a correct answer, and then asks what screen the student is to branch to next based on the "correctness" level of his previous response. Most authoring systems allow for student answers which fall in four categories: Correct answers, partially correct answers, anticipated wrong answers, and unanticipated wrong answers. These systems can ignore case (upper vs. lower) discrepancies between the student answer and the correct answer, can handle minor misspellings with little trouble, can provide feedback for those answers which are clearly wrong but were anticipated by the author to receive special feedback, and can provide a general catch all for wrong answers which the author could not anticipate coming. These authoring systems are extremely easy to use from an author standpoint. All the courseware developer needs to know is the flow of the lesson. Lesson mapping must still be done, but because the lesson is screen oriented one needs to know only which screens are to be seen, under what conditions, and in what sequence. All of the "programming" is then handled automatically by the authoring system. Clearly, authoring systems have significant advantages in the courseware development area. They are easy to use, designed for instructional applications, and exist for many of the popular types of microcomputers. They do, however, have a substantial number of limitations which vary from authoring system to authoring system. First there is cost. The developer must buy a copy of the system to author upon. Then, because, proprietary software is used to present the software back for the student, the user must acquire a presentation package in order to use the software. Some authoring system publishers may grant some sort of site license to either the authors or users, but one should not depend on such activity to continue. Second, most authoring systems do not have any mathematical capabilities. Therefore, the application in mathematics is highly limited. Most do not have random generation of exercises nor do they have the ability to compute the correct answer. Also, because they evaluate answers as strings instead of numeric values, answers which might actually be correct would be marked wrong. For example, if the answer is 20 and the student typed in 20. the decimal point would result in an incorrect answer. Accounting for all of the possible correct answers which might be misinterpreted could become a development nightmare. Third, many authoring systems presently do not allow an author to do anything with graphics other than the standard ASCII character set. Therefore, if the author wants a fancy picture s/he may be out of luck. Some of the authoring systems are now allowing the author to call up a picture created with a graphics editor but this requires that the developer becomes familiar with the graphics package and perhaps pay royalties or presentation fees on it also. Fourth, many of the authoring systems place severe constraints on where an author may place text, how questions can be asked, and where on the screen they can be placed, how responses are interpreted as being correct or incorrect, and the number of branches available on a set of incorrect responses. Fifth, because powerful authoring systems take up RAM space and disk space there may not be much room left in memory or on the disk for lengthy courses. Private Tutor, for example imposes a limit of a maximum of 60 screens available in a single lesson. If more are needed to author the lesson, the student must route himself manually to additional lessons from the menu. This is very cumbersome when the computer should be able to do it automatically. Clearly, one must weigh very carefully the capabilities of the authoring system one is considering and make absolutely certain that it will do everything which is required. Unfortunately, once the author is entering a lesson it may be too late to find out the system is inadequate. The author will probably have to start all over again in the lesson entry process or live with the limitations of the system.

AUTHORING SUPPORT TOOLS. Authoring tools which work in a particular language (most commonly BASIC) are now becoming popular in the development process. One can acquire, for example, hires character sets and graphics, already preprogrammed, which can be inserted directly into a program. MECC, Conduit, Beagle Brothers, and Synergistic produce hires characters, graphics images, standard student response evaluation routines and other utilities which can make the development process easier. The major drawback of these is that they limit what programs one can use them in due to copyright restrictions. An author may have to pay significant royalties in order to include them.

SUMMARY

There are many development tools on the market today which can reduce the effort required to code computer assisted instruction materials. One must be very careful to balance in terms of ease of use, cost, and power. It is very important to realize, however, that the process of instructional design which includes lesson flow, content, and student evaluation must still be properly developed. Generic languages, authoring languages, authoring systems, and authoring tools provide no assistance without proper instructional design. They can, however, reduce the more onerous task of getting well-designed instruction into the machine.

AUTHORING SYSTEMS

The following list of authoring systems is provided in Systems for Authoring Computer-Based Instruction, Technical report #LHNCBC85-1, Department of Health and Human Services, Public Health Services, National Institutes of Health.

Note: Only those computer brands are listed for which the system was primarily designed. Some systems may work with compatible computers; others may not. Some producers state their systems work on compatibles, relying on the compatibility claims of these computer manufacturers, while others have tested their systems on compatible machines. Producers should be contacted for further information about computer brand(s) and configuration.

MICROCOMPUTER AUTHORING SYSTEMS

ACCORD	Global Information Systems Technology	AT & T (also IBM for delivery); videodisc interface, color; graphics; text; questions; record keeping.
ADROIT	Applied Data Research	IBM; videocassette and videodisc interface; color; graphics; test; questions; record keeping.
AIS-II	McDonnel Douglas CBT Group	Gould minicomputer and network of delivery stations; videodisc; light pen and touch screen; color; graphics; test; questions; record keeping.
The Author	Raptor Systems	Apple, IBM, Texas Instruments-Pro, and Wang; test; questions; record keeping.
Author I	Radio Shack	TRS-80 Model III; test; questions; record keeping.
The Author Plus	Raptor Systems	IBM; color; graphics; text; questions; record keeping.

Authority	Interactive Training Systems	IBM; videodisc interface; light pen; color; graphics; text; questions; record keeping.
AVA	Bell and Howell	Apple; videocassette and videodisc interace; color; graphics; text; questions; record keeping
AVID	Advanced Interactive Systems	DEC Pro 350; videocassette; videodisc and random access slide projector interface; color; graphics; text; questions; record keeping.
CAIWARE 3D	Fireside Computing, Incorporated	TRS-80 (Models I, III, and IV); graphics, text; questions; record keeping.
CAI Plus	eduCAItor, Incorporated	IBM; graphics; text; questions; record keeping.
CAMELOT	Miami-Dade Community College	IBM, Sage, QUAY and others; text; questions; record keeping.
CAMPS	DACIS Software	Apple; text; questions; record keeping. Preformatted patient management simulations.
CAN-8	Honeywell Information Systems	Honeywell 610 personal computer and other 600 series multi-user systems; videocassette and videodisc interface; color; graphics; text; questions; record keeping.

CAST	Software Mart, Incorporated AT & T; videocassette and videodisc interface; text; questions; record keeping. Some programming required.
CASTE	Pangaro, Incorporated System uses Symbolics minicomputer to create lesson dialogs. A prototype version exists for the Apple microcomputer.
CDEX	Electronic Publishing System AT & T (UNIX authoring and conversation to operating system for different micros); color; text; questions; record keeping.
CDS	EIS, Incorporated IBM and Sony; videocassette, videodisc and audiodisc interface (or other devices); color; text; questions; record keeping. Some programming required.
CLAS	Touch Technologies, Incorporated Acorn, Apple, Commodore and IBM; videodisc interface (for Commodore); color; text; questions; record keeping.
Clinical Simulation Authoring System	Richard Trynda Apple; text; questions; record keeping; patient management simulations.
COURSEWARE AUTHORING SYSTEM	Courseware, Incorporated Apple; color; text; questions; record keeping.
CREATE-A-TEST	Cross Educational Software Apple; multiple-choice, matching, essay, fill-in-blank tests.

CREATOR	Computer Guidance, Incorporated Apple Macintosh. System under development.
DiscWriter	JAM IBM; videodisc interface; color; text; questions; record keeping.
Eazylearn	Miracle Computing IBM; videocassette interface; color; text; questions.
The EDUCATOR	Spectrum Training Corporation Apple, Burroughs, Honeywell, IBM, NCR, Texas Instruments, Wang, and other microcomputers; videocassette and videodisc interface; color; graphics; text; questions; record keeping.
EnBASIC	COMPress Apple; color; text; questions. Some programming required.
Eureka Learning System	Eiconics, Incorporated Apple; graphics; text; questions.
EZ Learner	Silicon Valley Systems Apple; text; questions.
ForceTen Authoring System	ForceTen Enterprises IBM; videocassette and videodisc interface; mouse; color; graphics; text; questions; record keeping.
Ghostwriter	CAVRI Systems, Incorporated Apple; videocassette interface; text; questions; record keeping.

Handy	International Business Machines System used internally for research.
HyperGraphics	HyperGraphics Corporation IBM; color; graphics; digitizing; text; questions.
IDeAS	BCD Associates IBM; videocassette and videodisc interface; color; graphics; text; questions; record keeping.
Insight	Whitney Educational Services Apple, IBM and Sony; videocassette and videodisc interface; color; text; questions; record keeping.
InterAct	Ashton ITC, Incorporated Apple and IBM; videocassette and videodisc interface; color; text; questions; record keeping.
The Instructor	BCD Associates Apple; videocassette interface; color; graphics; text; questions; record keeping.
LDS	M. David Merrill Apple; system under development and based on the component display theory of instructional design.
The Learning System	Micro Lab Apple; text; questions; record keeping.
LENICAL	Duncan Atwell Computerized Technology IBM; voice synthesizer and audio tape interface; light pen or mouse; color; graphics; digitizing; text; questions; record keeping.

Lesson Writer	Random House TRS-80; text; questions.
McGraw-Hill IAS	McCraw Hill, Incorporated IBM; videocassette interface; color; graphics; text; questions; record keeping.
Micro Instructor	Mosby Systems Apple; IBM; text; questions; record keeping. Courseware runs on machines configured for Pascal.
Micro PHOENIX	Goal Systems IBM; text; questions; record keeping. Main frame authoring and micro delivery.
Micro PLATO	Control Data Corporation CDC and IBM; videodisc interface; touch screen; color; graphics; text; questions; record keeping.
Micro Teach	Compumax Associates, Incorporated Atari; color; text; questions; record keeping.
Micro TICCIT	Hazeltine Corporation Data General and IBM; videodisc interface; light pen; color; grahics; digitizing; text; questions; record keeping.
Multiple Choice File	Compu-Tations, Incorporated Apple, Atari and IBM; multiple-choice tests; record keeping.
MVA	Bell and Howell IBM; videocassette, videodisc, and random access sound-slide interface; color; graphics; text; questions; record keeping.

Naturalwriter	Computer-Assisted Instruction, Incorporated IBM; videocassette and videodisc interface; color; graphics; text; questions; record keeping.
PASS	Bell and Howell Apple; videocassette and videodisc interface; color; graphics; text; questions; record keeping.
PCIS	International Business Machines IBM; test; questions; record keeping.
PLANIT	Frye Software Unlimited Works on variety of multi-user computers (including micros with fixed disks); text; questions; record keeping.
Private Tutor	International Business Machines IBM; color; text; questions; record keeping.
The PROF	Abdulla Abdulla Apple; videocassette, videodisc, and random access slide projector interface; text; questions; record keeping. Geared mainly for patient management simulations.
Pro-Producer	Digital Equipment Corporation VAX for authoring, delivery on VAX or DEC Pro 350 or IVIS; videodisc interface and touch screen (for IVIS); color; graphics; text; questions; record keeping. Some programming required.
PWP	Touch Technology, Incorporated Apple and IBM; videodisc interface (for Apple); touchscreen; color; text; questions; record keeping.

QUEST	Allen Communications Apple and IBM; videocassette and videodisc interface; color; graphics; text; questions; record keeping
Quick Quiz	Radio Shack TRS-80 Model III; multiple-choice tests; record keeping.
Regency Course Generator	Regency Systems Regency; videocassette or videodisc interface; touch screen; color; graphics; text; questions; record keeping.
SAL	Cordatum, Incorporated IBM; videocassette and videodisc interface; color; text; questions; record keeping.
SAM	Learncom Technology Systems IBM; videodisc and audiodisc interface; color; graphics; text; questions; record keeping.
Scholor/Teach 3	Boeing Computer Services IBM; videodisc and audiodisc interface; color; graphics; text; questions; record keeping.
Skillcraft Authoring System	Skillcraft Software, Inc. Apple; color; graphics; text; questions.
Storyboard/ Storyboard II	Multimedia Software Apple; videocassette, videodisc, and audiotape interface; text; questions; record keeping.

Study Quiz File	Compu-Tations, Incorporated	Apple, Atarti, Commodore, and IBM; completion and multiple-choice tests; record keeping.
SURPAS	Joseph Scandura	Apple; videocassette or videodisc interface; color; graphics; text; questions; record keeping.
Talk/Tutor graphics	Radio Shack	TRS-80 Color Computer; audio interface; color; graphics; digitizing; text; questions; record keeping.
Teaching Assistant	Minnesota Educational Computing Corporation (MECC)	IBM; multiple-choice, matching, true/false, short answer tests; record keeping.
TenCORE	Computer Teaching Corporation	IBM; color; graphics; text; questions; record keeping. Some programming required.
Tests Made Easy	Compu-Tations, Incorporated	Apple and IBM; essay and completion tests; record keeping.
TestRite	Class I Systems	Apple; IBM; TRS-80 (Model III and IV); multiple-choice, true/false, matching, and essay tests; record keeping.
Testwriter	Random House	TRS-80 Models III and IV; multiple-choice, true-false, matching, and essay tests; record keeping; item analysis.

Test Writer	Persimmon Software Apple; true-false, multiple-choice, matching, completion, and essay tests.
Torricelli Author	The Answer In Computers IBM; color; text; questions; record keeping.
Trainer 3000	Computer Systems Research, Incorporated IBM; color; text; questions; record keeping.
V/CDS	Bell and Howell Apple; videotape interface; text; questions.
Video Nova Authoring System	Video Nova Sony; videodisc and audio tape interface; touch screen; color; graphics; text; questions.
V:Link	Visage IBM; videodisc interface; color; graphics; text; questions; record keeping. Some programming required.
WISE	WICAT, Incorporated WICAT; videodisc interface; color; graphics; text; questions; record keeping.
ZES	Avant-Garde Creations Apple; color; graphics; text; questions; record keeping.

Session 20S
Spreadsheet/Graphics Package: Distance, Rate, & Time

Teacher Notes for Velocity/Acceleration Problem

OBJECTIVE

To present ideas of velocity and accelerated motion in a more concrete manner for students. To show two ormore methods of solution for a single problem.

DESCRIPTION

Students in beginning physics courses struggle to understand the concept of accelerated motion, but as algebra 1 students they were confronted with "word problem" sections that throw equations like $s = vt - 4.9t^2$ at them with little or not explanation. A spreadsheet analysis of an acceleration problem can set the stage for discussion of initial, final, and average velocities and velocity per unit time. A graphics package can allow quick and easy visual analysis of the same problem. Both methods lead to a number of interesting and important questions that can be readily answered. After this approach has been taken, students will have better success in analyzing those :word problem: sections in their algebra books.

SPECIAL NOTES

These lessons can take up to two or more class periods, depending on the interest ,level of the teacher and students. The teacher should answer the questions themself before asking them to the students. If in doubt, the physics teacher can be a wonderful resource; perhaps a perfect time for a guest lecturer or team teaching approach. The spreadsheet template may be made ahead of time or included as part of the lesson.

Velocity/Acceleration Problem for a spreadsheet

A police car is stopped at a red light. As the light turns green, a diesel truck hurtles past in the next lane traveling at a constant speed of 28.0 m/sec. If the police car, sirens blaring and lights flashing, accelerates at 4.0 m/sec^2, how many seconds will it take to catch the truck?

VEHICLE ONE: MOVING WITH CONSTANT VELOCITY

DISTANCE = AVERAGE * TIME TRAVELED VELOCITY

CONSTANT VELOCITY = 28 m/sec

TIME	VELOCITY	DISTANCE	TOTAL DISTANCE
0	28	0	0
1	28	28	28
2	28	28	56
3	28	28	84
4	28	28	112
5	28	28	140
6	28	28	168
7	28	28	196
8	28	28	224
9	28	28	252
10	28	28	280
11	28	28	308
12	28	28	336
13	28	28	364
14	28	28	392
15	28	28	420

VEHICLE TWO: ACCELERATING AT CONSTANT RATE FROM REST

DISTANCE = (1/2) * ACCELERATION * TIME * TIME TRAVELED

ACCELERATION = 4 m/sec^2

TIME	INITIAL VELOCITY	FINAL VELOCITY	AVERAGE VELOCITY	DISTANCE	TOTAL DISTANCE
0	0	0	0	0	0
1	0	4	2	2	2
2	4	8	6	6	8
3	8	12	10	10	18
4	12	16	14	14	32
5	16	20	18	18	50
6	20	24	22	22	72
7	24	28	26	26	98
8	28	32	30	30	128
9	32	36	34	34	162
10	36	40	38	38	200
11	40	44	42	42	242
12	44	48	46	46	288
13	48	52	50	50	338
14	52	56	54	54	392
15	56	60	58	58	450

Following down the total distance column on each table, you will find that both vehicles are at 392 meters from their starting position 14 seconds after the race began.

1. How fast was the policeman going when he caught the trucker? Is reaching that speed in that amount of time possible?

2. Could this problem be converted to English measurements? Would that be difficult?

3. What if the car caught the truck at a fraction of a second--say 13.4 seconds instead of 14. Can you think of some way to modify the spreadsheet to give more accurate time intervals?

4. Is there a veloocity the truck could be traveling at so that the policeman could not catch up to the truck at all?

5. What is the maximum acceleration reasonable for a police car?

6. Is it reasonable and prudent for a policeman to accelerate to such speeds in a city to catch an offender?

Velocity/Acceleration Problem Graphically

Now let's consider the same problem from a different point of view. Let's use a graphics package to plot the graphs of $d = vt$ and $d = 2at^2$. Where the two graphs intersect should give us the time and distance travelled.

In setting up these graphs, x will stand for time elapsed in seconds (Is that the dependent or independent variable?) and y will stand for the distance travelled in meters. If students had no notions about the size of the answers to begin with, they would probably not change the calibration of the axes. You would be able to discuss the shapes of the two graphs--a line and a parabola-- and why you were concerned with only observing the first quadrant. To save time, you may want to suggest calibrating the x axis from 0 to 15 and the y axis from 0 to 400.

1. Could you do this exercise again using velocity vs time graphs instead?
2. What equations would you choose?
3. Would the answers be the same?
4. What relation is there between the distance-time and velocity-time graphs?

Session 20S.1
Algebra, Geometry and Calculus

OBJECTIVE

To display graphically to students the relationship between the original function, its first and second derivative.

DESCRIPTION

A graphics program such as RELATION GRAPHER from Scharf Systems' "Chalkboard Graphics Tool Box I" will be used to graph a function, its first and second derivatives. The students and the instructor will search for relationships between the graphs of these three functions.

PROCEDURE

- Boot the disk for Scharf Systems' "Chalkboard Graphics Tool Box I".
- Select option #2, the program RELATION GRAPHER.
- Set the domain between -8 and 8 and the range between -5 and 9 with increment of 2 in both cases.
- Select option #6, "Any Function" and input the function $Y = (1/3)X^3 - X^2 - 3X + 6$ --- don't forget to input the function in the form F(X)=(1/3)*X^3-X^2-3*X+6).
- Select the plot speed by pressing "P" and choosing option #2, "Fast/Low".

Additional relationships in secondary mathematics are developed through graphics packages and a data base.

- Select option #1, "Add another relation", so that on the same set of axes, you can now graph the derivative $Y = X^2 - 2X - 3$. Don't forget the form in which you must enter the function.
- Look at the two graphs. Observe the zeros of the derivative and what happens on the original function at these points.
- Now do the same for the minimum point on the derivative.
- Make up "rules" that appear to relate these special points on the two functions.
- Once again, select option #1 so you may, on the same set of axes, graph the second derivative of the original function, namely $Y = 2X - 2$. Enter it in proper form for the program.
- Look at the zero of this function.
- Observe what happens at this point on the two previously drawn functions.
- Look at the interval on which the second derivative's value is negative. What significance does this appear to have in relation to the original function and the first derivative?
- Look for other relationships.
- Again attempt to make up "rules" that relate the three functions.

ACTIVITY TWO

OBJECTIVE

To analyze a function for its real zeros, its relative maxima and minima all on the same screen.

DESCRIPTION

This activity can easily be completed by referring to a similar activity in the "First Session - Part Two". The software package entitled "Chalkboard Graphics Tool Box I" by Scharf Systems Inc. will be used. A program in this package called FUNCTION ANALYZER will be employed to analyze the function $Y = X^3 + 3X^2 - 9X - 10$ (entered as F(X) = X^3+3*X^2-9*X-10) to find its real zeros to the nearest thousandth as well as the coordinates of its maximum and minimum points.

PROCEDURE

1. From the main menu, select #1, FUNCTION ANALYZER.

2. Once FUNCTION ANALYZER is loaded, the function must be entered immediately after the "F(X) =" - at the blinking cursor. Enter X^3+3*X^2-9*X-10 at this time. Then press <RETURN>.

3. You will now be requested to enter the maximum and minimum values for the domain and range as well as the increment for each.

   - At this time, it is suggested that you set the domain between -5 and 5 with an increment of 1 and the range between -20 and 20 with 4 as the increment. These are arbitrary and can easily be changed, but for the purpose of this demonstration, they are given so as to make the "first try" as illustrative as possible and save time during this demonstration.

4. You are then questioned as to whether or not you wish to change the plot speed - which, of course, will affect the resolution of the final graph. Pressing "P" permits you the choices "0" through "3". Since we're after a sketch, let's choose option #3 - "very fast/sketch" - and we'll get just that!!

5. Once the sketch is complete, observe that the leftmost real zero appears to be between "-4" and "-5".

   - Now touch "1" to display a table of values.

   - Once the table appears, start the table at -5 and stop at -4 with an increment of .1.

- Once the values in the table are displayed, it's clear that one of the real zeros is between -4.6 and -4.5.

- Now touch "1" to recalculate the table.

- This time, start the table at -4.6 and stop at -4.5 with an increment of .01.

- By scrolling the table down, it is clear that the real zero is between -4.51 and -4.50.

- Again recalculate the table between these values using an increment of .001.

- Now --- the real zero is shown to be between -4.506 and -4.505.

- One last try by recalculating the table between -4.506 and -4.505 with an increment of .0001 and scrolling the table down makes it clear that one of the real zeros is -4.505 to the nearest thousandth.

6. From the graph, it is clear that the other two zeros are also real with one a bit to the right of -1 and the other between 2 and 3. Using the procedure described above, these real zeros can be determined to the nearest thousandth.

7. Now let us attempt to find the relative minimum point.

   - It appears to be somewhere between 0 and 2.

   - Choose option #1 to "Recalculate the Table" and start at 0, stop at 2 with .1 as the increment.

   - Once the table is generated, scroll down and note that it appears as if the relative minimum is at 1.

   - This can be easily verified by methods of the calculus.

ACTIVITY THREE

OBJECTIVE

To demonstrate the analogy between the conics and absolute value relations.

DESCRIPTION

Again, "Chalkboard Graphics Tool Box I" by Scharf Systems Inc. will be used. This time, the program called RELATION GRAPHER will be employed. We will look at the graph of two absolute value relations and call attention to their relationship to two conics - the circle and the ellipse.

PROCEDURE

1. We wish to graph $|X| + |Y| = 6$. Select the RELATION GRAPHER from the main menu. Once the program is loaded into the computer's memory, choose option #5 --- the absolute value relation. Pick:

   A=1, B=0, C=1, D=0 and E=6

   - Choose the -10 and 10 with increment of 2 for the domain and the same bounds for the range.
   - As the speed of plot, choose option #2 - "Fast/Low" - and shortly, the diamond shaped graph is obtained.

2. Touch "1" to "add another relation". Choose option #2 - the conic. This time, pick:

   A=1, B=0, C=1, D=0 and E=36

   - Note that this time the values of the coefficients are the squares of the values chosen for the graph of the absolute value relation previously drawn.
   - Of course we obtain a circle that circumscribes the diamond.

3. Touch "2" to "Select a New Relation". Choose option #5 - the absolute value relation. This time we'll graph $2|X|+|Y|=6$ by entering:

   A=2, B=0, C=1, D=0 and E=6

- Use the same domain, range, increment and plot speed as in the last example.
- When the graph is complete, note that the diamond has had its width halved, but its height remains the same.
- Look at the equation to see if you could have predicted the result by observing the coefficients and constants in the equation.

4. Now touch "1" - "add another relation". Choose option 2 - the conic. Let all of the values of A, B, C, D and E be the squares of those used in the previous absolute value relation, namely:

   A=4, B=0, C=1, D=0 and E=36

- We quickly see that we obtain the ellipse that circumscribes the absolute value relation previously graphed.
- It should be obvious that if students learn how to graph various forms of the absolute value relation, they will have also learned a great deal about sketching the graphs of the corresponding conics.

ACTIVITY FOUR

NOTE

The basic concept for this activity was developed by Joyce Scalzitti, Summit Public Schools, Summit, New Jersey.

OBJECTIVE

To use the data base component of AppleWorks to familiarize students with properties of certain polygons and their interrelationships.

DESCRIPTION

Two data bases will be employed in this activity. One data base contains five particular quadrilaterals and a list of properties that either are or are not attributes of the figure named. The other data base contains polygons from triangles through dodecagons along with the number of sides, number of diagonals, sum of the angles, and the size of each interior and exterior angle if they're regular.

PROCEDURE

QUADRILATERALS DATA BASE

1. The data base component of AppleWorks is chosen and the labels and data are entered as shown below.

FIGURENAME: PARALLELOGRAM
OPP.SIDES.PAR: Y
OPP.SIDES.EQ: Y
ALL.SIDES.EQ: N
OPP.ANGLE.EQ: Y
ALL.ANGLES.EQ: N
DIAG.BISC.OTH: Y
DIAG.EQUAL: N
DIAG.PERPEN: N
DIAG.BISC.ANG: N
DIAG.CONG.2TRI: Y
DIAG.CONG.4TRI: N

FIGURENAME: RECTANGLE
OPP.SIDES.PAR: Y
OPP.SIDES.EQ: Y
ALL.SIDES.EQ: N
OPP.ANGLE.EQ: Y
ALL.ANGLES.EQ: Y
DIAG.BISC.OTH: Y
DIAG.EQUAL: Y
DIAG.PERPEN: N
DIAG.BISC.ANG: N
DIAG.CONG.2TRI: Y
DIAG.CONG.4TRI: N

```
FIGURENAME: RHOMBUS
OPP.SIDES.PAR: Y
OPP.SIDES.EQ: Y
ALL.SIDES.EQ: Y
OPP.ANGLE.EQ: Y
ALL.ANGLES.EQ: N
DIAG.BISC.OTH: Y
DIAG.EQUAL: N
DIAG.PERPEN: Y
DIAG.BISC.ANG: Y
DIAG.CONG.2TRI: Y
DIAG.CONG.4TRI: Y

FIGURENAME: SQUARE
OPP.SIDES.PAR: Y
OPP.SIDES.EQ: Y
ALL.SIDES.EQ: Y
OPP.ANGLE.EQ: Y
ALL.ANGLES.EQ: Y
DIAG.BISC.OTH: Y
DIAG.EQUAL: Y
DIAG.PERPEN: Y
DIAG.BISC.ANG: Y
DIAG.CONG.2TRI: Y
DIAG.CONG.4TRI: Y

FIGURENAME: TRAPEZOID
OPP.SIDES.PAR: N
OPP.SIDES.EQ: N
ALL.SIDES.EQ: N
OPP.ANGLE.EQ: N
ALL.ANGLES.EQ: N
DIAG.BISC.OTH: N
DIAG.EQUAL: N
DIAG.PERPEN: N
DIAG.BISC.ANG: N
DIAG.CONG.2TRI: N
DIAG.CONG.4TRI: N
```

2. The data can now be displayed in table format. In order to fit the table on the page, the labels above had to be changed to numbers. As a guide, you should realize the following:

1 stands for "opposite sides parallel
2 for "opposite sides equal"
3 for "all sides equal"
4 for "opposite angles equal"
5 for "all angles equal"
6 for "diagonals bisect each other"
7 for "diagonals equal"
8 for "diagonals perpendicular"
9 for "diagonals bisect the angles"
10 for "diagonals divide into 2 congruent triangles"
11 for "diagonals divide into 4 congruent triangles"

QUADRILATERALS TABLE

FIGURENAME	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
PARALLELOGRAM	Y	Y	N	Y	N	Y	N	N	N	Y	N
RECTANGLE	Y	Y	N	Y	Y	Y	Y	N	N	Y	N
RHOMBUS	Y	Y	Y	Y	N	Y	N	Y	Y	Y	Y
SQUARE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
TRAPEZOID	N	N	N	N	N	N	N	N	N	N	N

3. By questioning students, be certain that they understand the meaning of the "Y" and "N" designations that have been entered - or, better yet, have the students go through and set up the designated entries for each quadrilateral.

4. At this point several questions can be answered by having the data base program sort the data. Begin with the following procedure to find all of the quadilaterals from the list that have diagonals that bisect each other and are perpendicular.

- press <OPEN APPLE> R to sort data
- when categories show up, use the "down arrow" key to highlight DIAG.BISC.OTH
- press <RETURN>
- use the "down arrow" key to highlight "begins with"
- press <RETURN>
- type Y to indicate that you want the figure to have the aforementioned property

- press <RETURN>
- highlight "and"
- press <RETURN>
- when categories are displayed again, highlight DIAG.PERPEN
- press <RETURN>
- highlight "begins with"
- type Y
- press <RETURN> then <ESC>
- on the screen you should find the records of those quadrilaterals with diagonals which bisect each other and are perpendicular

5. Students should now be asked to use the data base to discover the quadrilaterals that have the following conditions:

   a. the diagonals form 4 congruent triangles
   b. all sides equal and both diagonals equal
   c. diagonals bisect vertex angles
   d. all sides equal and diagonals not equal
   e. all sides not equal and equal diagonals
   f. opposite sides equal and diagonals that don't bisect each other

6. Based on the work with this data base, determine whether the following are true or false:

   a. every square is a rhombus
   b. every rhombus is a square
   c. every rectangle is a square
   d. every square is a parallelogram
   e. every rhombus is a parallelogram

POLYGON DATA BASE

1. **The labels and data for this data base are entered as illustrated below:**

FIGURE:: N-GON
# SIDES:: N
DIAG:: .5N(N-3)
ANG.SUM:: 180(N-2)
INT.ANG.:: 180(N-2)/N
EXT.ANG.:: 180-(180(N-2)/N)

FIGURE:: TRIANGLE
# SIDES:: 3
# DIAG:: 0
ANG.SUM:: 180
INT.ANG.:: 60
EXT.ANG.:: 120

FIGURE:: QUADRILATERAL
# SIDES:: 4
# DIAG:: 2
ANG.SUM:: 360
INT.ANG.:: 90
EXT.ANG.:: 90

FIGURE:: PENTAGON
SIDES:: 5
DIAG:: 5
ANG.SUM:: 540
INT.ANG.:: 108
EXT.ANG.:: 72

FIGURE:: HEXAGON
# SIDES:: 6
# DIAG:: 9
ANG.SUM:: 720
INT.ANG.:: 120
EXT.ANG.:: 60

FIGURE:: HEPTAGON
# SIDES:: 7
# DIAG:: 14
ANG.SUM:: 900
INT.ANG.:: 128.6
EXT.ANG.:: 51.4

```
FIGURE:: OCTAGON
# SIDES:: 8
# DIAG:: 20
ANG.SUM:: 1080
INT.ANG.:: 135
EXT.ANG.:: 45

FIGURE:: NONAGON
# SIDES:: 9
# DIAG:: 27
ANG.SUM:: 1260
INT.ANG.:: 140
EXT.ANG.:: 40

FIGURE:: DECAGON
# SIDES:: 10
# DIAG:: 35
ANG.SUM:: 1440
INT.ANG.:: 144
EXT.ANG.:: 36

FIGURE:: 11-GON
# SIDES:: 11
# DIAG:: 44
ANG.SUM:: 1620
INT.ANG.:: 147.3
EXT.ANG.:: 33.7

FIGURE:: DODECAGON
# SIDES:: 12
# DIAG:: 54
ANG.SUM:: 1800
INT.ANG.:: 150
EXT.ANG.:: 30
```

2. The data, when displayed in table format, appears as below:

```
FIGURE:          # SIDES:  # DIAG:    ANG.SUM:   INT.ANG.:    EXT.ANG.:
------------------------------------------------------------------------------------------
N-GON            N         .5N(N-3    180(N-2)   180(N-2)/N   180-(180(N-2)/N)
TRIANGLE         3         0          180        60           120
QUADRILATERAL    4         2          360        90           90
PENTAGON         5         5          540        108          72
HEXAGON          6         9          720        120          60
HEPTAGON         7         14         900        128.6        51.4
OCTAGON          8         20         1080       135          45
NONAGON          9         27         1260       140          40
DECAGON          10        35         1440       144          36
11-GON           11        44         1620       147.3        33.7
DODECAGON        12        54         1800       150          30
```

3. At this point we wish to sort the data so that we can find the polygons in which the number of sides is greater than 3 and the number of diagonals is less than 10.

   Now, proceed as follows:

   - press <OPEN APPLE> R to sort the data
   - use the "down arrow" to highlight # SIDES
   - press <RETURN>
   - highlight "is greater than"
   - press <RETURN>
   - type 3
   - press <RETURN>
   - now highlight "and"
   - press <RETURN>
   - next highlight # DIAG
   - press <RETURN>
   - highlight "is less than"
   - press <RETURN>
   - type 10
   - press <RETURN>, then <ESC>
   - press <OPEN APPLE> Z to change the format and copy the answers

4. Now, as a further exercise, you're on your own to have the data base select the polygons in our list that meet the following criteria.

   a. the sum of the angles is greater than 550 degrees

   b. the measure of each interior angle of the regular polygon is less than 130 degrees

   c. the number of diagonals is 5 and the sum of the angles is 540 degrees

Session 21S Advanced Topics

LINES AND CURVES
(35 minute class)
SEQUENCES AND LIMITS
(35 minute lab)

OBJECTIVE: To see how the computer can assist in teaching about (1) lines and curves via parametric equations; (2) numerical sequences; and (3) limits of numerical sequences. Emphases will include the use of algebra to teach geometry, and the LISTing and modifying of short programs as a way of teaching and learning about lines, curves, sequences, and limits.

LINES (20 minutes)

Transparency 18. Two points determine a line. So, if (A,B) and (C,D) are two points, how can we best represent the set of points on their line?

(Referring to the transparency...) Students should think of T as Time. At Time = 0, the action starts with (X,Y) located at the point (A,B). As T advances to 1, the point (X,Y) moves along the line to the point (C,D).

Prompt students to raise questions like these:

1. Does T = .5 bisect the segment? Does T = 1/3 trisect it?
2. Will this program work for every choice of two on-screen points (A,B) and (C,D)?
3. If Line 20 were changed to FOR T=0 TO 2 STEP 1/16, would the segment be twice as long?

Using programs to assist in teaching about lines, curves, sequences, and limits.

4. If STEP 1/16 were changed to STEP 1/32, would the segment be denser?

5. If Line 40 were changed to Y = (B + (D-A)*T)^2, would the result be a parabola instead of a line? Can other curves be created this easily?

The answers to these questions are yes, yes, yes, yes, and yes.

Transparency 19. How easy would it be to draw this line on the computer's screen? On an IBM-PC, only one additional line of code is needed, shown here as Line 5. Unfortunately, four or five lines are needed for an Apple, as indicated in Program P23 on your Short Programs disk.

Now, what should we do with lines in the classroom that we have not been doing because we haven't been using parametric equations? How about a general formula and program for intersecting lines?

To derive such a program, as one might derive a formula, suppose one line joins the user-input points (A,B) and (C,D), and the other, points (E,F) and (G,H). These two lines then have parametric equations as seen here.

(Transparency 20. Read it.)

The value of T appears at Line 70 of the following program.

(Transparency 21.)

Run Program P24 three times using these inputs:

	1st time	2nd time	3rd time
A,B	0,0	13,-6	1,7
C,D	17,13	-5,3	3,11
E,F	-18,5	-14,6	-1,5
G,H	0,0	63,-27	4,15

Not counting the INPUT and PRINT statements, this program is only three lines long. The program sparks off some very useful corollaries, included on your Short Programs disk as Program P25, P26, and P27.

(Run Programs P25, P26, and P27. As time permits, note the Problems About Lines and their Solutions and Notes.)

CURVES (15 minutes)

The study of lines and curves using parametric equations is found in many precalculus and calculus textbooks. However, it is a subject that deserves more time than it usually receives.

One strong reason for this new emphasis is that to draw mathematical curves on a screen, parametric representation is the most natural to use. A good example for backing this claim is the right half of a circle - or any other curve for which Y is not a function of X.

Students are fascinated by computer graphics, and math teachers should be able to offer some enrichment topics that students are not likely to find from any other source.

Consider Program P23 on your Short Programs disk. With it students can graph any curve for which X stays between -13 and 13 and Y stays between -9 and 9.

(Run Program P23. It graphs the spiral marked with an asterisk on Transparency 22.)

Transparency 22.

Many students have drawn spirograph curves. One such curve is traced by a point on a wheel rolling around the inside of a circle. The locus of this point is a curve called a hypocycloid.

Let's make Program P23 draw a hypocyclod, using the parametric equations shown here (on transparency).

LIST 20-30. Type in

```
20  DEF FNX(T) = N*R*COS(T) + R*COS(N*T)

30  DEF FNY(T) = N*R*SIN(T) - R*SIN(N*T)

15  N = 10/3

16  R = 2
```

and then type RUN. If you get a SYNTAX ERROR, check Line 20 and Line 30. After the hypocycloid is finished, change N to 3, and run again. Comment that the ease of making such changes is what makes such programs so useful.

Whatever time remains may be spent discussing (with experimentation) changes in the stepsize I at Line 180 and the "overlap number" W at Line 190, as well as choices of X(T) and Y(T).

- END OF CLASS; BEGINNING OF LAB -

Classroom Program Writing (20 minutes)

Certain mathematical topics are so conducive to the activity of program writing that they may be effectively learned via classroom program writing.

One such topic is Arithmetic Progressions. For the next few minutes, let's model the use of program-writing as a tool for teaching about Arithmetic Progressions.

This kind of teaching can be done very successfully without having a computer in the classroom. Just writing programs on the chalkboard as they are developed through classroom discussion can be very effective.

For this present setting, however, let's do take time to type programs and run them.

Transparencies 23 and 24.

(Allow time for participants to type and run their work.)

These six steps show how the topic of Arithmetic Progressions can be developed through classroom program writing. Further development of this topic, either by classroom discussion or in the form of assigned homework, is suggested in the accompanying Program-Writing Problems About APs and GPs.

(Locate those problems and examine them until the 20 minutes are up.)

Limits (15 minutes)

Pass out copies of MATHDISK FOUR for participants to use. Be sure to collect them after 15 minutes.

Transparency 25. Read it.

Transparency 26. Tell participants to choose problems from this list. Encourage them to discuss among themselves ways to use programs like these as teaching tools. Emphasize that this particular disk comes with a workbook containing 200 problems. Copying, modifying, and re-saving individual programs onto student disks is encouraged.

PROBLEMS ABOUT LINES

Problems 1-8 require the use of Short Programs P22-P27.
Problems 9-13 are for programmers.

1. In each case, lines L and L' are given. Use Program P24 to find their point of intersection.

   a. L contains the points (1,4) and (2,7); L' contains the points (3,-5) and (19,23).

      Point of intersection: _______

   b. L is the line $X = 4$; L' is the line parallel to $Y = 3X + 1$ and having X-intercept 2.

      Point of intersection: _______

   c. L is the line through (2,-1) and perpendicular to the line $Y = -X/2$; L' is the line $5X + 3Y = 15$.

      Point of intersection: _______

2. The lines $Y = 0$, $Y = 3X$, and $6X + 5Y = 42$ form a triangle. Use Program P13 to find its vertices.

   Vertices: ____________________________

3. The center of mass (or balance point) of a triangular region is the point of intersection of the medians of the triangle. (A median is a line through a vertex and the midpoint of the opposite side.) Find the center of mass of the triangle of Problem 2.

   Center of mass: ________________

4. Continuing with the triangle of Problem 2, use Program P25 to find the point of perpendicular projection of each vertex onto the side opposite that vertex.

   Three points of projection: ____________________

5. Continuing Problem 4, the segment from each vertex to its perpendicular projection onto the opposite side of the triangle is called an altitude of the triangle. Use Program P24 to show that all three altitudes meet at a point. Is it the center of mass?

   Point of concurrence: ________________

6. Continuing Problem 3, use Program P26 to find the distance from the center of mass to each of the sides of the triangle.

   Three distances: ______________________________

7. Continuing Problem 3, use Program P22 to find the distance from the center of mass to each of the vertices of the triangle.

   Three distances: ______________________________

8. Suppose P and Q are points on the same side of a line L. Below are three steps for finding the shortest path from P to L to Q:

   Step 1. Let R be the point of reflection of point Q about line L.

   Step 2. Let X be the point of intersection of lines PR and L.

   Step 3. The segment PX followed by the segment XQ is the required shortest path.

   How long is the shortest path from the point (2,3) to the line $3X + 6Y = 12$ to the point (6,9)?

   Distance: _______________

In each of the problems numbered 9 to 13, you will see a brief description of a program. Write an original program that fits the description.

9. INPUT: Points (A,B) and (C,D)
   OUTPUT: Several segments parallel to the line joining (A,B) and (C,D)

10. INPUT: Points (A,B) and (C,D)
    OUTPUT: Several segments perpendicular to the line joining (A,B) and (C,D)

11. INPUT: Points (A,B), (C,D), and (E,F)
    OUTPUT: Segments (A,B) to (C,D) and (C,D) to (E,F)), and a segment emanating from (C,D) which bisects the angle there.

12. INPUT: Points (A,B), (C,D), and (E,F)
    OUTPUT: The lengths of the sides of the triangle whose vertices are these three points.

13. INPUT: Points (A,B), (C,D), and (E,F)
    OUTPUT: The word ABOVE if (E,F) lies about the line that passes through the first two points.

SOLUTIONS AND NOTES
for use with PROBLEMS ABOUT LINES

1. Point of intersection: (-9, -26)

   Objective: To strengthen students' understanding of parametric equations in terms of other already familiar line-equations.

   Follow-up: Lead students to discover how to tell the slope of a line from parametric equations: If the line is written as $X = I + JT$, $Y = K + LT$, then two points on it are (I,K) and (I+J, K+L), so the slope is L/J, unless $J = 0$, in which case the line is vertical.

2. Vertices: (0,0), (7,0), and (2,6)

   Objective: To give students further practice in finding two points on a line.

   Follow-up: Have students graph all three lines. Do the locations of the vertices on graph paper agree with outputs from Program P24?

3. Center of mass: (3,2)

   Objective: To provide a practical application of Program P24.

   Follow-up: The X coordinate of the center of mass is the average of the X coordinates of the three vertices of the triangle. Have students confirm this in the present case. Ask them if this method applies to the Y coordinates also.

4. Projections: (2, 0), (.7, 2.1), and (4.13, 3.45)

   Objective: To illustrate the meaning of projection of a point onto a line.

   Follow-up: Show by LISTing that Program P25 is essentially a corollary to Program P24; that the point of projection is computed as the point of intersection of two (perpendicular) lines.

5. Point of concurrence: (2, 5/3)

   Objective: To illustrate how adeptly the computer can contribute to ruler-and-compass geometry.

   Follow-up: Tell students that the point of concurrence of triangle altitudes is called the orthocenter and that it equals the center of mass only if the triangle is equilateral.

6. Three distances: 2, 2.214, and 1.793

   Objective: To have students discover that the center of mass is not equidistant from the sides, thus paving the way toward the follow-up:

   Follow-up: Ask students if they think there is a point that has the same distance from all three sides. Let them experiment, using drawings and Program P26. The answer is YES. The point, called the incenter, is the point of concurrence of the three angle bisectors.

7. Distances: 3.606, 4.472, and 4.123

   Objective: To have students discover that the center of mass is not equidistant from the vertices, thus paving the way toward the follow-up:

   Follow-up: Ask students if they think there is a point that has the same distance from all three vertices. Let them experiment. The answer is YES. The point, call the circumcenter, is the point of concurrence of the perpendicular bisectors of the sides of the triangle.

8. Shortest path: from (2,3) to (2.64,.68) to (6,2) Total distance: 2.41 plus 3.61 equals 6.02

   Method: Program P27 shows (.44,-.2) to be the reflection of (2,3) about the given line. Point X = (2.64,.68) is then found by Program P24 as the point of intersection of the given line with the line joining (.4,-.2) and (6,2).

   Objective: To illustrate to students what is meant by using programs as tools for problem solving. The main thing for students is to decide which tools are needed.

   Follow-up: Mention that segments PX and XQ may be regarded as the path of a light beam reflected from a surface: the angle of incidence equals the angle of reflection.

PROGRAM-WRITING PROBLEMS ABOUT ARITHMETIC PROGRESSIONS AND GEOMETRIC PROGRESSIONS

1. INPUT: A,D,N
   OUTPUT: The Nth term of the arithmetic progression having first term A and common difference D

2. INPUT: A, R, N
   OUTPUT: The Nth term of the geometric progression having first term A and common ratio R

3. INPUT: List of numbers X(1), X(2), ..., X(N)
   OUTPUT: If appropriate, PRINT "Your numbers form an A.P.," and print A and D.

4. INPUT: List of numbers X(1), X(2), ..., X(N) If appropriate, PRINT "Your numbers form a G.P.," and print A and R.

5. Use your programs to illustrate the following facts:

   a. If $B > 1$ and {A(N)} is an arithmetic progression, then {B^A(N)} is a geometric progression.

   b. If {G(N)} is a geometric progression, then {LOG(G(N)} is an arithmetic progression.

   (Feel free to modify your programs when the instructions say "Use your programs ...")

6. INPUT: A,B,N
   OUTPUT: N+1 numbers that are in arithmetic progression, starting with A and ending with B.

7. INPUT: A,D,X,Y
   OUTPUT: A list of those terms T of the arithmetic progression A, A+D, A+2D, ... that satisfy X <= T <= Y. Assume $D > 0$.

8. INPUT: A,R,X,Y
   OUTPUT: A list of those terms T of the geometric progression A, AR, AR^2, AR^3, ... that satisfy X <= T <= Y. Assume $R > 1$.

9. Suppose {A(N)} and {B(N)} are arithmetic progression that have at least one term number in common. Let C(N) the the Nth number that the two progressions have in common. Illustrate the fact that {C(N)} is an arithmetic progression. Can you determine its common difference?

SOLUTIONS AND NOTES for use with PROGRAM-WRITING PROBLEMS ABOUT ARITHMETIC PROGRESSIONS AND GEOMETRIC PROGRESSIONS

1. Solution:

```
10 INPUT "A,D,N = ";A,D,N 20 PRINT A + (N-1)*D
```

Notes: This could be the very first mathematical program that a student would ever write. It could therefore be one of the most important. Program P60 should be used for confirmation. Here are some enlightening inputs:

A,D,N = 1,1,16
A,D,N = 0,2,16
A,D,N = 1,2,16
A,D,N = -25,5,100
A,D,N = 7,0,10
A,D,N = 0,.25,16
A,D,N = 0,-.25,16
A,D,N = 25,-5,100

2. Solution:

```
10 INPUT "A,R,N = ";A,R,N 20 PRINT A*R^(N-1)
```

Notes: Enlightening inputs include these:

A,R,N = 1,2,16
A,R,N = 8,2,16
A,R,N = 1,.5,16
A,R,N = 1,.9,1000

3. Solution:

```
10 HOME:   REM *** A.P. TESTER
20 N = 6
30 FOR I = 1 TO N
40 PRINT "INPUT A("I")=";: INPUT " ";A(I)
50 NEXT I
60 A = A(1)
70 D = A(2) - A
80 FOR I = 3 TO N
90 IF ABS(A(N)-A(N-1)-D) > .000001
THEN 130
100 NEXT I
110 PRINT "YOUR NUMBERS FORM AN ARITH-"
"METIC PROGRESSION WITH FIRST TERM"
120 PRINT A" AND "COMMON DIFFERENCE "D
130 END
```

Notes: This problem is a good example of mathetical problem solving.

Why?

Because to write the program, the student must determine for sure what it means to be an arithmetic progression: the difference between any two consecutive terms must always be the same number.

4. **Solution:**

```
10 HOME: REM *** G.P. TESTER
20 N = 6
30 FOR I = 1 TO N
40 PRINT "INPUT A("I")=";: INPUT " ";A(I)
50 NEXT I
60 A = A(I)
70 R = A(2)/A(1)
80 FOR I = 3 TO N
90 IF ABS(R - A(N)/A(N-1)) > .00001
THEN 130
100 PRINT "YOUR NUMBERS FORM A GEOMETRIC
110 PRINT "PROGRESSION WITH FIRST TERM"
120 PRINT A" AND "COMMON RATIO "R
130 END
```

Notes: The essence of geometric progressions is the notion of common ratio. It is hard to imagine a surer way for a student to gain the right sort of understanding than by writing a program that determines if given numbers are in geometric progression.

5. **Solution:**

a. A typical illustration is given by A,D,B = 1,1,2. Here, the A.P. 1,2,3,4, ... is transformed into the G.P. 2,4,8,16, ...

{EXP(A(N))} is another illustration.

b. Typical illustration: A,R = 10,10. In this case, the arithmetic progression has common difference LOG(10).

LOG means the natural logarithm (base e), but the theorem here holds for any base.

6. **Solution:**

```
10 HOME: REM *** INSERT TERMS
20 INPUT "A,B,N = "; A,B,N
30 FOR I = 0 TO N
40 PRINT I, A + I*(B-A)/N
50 NEXT I
```

Notes: The problem-solving part of this problem -hence the crucial part from the teacher's point of view, is that the student should figure out that "common difference" means "equally spaced"; hence the formulation I*(B-A)/N.

7. **Solution:**

```
10 HOME: REM *** A.P. BETWEEN X AND Y
20 INPUT "A,D,X,Y = "; A,D,X,Y
30 M1 = 1 + (X-A)/D: M = 1 + INT(M1)
40 IF M = M1 + 1 THEN M = M - 1
50 N = 1 + INT((Y-A)/D)
60 FOR I = M TO N
70 PRINT A + (I-1)*D
80 NEXT I
```

Notes: As a follow-up students may make the program user-friendly, so that every reasonable choice of A,D,X,Y will cause a suitable output. Here are some test-cases:

A,D,X,Y = 1,-1,1,6
A,D,X,Y = 5,0,3,7
A,D,X,Y = 1,2,5,2
A,D,X,Y = 1/3,2/3,6,9

8. Solution:

```
10 HOME: REM *** G.P. BETWEEN X AND Y
20 INPUT "A,R,X,Y = "; A,R,X,Y
30 M1 = 1+LOG(X/A)/LOG(R): M = 1+INT(M1)
40 IF M = M1 + 1 THEN M = M - 1
50 N = 1 + LOG(Y/A)/LOG(R)
60 FOR I = M TO N
70 PRINT A*R^(I-1)
80 NEXT I
```

Notes: As a follow-up students may make the program user-friendly, so that every reasonable choice of A,R,X,Y will cause a suitable output. Here are some test-cases:

A,R,X,Y = 1,.5,7,4
A,R,X,Y = 1,1,4,7
A,R,X,Y = 0,-2,8,16
A,R,X,Y = 4,0,1,5

9. Solution: The program written for Problem 3 can be used, but some students will want to write a program that generates {C(N)} from any user-input {A(N)} and {B(N)}.

If D1 and D2 are the common differences for the given progressions, then the common difference for {C(N)} is 1/(1/D1 + 1/D2). This is a good example of an appearance of the harmonic mean of two numbers.

Students who master this problem should carry out a similar pursuit for the sequence {C(N)} defined by C(N) = A(B(N)) [or the other composite, B(A(N)).]

LIST OF TRANSPARENCIES FOR SESSION 21S

ADVANCED TOPICS

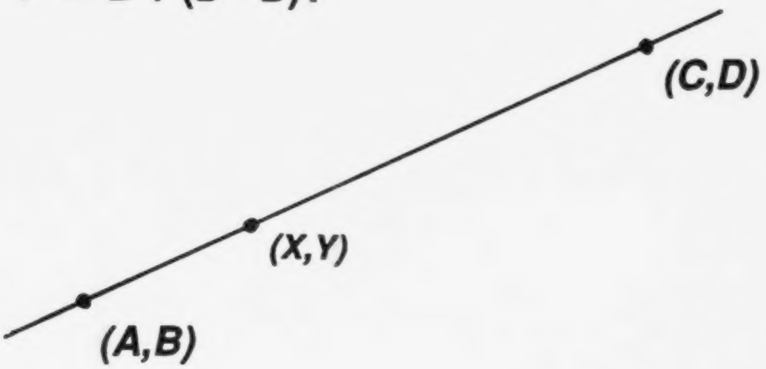
(LINES AND CURVES, SEQUENCES AND LIMITS)

18. THE LINE THROUGH POINTS (A,B) AND (C,D)
19. GRAPHICS ADDITION TO PREVIOUS PROGRAM
20. INTERSECT ANY TWO LINES
21. PROGRAM P24: INTERSECT LINES
22. CURVES AND THEIR PARAMETRIC EQUATIONS
23. MODEL FOR TEACHING ABOUT A.P.'S (STEPS 1-3)
24. (CONTINUATION - STEPS 4-6)
25. THE FIRST WORD IN CALCULUS
26. PROBLEMS TO SOLVE USING MATHDISK FOUR

THE LINE THROUGH POINTS (A,B) AND (C,D)

$X = A + (C - A)T$

$Y = B + (D - B)T$



PROGRAM: T GOES FROM 0 TO 1

```
10 A = 12: B = 6: C = -8: D = -8
20 FOR T = 0 TO 1 STEP 1/16
30 X = A + (C-A)*T
40 Y = B + (D-B)*T
50 PRINT T,X,Y
60 NEXT T
```

Transparency 18.

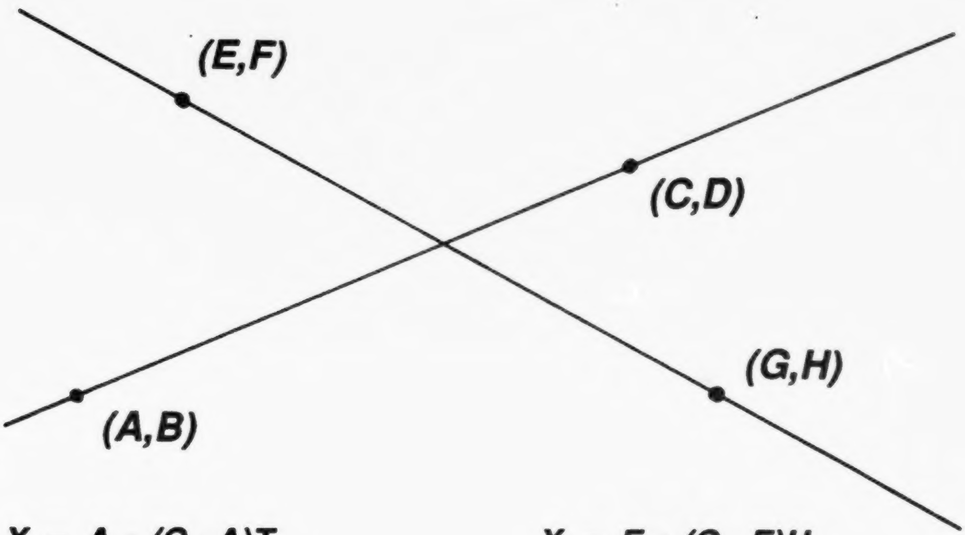
GRAPHICS ADDITION TO PREVIOUS PROGRAM

(FOR IBM-PC)

```
5   CLS:  SCREEN 2:  KEY OFF: WINDOW (-14,-10)-(14,10)

10  A = 12: B = 6: C = -8: D = -8

20  FOR T = 0 TO 1 STEP 1/16

30  X = A + (C-A)*T

40  Y = B + (D-B)*T

50  PRINT T,X,Y: PSET (X,Y)

60  NEXT T
```

Transparency 19.

INTERSECT ANY TWO LINES



$X = A + (C - A)T$
$Y = B + (D - B)T$

$X = E + (G - E)U$
$Y = F + (H - F)U$

IF THESE LINES ARE NOT PARALLEL, THEN FOR SOME T AND U , THE X COORDINATES ARE EQUAL, AND SO ARE THE Y COORDINATES. EQUATING THE X'S AND EQUATING THE Y'S GIVES

$$(C - A)T + (E - G)U = E - A$$

$$(D - B)T + (F - H)U = F - B .$$

THIS SYSTEM OF TWO EQUATIONS IN TWO UNKNOWNS HAS A SOLUTION IF AND ONLY IF THE DETERMINANT

$$Z = \begin{vmatrix} C - A & E - G \\ D - B & F - H \end{vmatrix}$$

IS NONZERO, AND IN THIS CASE, THE VALUE OF T IS GIVEN BY CRAMER'S RULE (LINE 70 OF PROGRAM P24).

Transparency 20.

PROGRAM P24: INTERSECT LINES

```
10  INPUT  "INPUT  A,B = "; A,B

20  INPUT  "INPUT  C,D = "; C,D

30  INPUT  "INPUT  E,F = ";  E,F

40  INPUT  "INPUT  G,H = "; G,H

50  Z = (C-A)*(F-H) - (E-G)*(D-B)

60  PRINT: IF Z=0 THEN PRINT "THESE LINES ARE
            IDENTICAL OR PARALLEL.": END

70  T = ((E-A)*(F-H) - (E-G)*(F-B))/Z

80  X = A + (C-A)*T:  Y = B + (D-B)*T

90  PRINT "THE LINE JOINING  (A,B)  AND  (C,D)
            INTERSECTS THE LINE JOINING  (E,F)
            AND  (G,H)"

100  PRINT "IN THE POINT  ("X","Y")."
```

Transparency 21.

CURVES AND THEIR PARAMETRIC EQUATIONS

CURVE	X	Y
LINE	A + (C - A)*T	B + (D - B)*T
CIRCLE	R*COS(T)	R*SIN(T)
*SPIRAL	SQR(T)*COS(T)	SQR(T)*SIN(T)
HYPOCYCLOID	N*R*COS(T) + R*COS(N*T)	N*R*SIN(T) - R*SIN(N*T)
EPICYCLOID	N*R*COS(T) - R*COS(N*T)	N*R*SIN(T) - R*SIN(N*T)

FOR DETAILS AND MORE CURVES, SEE
WORKBOOK FOR MATHDISK TWO.

Transparency 22.

MODEL FOR TEACHING ABOUT ARITHMETIC PROGRESSIONS

STEP 1. WRITE ON CHALKBOARD:

INPUT A,D AND THEN PRINT THE NUMBERS
A, A+D, A+2D, ..., UP TO A+10D .

STEP 2. USING STUDENTS' SUGGESTIONS, TRANSLATE STEP 1. THIS (OR SOMETHING LIKE IT) SHOULD BE WRITTEN ON THE CHALKBOARD. IT ***MAY*** ALSO BE TYPED INTO A COMPUTER, BUT MANY TEACHERS PREFER USING PROGRAM DEVELOPMENT AS A TEACHING TOOL ***WITHOUT*** ACTUALLY USING THE COMPUTER !

```
10 INPUT "A,D = "; A, D
20 FOR X = A TO A+10*D STEP D
30 PRINT
40 NEXT X
```

STEP 3. WRITE ON CHALKBOARD:

PRINT THE RUNNING SUM

A, A + A+D, A + A+D + A+2D, A + A+D + A+2D + A+3D, ETC.

Transparency 23.

(CONTINUATION OF MODEL)

STEP 4. USING STUDENTS' SUGGESTIONS, TRANSLATE STEP 3:

```
10  INPUT  "A,D = "; A,D
20  FOR X = A TO A+10*D STEP D
25  S = S + X
30  PRINT X,S
40  NEXT X
```

STEP 5. WRITE ON CHALKBOARD:

$$SUM = A + A+D + A+2D + \ldots + A+(N-1)D$$

$$SUM = NA + (1 + 2 + 3 + \ldots + N-1)D$$

$$SUM = NA + N(N-1)D/2$$

$$\underline{SUM = N[A + (N-1)D/2]}$$

MAKE THE COMPUTER "PROVE" THIS FORMULA.

STEP 6. USING STUDENTS' SUGGESTIONS, TRANSLATE THE FORMULA:

```
10  INPUT  "A,D = "; A,D
20  FOR X = A TO A+10*D STEP D
23  N = N+1
25  S = S+X                       : REM   ACTUAL SUM
27  S1 = N*(A + (N-1)*D/2)        : REM   FORMULA  SUM
30  PRINT  X,S,S1
40  NEXT X
```

Transparency 24.

THE FIRST WORD IN CALCULUS

QUESTION: ***IN ONE WORD, WHAT IS THE DIFFERENCE BETWEEN ALGEBRA AND CALCULUS?***

ANSWER: ***LIMITS***

EXPLANATION: CALCULUS CONSISTS MOSTLY OF

DERIVATIVES, INTEGRALS, *AND* ***SERIES,***

AND EACH OF THESE IS DEFINED AS A LIMIT.

THE DEFINITION OF LIMIT WAS GIVEN BY AUGUSTIN CAUCHY 100 YEARS AFTER THE DISCOVERERS OF CALCULUS REALIZED THAT THEY DID NOT HAVE AN ADEQUATE UNDERSTANDING OF DERIVATIVES, INTEGRALS, AND SERIES.

UNDERSTANDABLY THEN, MANY STUDENTS FIND THE DEFINITION AND BASIC FACTS ABOUT LIMITS TO BE DIFFICULT AT FIRST.

Transparency 25.

PROBLEMS TO SOLVE USING

MATHDISK FOUR

1. USE *ALL THREE METHODS* (PROGRAM 144) TO FIND THE AREA TRAPPED BETWEEN THE X AXIS AND ONE ARCH OF Y = SIN(X).

2. USE *DISTANCE BETWEEN CURVES* TO FIND THE DISTANCE BETWEEN THE TOP AND BOTTOM SEMICIRCLES OF THE CIRCLE X^2 + Y^2 = 1.

3. USE *RECURRENCE* TO PRINT THE FIRST 10 TERMS OF EACH OF THE FOLLOWING SEQUENCES:

   A) THE FIBONACCI SEQUENCE: 1, 1, 2, 3, 5, . . .

   B) THE ARITHMETIC SEQUENCE 2, 5, 8, 11, . . .

   C) THE GEOMETRIC SEQUENCE 8, 16, 32, 64, . . .

   D) THE FACTORIAL SEQUENCE: 1, 2, 6, 24, . . .

   E) THE 3rd DIAGONAL OF PASCAL'S TRIANGLE: 1, 3, 6, 10, . . .

   F) THE SQUARES: 1, 4, 9, 16, . . .

Transparency 26.

Session 22M
Special Topics

Exploring Fibonacci Sequences

Objective

The student will manipulate the seed numbers in a Fibonnaci sequence to determine relationships between the seed numbers and the resulting sequence.

Description

This program asks the student to input the first two numbers of the Fibonnaci sequence they wish to explore. The computer generates the rest of the sequence out to 40 numbers.

Procedure

Load the program called "Fibonnaci Sequences". Input 1 as the first seed number and 1 as the second seed number. Verify by hand for the first 10 digits that the computer accurately computes the Fibonacci sequence. Copy down the first 10 numbers of the sequence on the data sheet.

Rerun the program with 2 and 2 as the seed numbers. Copy down the first 10 numbers. Is there a relationship between the first set and the second set?

This session is a potpourri of special topics utilizing the computational power of the microcomputer to demonstrate theorems about the Fibonacci Sequence, magic squares, and inflation and interest.

What is this relationship? (Hint: use the FACTOR program to help you find it)

Rerun the program with 3 and 3 as the seed numbers. Copy down the first 10 numbers in the sequence. Does your relationship hold?

Form a generalized rule expressing the relationship between the two seed numbers and the 1 and 1 Fibonacci sequence.

Rerun the program with the seed numbers of 1 and 2. What do you get for your sequence? (Copy down the first 10 numbers in the sequence)

Rerun the program with the seed numbers of 1 and 3. What do you get for your sequence? (Copy down the first 10 numbers in the sequence).

Is there a relationship between the 1,3 sequence and the 1,1 sequence? (Hint try dividing term by term and look at the remainder)

Try a 1 and 4 seed. Is their any relationship to the 1 and 1 sequence? Try it out?

Try a 2 and 5 seed. Is their any relationship to the 1 and 1 sequence? How about the 2 and 2 sequence? Copy all of your data down on the data sheet and spend some time looking for various relationships. Formulate hypotheses and test them out.

Magic Squares

Objective

To manipulate 3 * 3 magic squares and derive various rules about them.

Description

This spreadsheet template allows the student to put in the 9 numbers in a magic square. It then adds the rows, columns and diagonals to determine if the square is a real magic square. If it is, then all of the rows, columns, and diagonals add up to the same number.

Procedure

Load the spreadsheet template "Magic Square2". Use it to derive a magic square.

Transpose the magic square so the rows become the columns and the columns become the rows. Is the result a magic square?

Take a constant term and add it to every cell in a magic square. Is the result a magic square?

Take a constant term and first subtract it, divide it, multiply it by every cell in the magic square. Is each result a magic square?

Take two magic squares. Add them cell by cell so that the first cell of the first square is added to the first cell of the second square. Is the result a magic square?

Repeat the process with two magic squares only this time multiply the squares cell by cell, subtract the squares cell by cell, divide the squares cell by cell. Are the results magic squares?

Definition

Scalar multiplication: Given a magic square A and a real number "a", a*A means to multiply "a" by every cell in A.

Definition

Scalar operation: Substitute addition, subtraction, or division for multiplication in the definition above.

Definition

A magic square A is said to be "derivable" from another magic square B if either there is a term "a" such that a scalar operation of a and A which gives magic square B, or if A is a transposed version of a scalar operation on B.

Determine whether the following rule is true or false: Given two 3 * 3 magic squares A and B. One is always derivable from the other.

Computing Inflation

Objective

To manipulate the interest rate and principle and observe the relationship between them over a fixed period of time.

Description

The program allows the user to put a value for the principle into the spreadsheet. Then the user puts in a value for the interest or inflation rate as a decimal (e.g. 5%=.05). The user can then observe what the interest is worth year by year and the total value of the interest plus principle year by year.

Procedure

Load the spreadsheet "Inflation". Put in a value for the principle in cell B2. In B3 put in an interest rate. Now watch the values year by year.

Now put in $100 for the principle and .05 for the interest rate. How much is the interest worth after 6 years?

After how many years is the interest worth $45.00?

When is the total value equal to 137.04?

Does doubling the rate of interest double the value of that interest? Put in $100 and 10% interest. After 10 years how much larger is the interest over the final 5% interest?

What is the ratio of the amount of interest in dollars after 10 years (earning 10%) versus the amount of interest in dollars after 10 years (earning 5%)?

Rerun the spreadsheet. This time the principle is $100 and the interest rate is 8%. Copy and record the final amount of interest in dollars after 10 years.

Run the program with the same $100 only this time with 16% interest rate. What is the ratio of the two final amounts of interest? (the amount earning 16% versus the amount earning 8%). Is this the same ratio as derived before?

(for advanced mathematics students: Put in $100 as the principle. Put in 5% as the interest. Record the following data in a table: year, amount of interest. Now insert that data into the DATA FIT program. Try to derive an equation that represents the data as well as possible. Run the program with $200 and 5% interest. Does the same function serve as a good estimator of the second set of data also? Modify it so that it does represent the second data.)

Run the program with $200 and 5% interest. Rerun the program with $200 and 10% interest. Find the ratio of the amount of interest after 10 years at 10% versus the amount of interest after 10 years at 5%. Is this ratio the same as the one found first?

Investing and interest

Objective

To observe that money invested monthly over a period of time accumulates to significant value. This value is dependent on the amount invested each month and the interest rate. The student will observe that increasing the interest rate increases the value of the investment faster than increasing the amount invested. (i.e. doubling the interest is better than doubling the principle)

Description

This spreadsheet template asks the student to put in the amount of money to be invested each month for 30 years. It then asks for the rate of return on this investment per year compounded monthly. The rate is entered as a decimal equivalent to percent. (e.g. 5% is entered as .05). The program then shows what that money is worth after 10 years and after 30 years. The program also shows what that first year of money (assuming investment ceases after the first year) is worth after each year and after year 10 and year 30.

Procedure

Load the spreadsheet called ANNUITY2. Put in $100 in for the investment and 9% for the rate of return. How much money is the investment worth at the end of the first year?

How much was actually invested in the first year?

How much is the actual interest accumulated in the first year?

Compute the actual interest as a percent of the amount invested during the first year. Why is this amount less than 9% if we were supposed to be getting a 9% return on our investment?

How much is that investment worth after 10 years (assuming we continue to invest monthly for 10 years)?

How much is that investment worth after 30 years (assuming we continue to invest monthly for 30 years)?

How much have we invested during the 30 year period of time?

What is the amount of interest accumulated over 30 years?

What percent is the interest of our total investment?

If we were to find an investment which paid double the interest (=18%). Would we double our investment after 30 years? Would we double our interest after 30 years?

Compute what the interest would be on our investment at 18% over 30 years. What percent of our investment is this?

If instead of doubling our interest we doubled the amount invested instead, would we do as well as doubling our interest? Determine what our investment would be worth after 30 years.

Use this amount to figure what our interest would be.

Then compute the percent this interest is of our total investment.

Which is better, doubling the interest or doubling the principle we invest in terms of having money at the end of the thirty year period?

Now invest $100 at 9% interest. Go down to row 22 and look across. One interpretation of the row is that if you invest for 1 year only and cease investment but let the existing investment stay in the annuity, after year 1 that money is worth 1260. After year 2 it is worth 1374; after year 3 it is worth 1497, etc. When does the money begin accumulating value quickly? Is it at the beginning of the investment period or at the end of the investment period?

Construct a graph with the x-axis being the year and the y-axis being the investment values in row 22. Is the graph a straight line? What happens as the graph approaches year 30.

Another interpretation of row 22 is that you continue to invest monthly for the entire 30 year period. After year 1 the money you have invested in year 1 is worth 1260. After year 2 the money you invested in the first year is now worth 1374 (why?) and the amount invested this year is 1260. After year 3 the amount invested the first year is worth 1497, the amount invested the second year is worth 1374, and the amount invested in the third year is worth only 1260.

Modify the spreadsheet so that it determines how much interest has been accrued after each year. Then plot the interest accrued year by year and determine the total amount of interest earned.

Assuming that you could get an interest rate of 15%, how much would have to be invested monthly to end up with over $1,000,000 total (investment and interest) at the end of 30 years? (Care to be a millionaire???)

Session 23

From Classrooms to Careers

Nobody can tell us with 95% confidence about the career opportunities that await our students. However, statistical methods do provide a broad view of future careers, and it is a view that we must take very seriously. Our national well-being depends on it, not to mention our well-being as mathematics educators. Report after report, with titles like "A Nation At Risk," have warned us that we are going to have to bridge the gap between classrooms and careers.

One part of the view is already clear: during our emerging Information Age, careers will go hand in hand with computers and problem solving. Let's examine these three - careers, computers, and problem solving, individually at first, and then all together. We're going to find some not-too-subtle hints about using computers in mathematics classrooms.

(Show Transparency 1.) Some jobs are going. Here is the first of three summaries from a 1982 issue of Newsweek. These jobs are not especially technological or mathematical. Two of them that are of some interest, however, are graduate assistants and secondary-school teachers. More about these two later.

Future jobs demand more problem solving, statistics, and computers in our schools, now.

(Transparency 2.) Other jobs are growing. At least half of the jobs listed here definitely depend on an ability to deal with numbers, numerical tables, and statistics. These jobs will require a well developed ability to organize and classify numerical data.

(Transparency 3.) But the future, according to Newsweek, is in this list. How many of us would have predicted that industrial robot production would head the list?

Needless to say, this third list represents millions of future employees who will be using computers, applying mathematics, and solving problems. Right now they are sitting in our classrooms.

(Transparency 4.) Does your school keep the Occupational Outlook Quarterly out where students will see it? The Spring 1984 issue is especially informative. It consists of projected work forces in every category. Let's take a look.

Here you see five columns. Column 1 classifies the second largest profession, which is engineering, into several huge categories.

Column 2 gives employment figures for 1982, and Column 3 gives the estimated percentage change during the 13-year period from 1982 to 1995. Column 4 gives the expected increase in jobs during those 13 years.

Column 3 is the key. The average percentage change in employment over all areas of employment is 25%. So, you can see that engineering, most especially electrical engineering, looks good.

(Transparency 5.) Next let's look at mathematics itself, and one of its closest cousins, statistics. There are presently almost two statisticians for every mathematician, and growth rates in these areas will be about average.

Much larger are the populations and growth rates in computer systems analysis, computer programming, and electrical and electronics technology.

(Transparency 6.) Engineering is the second largest profession. The largest is teaching. Here are projections from the Occupational Outlook Quarterly.

Again, let's pay special attention to Column 3. We expect the growth rate for K-6 teaching to exceed the overall occupational growth rate of 25%. On the other hand, we expect only a 13% increase in secondary level teaching. This figure is for all subjects, but the same article goes on to say that more "favorable opportunities will exist for persons qualified to teach special education, vocational subjects, mathematics, sciences, and bilingual education."

Look at that astounding negative 15% growth rate for college teachers. A loss of one-hundred eleven thousand jobs. Nevertheless, an explanatory note predicts "good job prospects for engineering and computer science faculty."

(Remove Transparency 6.) The rest of this particular Job Outlook makes interest reading, too. The people who really need to study it are the people now in school, and their teachers. Again and again, they will see that the careers of the future have a great deal to do with increased automation and dependence on computers.

The Occupational Outlook Quarterly is the sort of thing students should see when they enter their school library or computer lab. One recent issue, for example, shows all the job categories that comprise the laser industry. It could be effectively posted on a bulletin board in a math classroom.

To subscribe to the Occupational Outlook Quarterly, just call the U. S. Government Superintendent of Documents. The number is 202-783-3238.

While discussing careers, we really must focus more closely on one of them in particular, if only because of the sheer number of our present students to which this career option offers itself. I am speaking about teaching mathematics.

All of us here today are aware that many states already have serious shortages of qualified mathematics teachers. How could this have happened?

Three often cited reasons were written by Clyde A. Paul in The Mathematics Teacher. Maybe you have read the article. It is entitled "Bald Eagles, Sperm Whales, and Mathematics Teachers."

Mr. Paul begins with a reminder that eagles and whales are endangered species. He then gives three reasons for the comparable plight of math teachers:

(Transparency 7. Read it.)

We often hear that the demand for teachers is a simple function of the number of students; that after the big baby boom and subsequent boomlets, fewer teachers are needed and fewer are available. However, the function in question is not as simple as many seem to think.

The bottom line on this subject has been stated this way: "The decline in students preparing to become mathematics teachers has far outstripped decline in enrollments in secondary schools." This is quoted from ERIC Mathematics Education Fact Sheet No. 3, 1982

Let's turn now to the subject of computers.

In 1980 the Board of Directors of NCTM set forth a document entitled "An Agenda for Action - Recommendations for School Mathematics of the 1980's". The first three of those recommendations are especially germane to our subject. Here are those first three:

(Transparency 8. Read it.)

The Agenda for Action goes on to articulate several principles for the use of computers in mathematics classrooms. Included are these:

(Transparency 9. Read it.)

(Transparency 10. Read it.)

None of these principles is really surprising, and so you might be wondering, why spend time rehashing the obvious? The answer is this: we in mathematics education need - and here we have - well formulated consensus statements for dealing with the rest of our society. That means students, parents, other teachers, future teachers, administrators, budget planners, and legislators. In a time when computers and the needs of society are hammering away at the mathematics curriculum, reshaping it for better or for worse, with pieces seeming to fly off in every direction and new material being slapped into place - with all of this going on, we need to remember our Agenda for Action In particular, we need to quote from it when dealing with the use of computers in mathematics classrooms.

To summarize from the Agenda then, computers will play an increasingly important role - one that will support the single most salient Action named in the Agenda document. It is an Action that you can say in just two words. In fact, this particular Action has become something of an NCTM slogan for the 1980's. Do you have those two words on the tip of your tongue?

(Pause. Show Transparency 11.)

Now the first problem about problem-solving is this: What is it? What do we mean by a "problem"? And what do others mean by a "problem"? For example, what does our society mean by "problems" when they demand that we do a better job of preparing the next generation to solve problems in the real world? Or, what does an educational psychologist mean by "problem solving," in terms of what students at different levels are really ready to solve?

These questions are addressed in a convenient review of the literature. The review occupies just one sheet of paper, and I'm going to show you exactly what it is and where you can get a copy:

(Transparency 12)

This fact sheet focuses right down on our most serious problem about problem-solving, no matter whose definition of problem is used.

(Transparency 13. Read it.)

We've looked briefly at careers, computers, and problem solving, individually. Now let's put them together. Let's approach problem solving from the perspective of careers and computers. What kinds of problem solving most demand our attention in mathematics classrooms?

(Transparency 14. Read it.)

(Pause)

Wouldn't it be nice to have a computer that would just combine these key elements and decide the best kinds of problems for solving our problem-solving problem? The computer could think about all the implications of all six of these characteristics. The computer could analyze huge data banks on student ability levels and strike the right balance with skills that are going to be most needed in future careers. The computer could just solve our problem of problem solving.

Well, why not? Here's a computer. Let's try it.

(Run Program SOLVE.)

(Transparency 15.)

The appearance of traditional math as part of the answer comes as no surprise. But what about statistics? Is its appearance here consistent with the Agenda for Action?

(Transparency 16.)

There are many questions to be raised: where to fit statistics into curricula, how best to use the computer in the teaching of statistics, and so on. Here are a few sources of answers:

(Transparency 17. Read it.)

(Transparency 18. Read it.)

As further evidence of the need for more statistics within mathematics curricula, let's consider some conjectures about statistics. Consider this to be True-False quiz. Which of these statements would you mark with a big T ?

(Transparency 19. Uncover it one statement at a time while reading the following.)

STATEMENT 1.	Computers have more to offer to the teaching of statistics than to the teaching of algebra, geometry, trigonometry, and calculus.
Rationale:	Statistics uses more numbers. Lots more. A teacher can solve

	quadratic equations and draw similar triangles on a blackboard. But computing the standard deviation of 50 data - that's something for a computer.
STATEMENT 2.	Statistics is more prominent in the Information Age than are algebra, geometry, trigonometry, and calculus.
Rationale:	Of those five areas of mathematics, which do you see most often in your newspaper? In company reports. The stock market. SAT scores. Agricultural yields. Maybe the best we can say for algebra here is that it underlies statistics. But it is statistics that's most visible.
STATEMENT 3.	Statistics describes real-world problems more directly than do algebra and the others.
Rationale:	Real-world descriptions are in fact the primary purpose of that branch of statistics that is named descriptive statistics.
STATEMENT 4.	Statistics plays a more prominent role in decision-making in business, industry, health-sciences, and other career categories than do algebra and the others.
Rationale:	As you know, along with descriptive statistics is the other branch, called inferential statistics. It includes methods of decision-making based on experimental results and past records.
STATEMENT 5.	Students should learn to use computer spreadsheets and databases, in conjunction with their learning of statistics.
Rationale:	Spreadsheets and databases are standard forms of Information Age communication.
STATEMENT 6.	At this very moment, out there in the world of careers, more computers are doing statistics than are doing algebra.

Let's turn now to the question of computer programming in connection with high-school mathematics.

Certainly, no part of the mathematics curriculum should be replaced by computer programming as an end in itself. And most certainly not by computer literacy.

On the other hand, thousands of students are good at programming, and they should be writing original programs that promote their mathematical problem-solving skills. Otherwise, these students will be greatly disadvantaged in competition for many of the careers that we examined a short while ago.

Here are some of the characteristics of program-writing that make it an ideal kind of problem-solving.

(Transparency 20.)

ONE. At the outset of writing a program, the student must figure out exactly what is given, and what is to be found. The recognition of these two, and the distinction between them, is often said to be half the battle by those who teach problem-solving.

TWO. The student must regard the problem to be solved by his or her program as a whole, then break it into major components, and then order these components into a logical sequence.

This rendering of the problem into a sequence of blocks is of enormous problem-solving value for the kind of problem-solving needed on the job. The reason for this is that most real-world problems belong within some sort of system, some ordering of blocks within so-called block diagrams. The interdependencies within block diagrams and organizational heirarchies is remarkably similar to the interdependencies of parts of a computer program. Students who work with subroutines are in effect working with blocks in a block diagram. Therefore they are developing problem-solving skills that apply to real-world problems.

Moreover, the activity of debugging a program is remarkably isomorphic to the kind of problem solving known in the real world as trouble-shooting.

Students who write programs learn to think in terms of systems of interconnected components. That is an essential feature of real-world problem-solving that is missing from most of the problems now being assigned to our mathematics students.

THREE. Language. Students who write programs learn that problem-solving depends on precise usage of language. Why? Because if any little thing is syntactically wrong, the program will not run properly. How often have our students taken another step forward because the computer told them they had made a SYNTAX ERROR?

Anything that promotes precise use of language among students should be supported by mathematics teachers. Anything that builds up students' ability to translate problems into mathematical symbols - as programming certainly does, should be exemplified and assigned by mathematics teachers.

FOUR. The computer is a very patient teacher. It never tires of letting the student try and try and try again.

FIVE. In the hands of a student who is a programmer, the computer is an obedient and powerful servant. For students who are "what-if-ers" and discovery-learners, the computer can become an extension of their own minds.

Lynn Arthur Steen, President of the Mathematical Association of America in 1985, wrote this: "Only when the computer is used as an instrument of discovery will it truly aid the learning of mathematics."

SIX. Some students who become programmers become so attached to their art that they go far beyond the limitations of any course. These students are like football players who practice after school, thereby going way beyond the requirements of a physical education course. If only our student programmers could attract the positive reinforcement that school athletes and musicians receive.

The extent to which some students are drawn to programming deserves further comment. Sherry Turkle wrote a book that has a great deal to say about this. The book is entitled The Second Self: Computers and the Human Spirit.

(Transparency 21.)

I quote from page 19. "Some adolescents adopt the computer as their major activity, throwing themselves into programming the way others devote themselves to fixing cars. But there is a more subtle and widespread way that computers enter the adolescent's world of self-definition and self-creation. A computer program is a reflection of its programmer's mind. If you are the one who wrote it, then working with it can mean getting to know yourself differently."

And from page 15. "For many, computer programming is experienced as creating a world apart. Some create worlds that are highly predictable and use their experiences in them to develop a sense of themselves as capable of exerting firm control. Others have different needs, different desires, and create worlds whose complexity is always on the verge of getting out of hand, worlds where they can feel themselves to be wizards of brinkmanship."

Let's return now to program-writing as problem solving. Specifically, let's consider the writing of mathematical programs."

I shall borrow an example from Donald O. Norris's article in The Mathematics Teacher.

(Show Transparency 22.)

Norris asks us to "compare the mental activity required to solve a quadratic equation using the formula with the mental activity required to program the computer to use the formula. In the first instance, the problem can be reduced to rote memorization, as it usually is; and this does not involve the use of logic or problem-solving techniques except in a very meager way. However, the computer program required to solve the problem requires an understanding of the formula, the ability to distinguish all the possible cases that might arise, and the ability to explain to an idiot (the computer) the precise instructions it must follow to obtain the desired results."

"My point is this," Norris continues, "It requires a tremendous amount of problem-solving ability to make a computer solve a problem. It requires the kind of analytical thinking we want our students to learn."

(Pause.)

It's time now to summarize. We looked first at careers, then computers, and then problem solving. We looked at these three individually, and then we zoomed in on that number-one priority called problem-solving, from the perspective of careers and computers. What did we see?

We saw statistics, and the programming of computers. These two stand out clearly, like two mountains that rise higher than neighboring hills that bear names like mathematical modeling, discrete math, and algorithmic math. These lower hills can be better seen and better approached by first climbing part way up the two big mountains of statistics and mathematical programming.

To close, I'd like to return once again to the Occupational Outlook Quarterly, this time to the Summer of 1984.

(Transparency 23.) Here we see a table of percentages. The percentages in each row add up to 100%. The name of the table is a question: "How frequently did graduates use the course content of their major field?" The population for this survey was the national population of college graduates of 1980 one year after they were graduated. The sample size was 9300.

Notice that the five percentages for Mathematics majors are very much like those for Engineering majors. On the other hand, these figures for Computer and information sciences are significantly higher in the first two columns: 75% of these graduates said they used their major-field course content "almost always" or "frequenly." For Engineering and Mathematics majors, this figure drops to 57% and to 51%.

(Transparency 24.) For the same population, here are figures for job satisfaction. To end this session on a happy note, let's look at the first column, labeled "Very Satisfied," and the row labeled "Mathematics." No field has a higher percentage of very satisfied employees. It appears that mathematics was one of the most satisfying career options that students could choose just a few years ago. Let's be sure it stays that way.

List of Transparencies

1. SOME JOBS ARE GOING ...
2. OTHERS ARE GROWING ...
3. BUT THE FUTURE IS HERE.
4. JOB OUTLOOK: ENGINEERS
5. JOB OUTLOOK: MATHEMATICIANS AND OTHERS
6. JOB OUTLOOK: TEACHERS
7. THREE REASONS
8. AN AGENDA FOR ACTION - RECOMMENDATIONS 1-3
9. THE AGENDA - "ALL STUDENTS SHOULD ... "
10. THE AGENDA - "SCHOOLS SHOULD PROVIDE ... "
11. PROBLEM SOLVING
12. ERIC FACT SHEET
13. PROBLEM SOLVING IS ... MULTISTEP
14. WHAT KINDS OF PROBLEMS DO WE NEED?
15. HYPOTHESES AND CONCLUSIONS (INPUT AND OUTPUT)
16. THE AGENDA - PRIORITIES AND EMPHASES
17. THE STATISTICS TEACHERS NETWORK
18. ARTICLES ON CLASSROOM STATISTICS
19. TRUE OR FALSE
20. "PROGRAM-WRITING IS PROBLEM-SOLVING"
21. THE SECOND SELF
22. NORRIS'S EXAMPLE
23. HOW FREQUENTLY DID GRADUATES USE THE COURSE CONTENT?
24. HOW SATISFIED WERE GRADUATES WITH THEIR JOB?

SOME JOBS ARE GOING ...

Occupation	Percent DECLINE in employment
Shoemaking-machine operators	19.2
Farm laborers	19.0
Railroad-car repairers	17.9
Farm managers	17.7
Graduate assistants	16.7
Housekeepers, private household	14.9
Child-care workers, private household	14.8
Maids and servants, private household	14.7
Farm supervisors	14.3
Farmers, owners and tenants	13.7
Timber-cutting and logging workers	13.6
Secondary-school teachers	13.1

Transparency 1.

OTHERS ARE GROWING ...

Occupation	Percent GROWTH in employment
Data-processing-machine mechanics	157.1
Paralegal personnel	143.0
Computer-systems analysts	112.4
Computer operators	91.7
Office-machine servicers	86.7
Tax preparers	77.9
Computer programmers	77.2
Aero-astronautic engineers	74.8
Employment interviewers	72.0
Fast-food restaurant workers	69.4
Child-care attendants	66.5
Veterinarians	66.1

Transparency 2.

BUT THE FUTURE IS HERE.

Occupation	Estimated employment by 1990
INDUSTRIAL-ROBOT PRODUCTION	800,000
GERIATRIC SOCIAL WORK	700,000
ENERGY TECHNICIANS	650,000
INDUSTRIAL-LASER PROCESSING	600,000
HOUSING REHABILITATION	500,000
HANDLING NEW SYNTHETIC MATERIALS	400,000
ON-LINE EMERGENCY MEDICAL	400,000
HAZARDOUS-WASTE MANAGEMENT	300,000
GENETIC ENGINEERING	250,000
BIONIC MEDICAL ELECTRONICS	200,000
LASER, HOLOGRAPHIC AND OPTICAL-FIBER MAINTENANCE	200,000

Transparency 3.

JOB OUTLOOK

Occupation	Estimated employment 1982	Percent change in employment 1982-95	Numerical change in employment 1982-95
AEROSPACE ENGINEERS	44,000	41	18,000
CHEMICAL ENGINEERS	56,000	43	24,000
CIVIL ENGINEERS	155,000	47	73,000
ELECTRICAL ENGINEERS	320,000	65	209,000
INDUSTRIAL ENGINEERS	160,000	42	67,000
MECHANICAL ENGINEERS	209,000	52	109,000
ALL ENGINEERS	**1,204,000**	**49**	**584,000**

Transparency 4.

JOB OUTLOOK

Occupation	Estimated employment 1982	Percent change in employment 1982-95	Numerical change in employment 1982-95
COMPUTER SYSTEMS ANALYSTS	254,000	85	217,000
MATHEMATICIANS	11,000	28	3,000
STATISTICIANS	20,000	28	5,700
COMPUTER PROGRAMMERS	266,000	77	205,000
ELECTRICAL AND ELECTRONICS TECHNICIANS	366,000	61	222,000
BIOLOGICAL SCIENTISTS	52,000	36	19,000

Transparency 5.

JOB OUTLOOK

Occupation	Estimated employment 1982	Percent change in employment 1982-95	Numerical change in employment 1982-95
KINDERGARTEN AND ELEMENTARY SCHOOL TEACHERS	1,366,000	37	511,000
SECONDARY SCHOOL TEACHERS	1,024,000	13	128,000
COLLEGE AND UNIVERSITY FACULTY	744,000	- 15	- 111,000

Transparency 6.

THREE REASONS (1979)

1. MORE FEMALES ARE BYPASSING THE TRADITIONAL OCCUPATION OF TEACHING IN ORDER TO TRAIN FOR POSITIONS FORMERLY DOMINATED BY MALES.
2. MORE STUDENTS ARE SELECTING COLLEGE TRAINING THAT WILL PROVIDE FINANCIAL REWARDS RATHER THAN PERSONAL SATISFACTION.
3. STUDENTS FROM THE MIDDLE AND LOWER ECONOMIC CLASSES ARE REALIZING THAT BLUE-COLLAR APPRENTICESHIP PROGRAMS PROVIDE QUICKER, AND MORE SUBSTANTIAL, FINANCIAL REWARDS THAN A TEACHER-TRAINING PROGRAM.

THE NEGATIVE PUBLICITY GIVEN THE EDUCATIONAL JOB MARKET HAS ALSO PLAYED AN IMPORTANT ROLE IN THIS REDUCTION OF PROSPECTIVE TEACHERS. PUBLIC PESSIMISM ABOUT THE WISDOM OF CHOOSING TEACHING AS A CAREER IS STILL BEING REINFORCED.

Transparency 7.

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National Council of Teachers of Mathematics

Transparency 8.

An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that—

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

FROM *THE AGENDA* -

ALL STUDENTS SHOULD HAVE ACCESS TO CALCULATORS AND INCREASINGLY TO COMPUTERS THROUGHOUT THEIR SCHOOL MATHEMATICS PROGRAM.

THE USE OF ELECTRONIC TOOLS SUCH AS CALCULATORS AND COMPUTERS SHOULD BE INTEGRATED INTO THE CORE MATHEMATICS CURRICULUM.

CALCULATORS AND COMPUTERS SHOULD BE USED IN IMAGINATIVE WAYS FOR EXPLORING, DISCOVERING, AND DEVELOPING MATHEMATICAL CONCEPTS AND NOT MERELY FOR CHECKING COMPUTATIONAL VALUES OR FOR DRILL AND PRACTICE.

Transparency 9.

FROM *THE AGENDA* -

SCHOOLS SHOULD PROVIDE CALCULATORS FOR USE IN ELEMENTARY AND SECONDARY SCHOOL CLASSROOMS.

EDUCATORS SHOULD TAKE CARE TO CHOOSE SOFTWARE THE FITS THE GOALS OR OBJECTIVES OF THE PROGRAM AND NOT TWIST THE GOALS AND DEVELOPMENTAL SEQUENCE TO FIT THE TECHNOLOGY AND AVAILABLE SOFTWARE.

AS HOME COMPUTERS COME INTO WIDER USE, HOMEWORK SHOULD BE ASSIGNED THAT CAN TAKE ADVANTAGE OF THEIR POTENTIAL IN PROBLEM SOLVING.

Transparency 10.

PROBLEM SOLVING

Transparency 11.

23

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National Council of Teachers of Mathematics

Transparency 12.

ERIC

Clearinghouse for Science, Mathematics, and Environmental Education

1200 Chambers Road, Third Floor
Columbus, Ohio 43212
(614) 422-6717

ERIC/SMEAC Mathematics Education Fact Sheet Number 2 1981

The Problem of Problem Solving

PROBLEM SOLVING IS ... MULTISTEP

PROBLEM SOLVING IS OFTEN EQUATED WITH SOLVING VERBAL TEXTBOOK PROBLEMS, BUT THIS WAS NOT THE TYPE OF PROBLEM THAT CAUSED DIFFICULTY. IN FACT, STUDENTS DID "REASONABLY WELL" ON ONE-STEP PROBLEMS SIMILAR TO THOSE FOUND IN TEXTBOOKS. WHEN THE CORRESPONDING COMPUTATIONAL SKILLS HAD BEEN ATTAINED, FINDING THE SOLUTION TO SUCH PROBLEMS PRESENTED LITTLE DIFFICULTY.

HOWEVER, THE MAJORITY OF STUDENTS AT ALL AGE LEVELS HAD DIFFICULTY WITH ANY PROBLEM REQUIRING SOME ANALYSIS OR THINKING.

THUS, "ALMOST EVERY PROBLEM THAT COULD NOT BE SOLVED BY A ROUTINE APPLICATION OF A SINGLE ARITHMETIC OPERATION CAUSED A GREAT DEAL OF DIFFICULTY."

PERFORMANCE ON MULTISTEP PROBLEMS WAS LOWER THAN . . .

Transparency 13.

WHAT KINDS OF PROBLEMS DO WE NEED?

1. **DO-ABLE PROBLEMS THAT REQUIRE AND PROMOTE MATHEMATICAL THINKING**

2. **"INFORMATION AGE" PROBLEMS: FINDING, READING, PROCESSING, INTERPRETING, AND PRESENTING NUMERICAL DATA**

3. **PROBLEMS THAT USE THE COMPUTER AS A TOOL**

4. **MULTISTEP PROBLEMS**

5. **PROBLEMS THAT DEVELOP BASIC SKILLS NECESSARY FOR REAL-WORLD PROBLEM SOLVING. THESE SKILLS INCLUDE *SYMBOL-SENSE* AND *ESTIMATING* .**

6. **PROBLEMS THAT LEND THEMSELVES TO EXPANSION, EXPERIMENTATION, AND DISCOVERY LEARNING**

Transparency 14.

HYPOTHESES
(= INPUT)

1. DO-ABLE
2. INFORMATION AGE
3. COMPUTER
4. MULTISTEP
5. SYMBOL SENSE & ESTIMATION SKILLS
6. DISCOVERY-LEARNING

CONCLUSIONS
(= OUTPUT)

TRAD. MATH
STATISTICS
PROGRAMMING

Transparency 15.

FROM *THE AGENDA* -

CHANGES IN THE PRIORITIES AND EMPHASES IN THE INSTRUCTIONAL PROGRAM SHOULD BE MADE IN ORDER TO REFLECT THE EXPANDED CONCEPT OF BASIC SKILLS.

THERE SHOULD BE INCREASED EMPHASES ON SUCH ACTIVITIES AS -

- **LOCATING AND PROCESSING QUANTITATIVE INFORMATION**
- **COLLECTING DATA**
- **ORGANIZING AND PRESENTING DATA**
- **INTERPRETING DATA**
- **DRAWING INFERENCES AND PREDICTING FROM DATA**

Transparency 16.

THE STATISTICS TEACHER NETWORK

THE STATISTICS TEACHER NETWORK IS A NEWSLETTER OF THE AMERICAN STATISTICAL ASSOCIATION AND THE NCTM JOINT COMMITTEE ON THE CURRICULUM IN STATISTICS AND PROBABILITY.

TO GET ONTO THE MAILING LIST, WRITE TO

BETH BRYAN
DEPARTMENT OF MATHEMATICS
AND COMPUTER SCIENCE
AUGUSTA COLLEGE
AUGUSTA, GA 30910

Transparency 17.

ARTICLES ON CLASSROOM STATISTICS

1. TEACHING DESCRIPTIVE AND INFERENTIAL STATISTICS USING A CLASSROOM MICROCOMPUTER, by BETTY COLLIS, MATHEMATICS TEACHER, MAY 1983, pages 318-322.

2. THE CLASS SURVEY: A PROBLEM-SOLVING ACTIVITY, by CLAIRE M. NEWMAN and SUSAN B. TURKEL, ARITHMETIC TEACHER, MAY 1985, pages 10-12.

3. DEVELOPING CONCEPTS IN PROBABILITY AND STATISTICS - AND MUCH MORE, by JAMES V. BRUNI and HELENE J. SILVERMAN, ARITHMETIC TEACHER, FEB. 1986, pages 34-37.

Transparency 18.

TRUE OR FALSE

1. COMPUTERS HAVE MORE TO OFFER TO THE TEACHING OF STATISTICS THAN TO THE TEACHING OF . . .

2. STATISTICS IS MORE . . . TO THE INFORMATION AGE THAN . . .

3. STATISTICS IS MORE VISIBLY DESCRIPTIVE OF OF REAL-WORLD PROBLEMS THAN . . .

4. STATISTICS IS USED MORE IN REAL-WORLD DECISION-MAKING THAN . . .

5. STATISTICS STUDENTS SHOULD USE COMPUTER SPREADSHEETS AND DATA BASES IN CONJUNCTION WITH THEIR LEARNING OF STATISTICS.

6. AT THIS VERY MOMENT, MORE COMPUTERS ARE DOING STATISTICS THAN . . .

Transparency 19.

"PROGRAM WRITING
IS
PROBLEM SOLVING"

1. WHAT IS GIVEN, AND WHAT IS TO BE FOUND

2. THE WHOLE AND ITS PARTS, ORDERED SYSTEMATICALLY; BLOCK DIAGRAMS AND INTERRELATIONSHIPS AMONG BLOCKS

3. PRECISE LANGUAGE AND 'SYMBOL SENSE'

4. COMPUTERS ARE VERY PATIENT

5. COMPUTERS ARE VERY OBEDIENT

6. PROGRAMMING AS A DEEPLY CREATIVE EXPERIENCE

Transparency 20.

THE SECOND SELF

COMPUTERS AND THE HUMAN SPIRIT

by Sherry Turkle

SIMON AND SCHUSTER • NEW YORK

Transparency 21.

AN EXAMPLE

FROM D. O. NORRIS: "LET'S PUT COMPUTERS INTO THE MATHEMATICS CURRICULUM,"

MATHEMATICS TEACHER, JAN. 1981, PAGES 24-26.

"NOW SUPPOSE YOU ASKED STUDENTS WHO HAVE LEARNED THE QUADRATIC EQUATION TO WRITE A PROGRAM . . . THEY WOULD HAVE TO DO AN ANALYSIS OF THE FOLLOWING SORT:

```
1.      READ IN  A, B, C

2-4.    (COMPUTE  D = B^2 - 4AC
           IF   D > 0   THEN ...
           IF   D = 0   THEN ...
           IF   D < 0   THEN ... )

5.      PROVIDE THE CAPABILITY TO REPEAT THE
        PROGRAM ON A NEW SET OF DATA.
```

Transparency 22.

HOW FREQUENTLY DID GRADUATES USE THE COURSE CONTENT OF THEIR MAJOR FIELD?

FIELD	ALMOST ALWAYS	FRE-QUENTLY	SOME-TIMES	RARELY	NEVER
ACCOUNTING	47	26	17	7	4
AGRICULTURE AND NATURAL RESOURCES	19	36	22	10	13
ART	23	30	6	14	27
BIOLOGICAL SCIENCES	24	16	25	16	19
BUSINESS AND MNGMENT, EXCL. ACCOUNTING	11	28	35	20	7
CHEMISTRY	51	12	17	11	9
COMMUNICATIONS	20	30	26	14	10
COMPUTER AND INFORMATION SCIENCES	43	32	20	3	2
ECONOMICS	4	18	30	33	15
EDUCATION, EXCL. PHYSICAL EDUC.	43	25	18	8	7
ENGINEERING	29	28	31	11	2
ENGLISH	16	23	26	23	12
HISTORY	8	5	19	31	37
HOME ECONOMICS	34	25	22	0	20
MATHEMATICS	25	26	30	13	6
NURSING	69	20	7	3	0
PHYSICAL EDUCATION	36	19	17	14	14
POLITICAL SCIENCE	1	18	18	29	34
PSYCHOLOGY	10	26	35	22	6
SOCIOLOGY	0	26	33	24	16
ALL GRADUATES	27	25	23	15	10

Transparency 23.

HOW SATISFIED WERE GRADUATES WITH THEIR JOBS?

FIELD	VERY SATISFIED	SOMEWHAT SATISFIED	NOT AT ALL SATISFIED
ACCOUNTING	47	42	11
AGRICULTURE AND NATURAL RESOURCES	49	43	8
ART	62	33	5
BIOLOGICAL SCIENCES	38	45	17
BUSINESS AND MNGMENT, EXCL. ACCOUNTING	42	46	12
CHEMISTRY	64	32	4
COMMUNICATIONS	38	46	16
COMPUTER AND INFORMATION SCIENCES	61	36	3
ECONOMICS	40	36	24
EDUCATION, EXCL. PHYSICAL EDUC.	55	37	8
ENGINEERING	63	31	6
ENGLISH	32	55	13
HISTORY	39	43	18
HOME ECONOMICS	54	28	17
MATHEMATICS	64	28	8
NURSING	49	46	5
PHYSICAL EDUCATION	44	44	11
POLITICAL SCIENCE	30	53	17
PSYCHOLOGY	38	44	18
SOCIOLOGY	28	56	16
ALL GRADUATES	47	42	11

Transparency 24.

Session 24
Symbol Manipulators

What They Are

Symbol manipulators are to the rest of mathematics what hand-held calculators are to whole number and decimal arithmetic. Imagine the ability to have polynomials factored instantly, equations solved for any variable requested, and algebraic expressions simplified at the touch of a button. No longer will teachers have to spend countless hours of class time trudging through mindless procedures which students forget as soon as they leave the mathematics classroom. Instead teachers will be able to emphasize key ideas and will be able to use the computational power of the computer to explore algebraic relationships. Some symbol manipulators will have the ability to graph algebraic expressions. Discoverable generalizations involving linear inequalities, quadratic equations, and polynomial functions can be covered in class without getting bogged down in difficult computations.

For example, questions can be asked like: Is the sum of two linear equations always a linear equation? In plotting a linear inequality, which region will always be shaded (with respect to the boundary line) in an expression like $y > mx + b$?

The computer will be used as a tool to solve problems which are generally too difficult or time consuming to presently dwell upon. For example, suppose two linear equations are given and the student is to find the solution point. Traditional methods would have the student use one of several techniques which require an extensive amount of algebraic manipulation such as substitution. With a symbol manipulator the intermediate calculations such as solving for one variable in terms of another can be done on the computer and the solution displayed on the screen. This solution can then be substituted directly into the other equation which can then be simplified on the computer and solved. Substituting one known variable back into one of the original equations can easily generate the value of the other variable by using the symbol manipulator. Furthermore, the students can easily see that which equation is used for this substitution is irrelevant, one gets the same answer either way. By removing the drudgery of computation, the teacher and student will complete many more problems and discover algebraic relationships without getting bogged down and losing the point.

Does this sound too good to be true? Well, it is available now. Various symbol manipulators are presently available. Examples are MuMath, TK Solver, MACSYMA, Eureka, and the Advanced Mathematics Utility (AMU).

How they can be used in the classroom?

How these products are used in the classroom are limited only by the imagination of the teacher and students. One effective application will be the use of the computer as an "electronic Blackboard" capable of displaying algebraic manipulations and graphs to students through a large screen projection system. The teacher will demonstrate key ideas by typing information into the computer and displaying the results to the entire class. The entire class will be able to observe outcomes, make inferences, and test out algebraic hypotheses. By watching the results and suggesting avenues to explore, the students can discover algebraic rules which are usually only stated directly and then verified in traditional mathematics classrooms.

For example, mathematics teachers typically tell the students to add -4 to both sides of the expression first before dividing by 3 in order to solve the following equation for x:

$$3x + 4 = 7$$

What would happen if the student divided by 3 first? Typically we don't show this aproach because the computation is more difficult. Does this approach always result in the same answer as the more traditional approach? A symbol manipulator can allow the student to explore this idea.

Another approach would be to set up a series of individualized explorations which the student can work through individually at the computer. These laboratory activities would be to secondary mathematics what science laboratories are to Biology, Chemistry, and Physics. In essence the student would get to manipulate variables and expressions to either verify known results or discover new ones.

How symbol manipulators can change what we do in the classroom.

Symbol manipulators can change what we do in the classroom. They can make our classes more inductive and exploratory; they can allow us to manipulate algebraic expressions regardless of how complicated the expressions are or how large the coefficients become. We will have a less procedural curriculum (i.e.: how to do things like procedures for solving quadratic equations) and a more conceptual/relational curriculum (e.g.: what are the coefficients and constants in a quadratic equation which will likely result in real solutions!)

Students will be more inclined to try our various hypotheses because the computer will do the routine work. Students may even take a more active interest in mathematics because the "symbol manipulation" will be handled directly by the computer. They will be freer to pose "what if..." questions like: What would happen if I added those two linear equations; I wonder what I would get? or I wonder what would happen if I factored that expression first...?" Since some symbol manipulators allow functions to be graphed, functions could be placed into the manipulator in virtually any form, solved for y in terms of x, and then graphed. No longer will students or teachers have to algebraically manipulate expressions by hand before they are graphed by function plotters.

Examples of ideas which could use a symbol manipulator/function plotter:

1. Do two linear equations when added always yield a linear equation?

2. What type of equation does the product of two linear equations have? Is it always the same? What is the relationship between the solutions of this resultant equation and the coefficients and constants of the original linear equations?

3. What do expressions have in common that have $x = 5$ and $x = -3$ as asymptotes?

4. What does the expression $(\sin(x))/|x|$ converge to as x converges to 0?

5. Solving systems of linear equations.

6. What do you know about the coefficients of quadratics which have real roots?

7. Solving systems of equations some of which are non-linear.

8. Computing the derivative of a function at any point.

9. Demonstrating that the slopes of the tangent lines having a rightmost common point converge to the slope of the derivative at that point.

10. Solving word problems. The student has to set up the equation of the problem and then use the symbol manipulator to algebraically solve the problem. Finally, the solution is tested in the equation by using the symbol manipulator.

Session 25
Curriculum Implications

Curriculum Implications

Recommendations

"Basic mathematical skills	NCSM
Agenda for Action"	NCTM
"The mathematical sciences curriculum K-12 What is still fundamental and what is not."	CBMS
Impact of computer technology on school mathematics	NCTM

Considerations

Skills for future
Calculator
Computer

Changing the curriculum

Government
Test bureaus
Publishers
School boards
Mathematics teachers ??

Technology Considerations

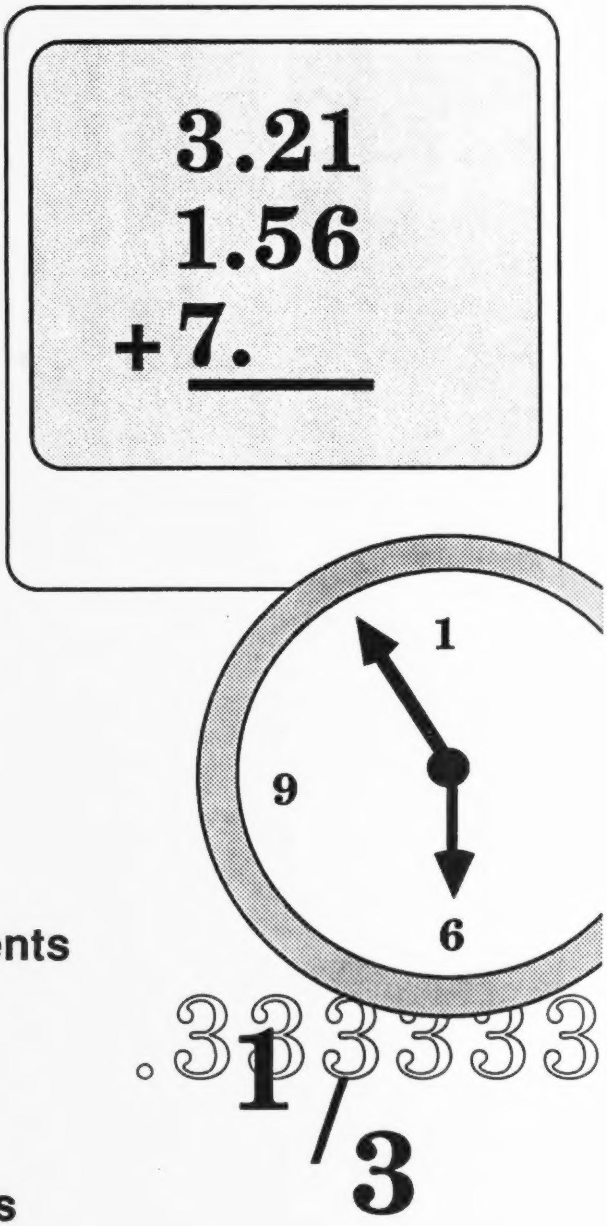
Teaching methods
Content
Sequence
Topics

Transparency 1

K - 8 Mathematics

Topics

- Operations
- Rules
- Scientific notation
- Rounding
- Errors
- Estimation
- Algorithms
- Models
- Random
- Arrays
- Decimals to fractions
- Fractions to decimals
- Repeating decimals
- Number theory
- Divisibility
- Surveying measurements
- Averages
- Unit conversions
- Telling time
- Trigonometry functions



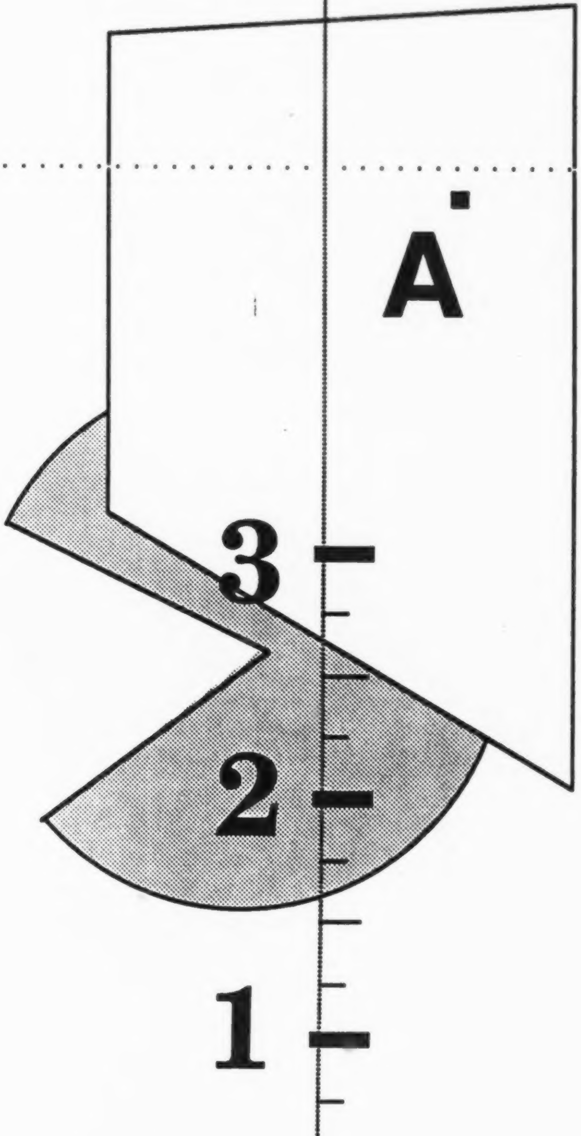
PROBLEM SOLVING

K - 8 Mathematics

Topics

Angles -	right	obtuse	acute	
Positions -	circle	triangle	square	rectangle

Patterns
Coordinates
Polygons
Perimeter formula
Circle formula
Degrees
Points
Line Segments
Variables
Logic
Symmetry
Negative Numbers
Planes
Size



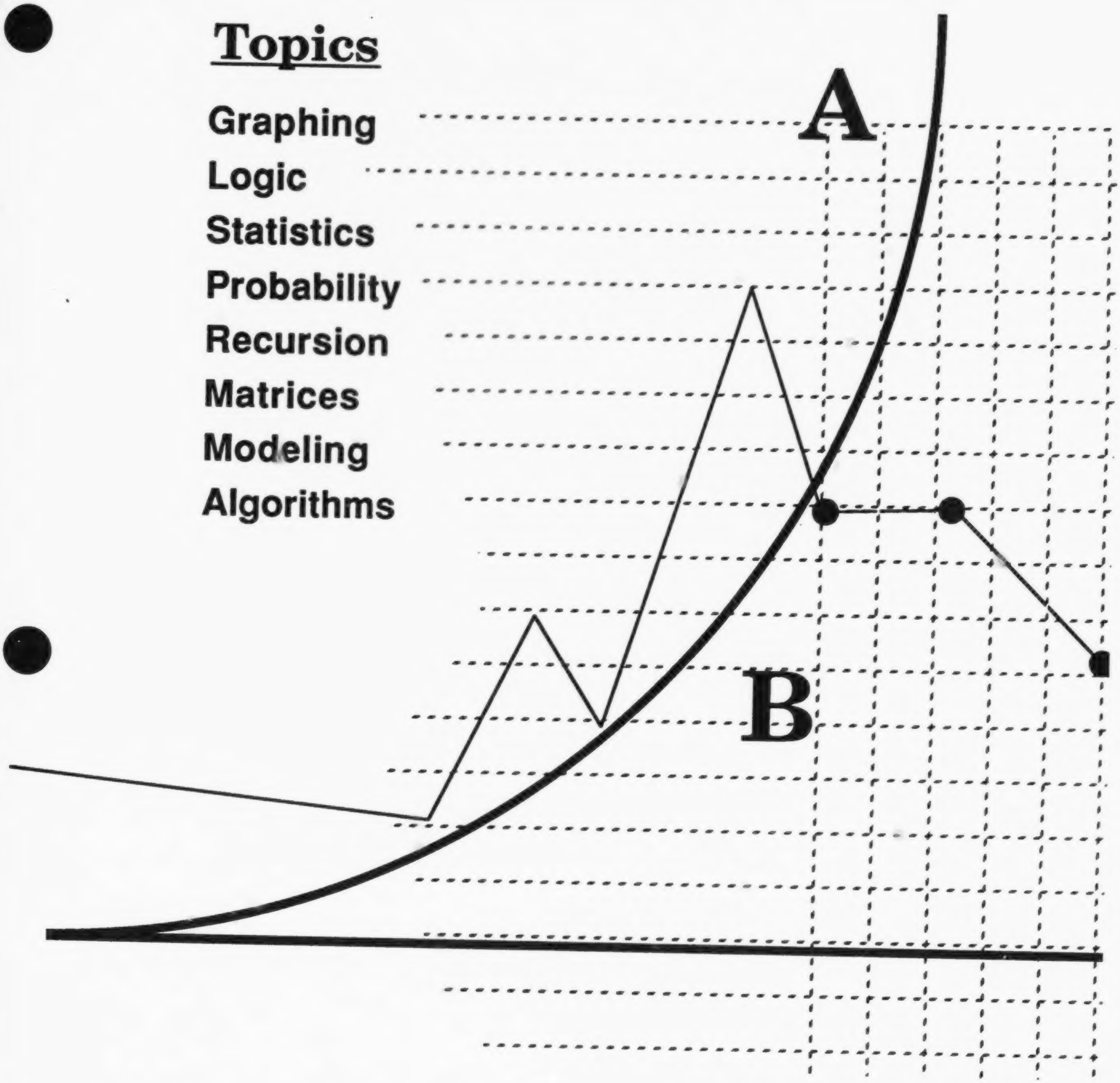
PROBLEM SOLVING

Transparency 3

General Mathematics

Topics

- Graphing
- Logic
- Statistics
- Probability
- Recursion
- Matrices
- Modeling
- Algorithms



PROBLEM SOLVING

National Council of Teachers of Mathematics 25 5

Algebra

Topics

Integrating with geometry and analysis
Functions
Graphing
Exploration
Experimentation
Finite methods
Numerical analysis
Algorithms
Real problems
Matrices

$a + 2b = 36$

$3x + 2y = 13$

$$5x - 2y = 11$$

nuMath

SemCalc

PROBLEM SOLVING

Transparency 5.

Geometry

Topics

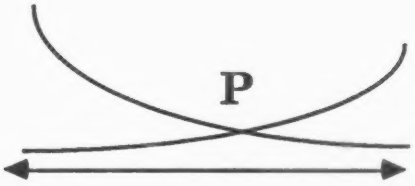
Coordinates

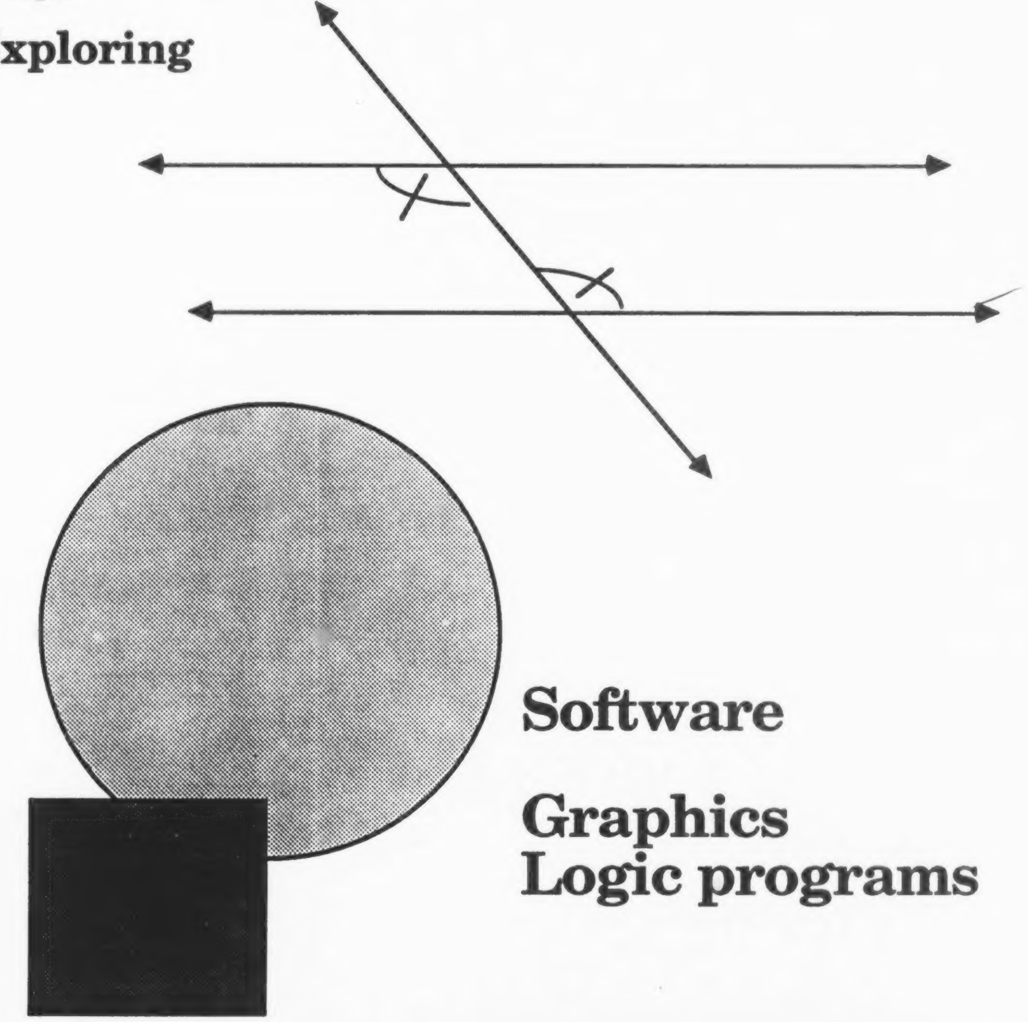
Vectors

Trigonometry

Logic

Exploring





Software

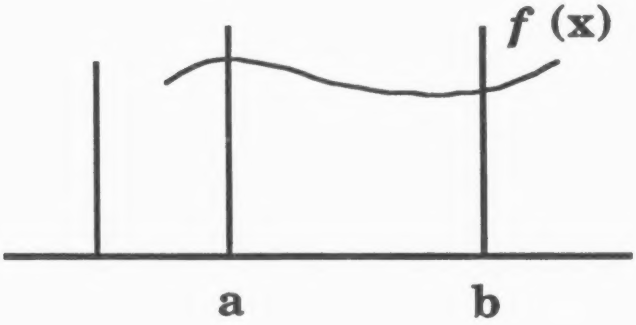
Graphics
Logic programs

PROBLEM SOLVING

Calculus

Topics

- Exploration
- Functions
- Limits
- Derivatives
- Integration - numerical methods
- Numerical approximations
- Realistic problems
- Graphing



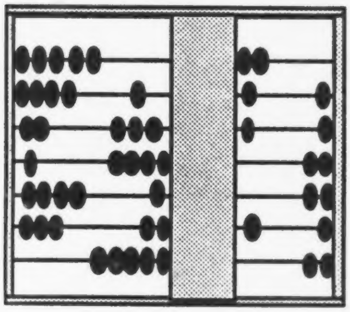
Software

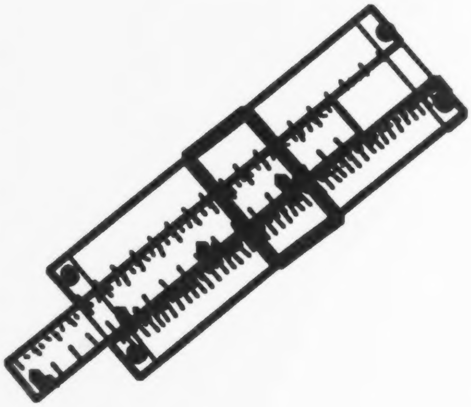
Transparency 7

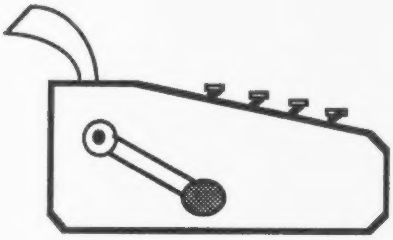
PROBLEM SOLVING

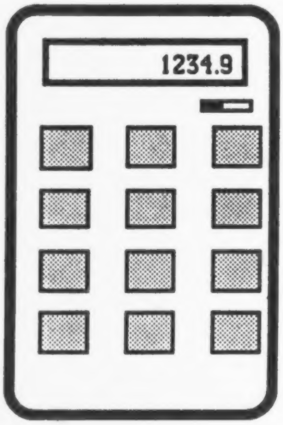
Session 30
A Look at The Future

PAST



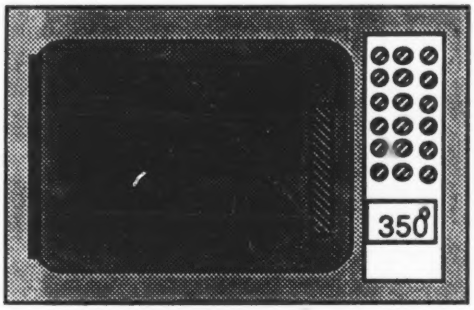




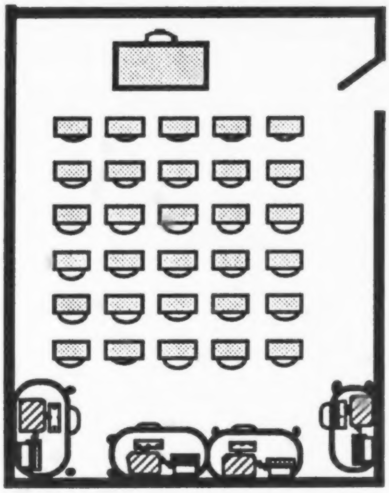


Transparency 1

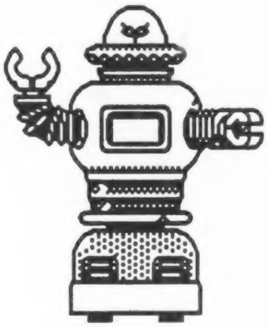
PRESENT



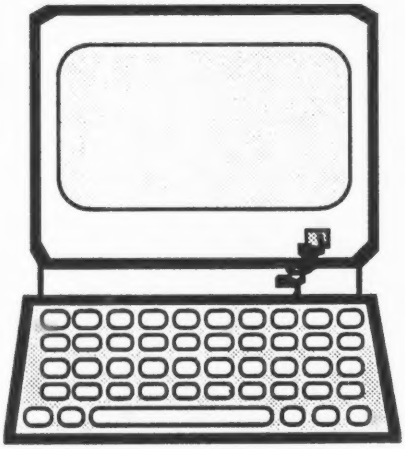
Push Button Appliances



Computer Lab



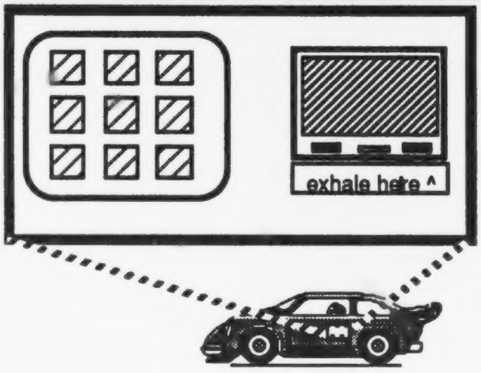
Robot



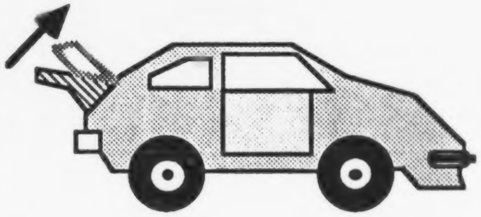
Microcomputer

Transparency 2

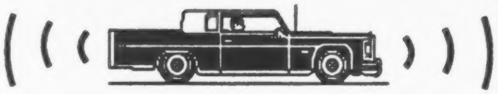
AUTOMOTIVE



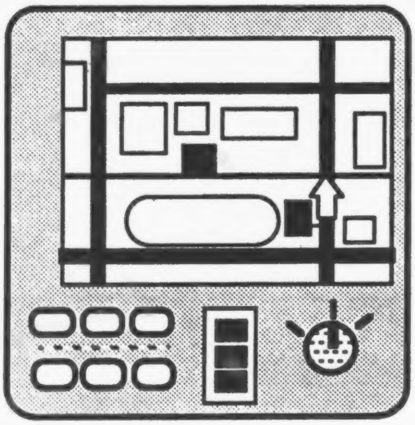
Push Button Door
with Breathalyzer



Adjustable
Aerodynamics



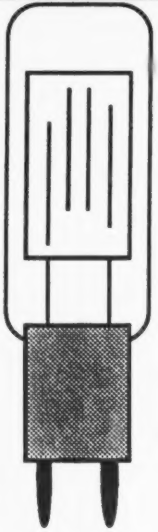
Front and Rear
Electronic Driving
(stop, etc.) Aids



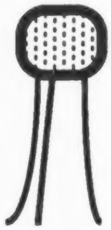
Screen Display Map

Transparency 3

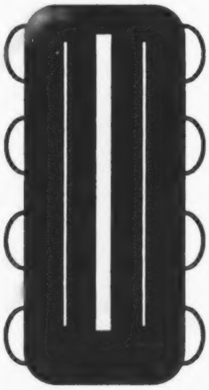
TECHNOLOGY



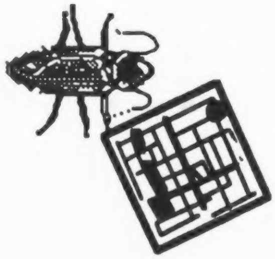
TUBE



TRANSISTOR

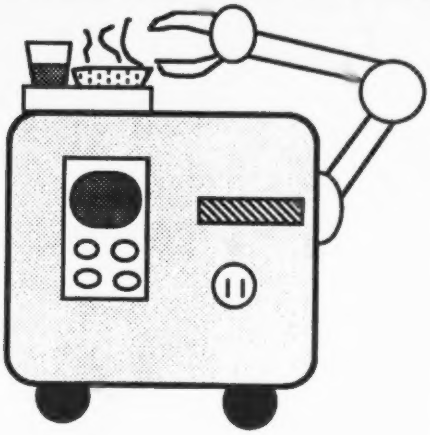


Integrated Circuit

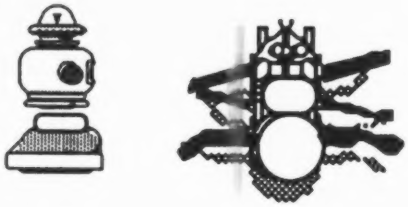


Chip

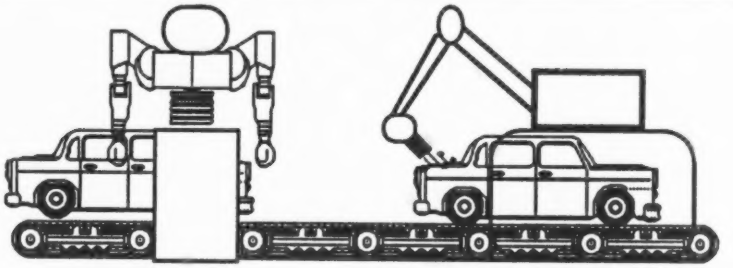
ROBOTS



Home



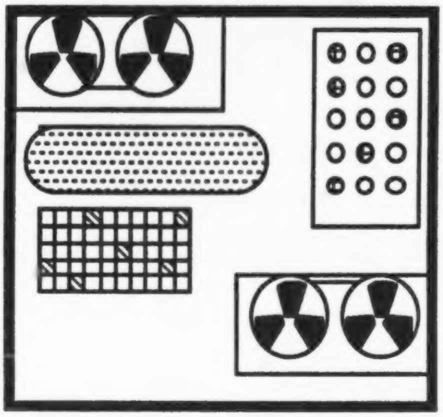
Entertainment



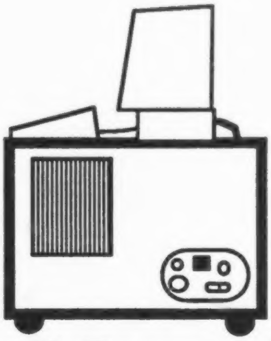
Assembly / Manufacturing

Transparency 5

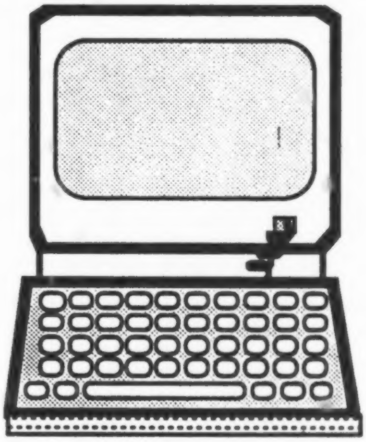
COMPUTERS



First Generation



Second Generation

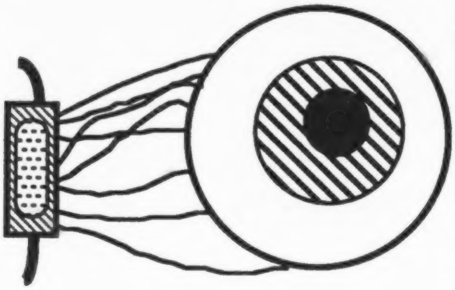


Third Generation

Fourth Generation

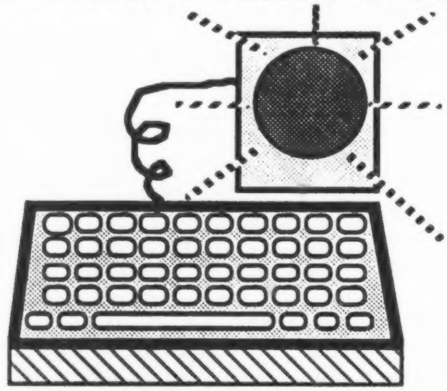
Transparency 6

MEDICAL

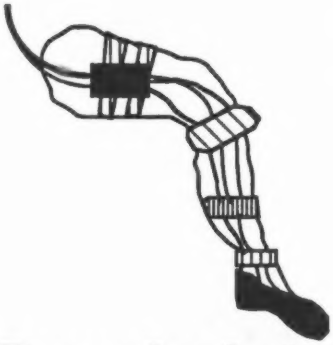


Blind to See

Deaf to Hear



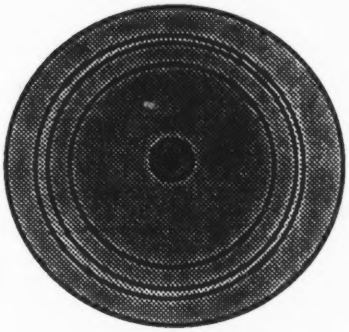
Mute to Speak
(Voice Synthesis)



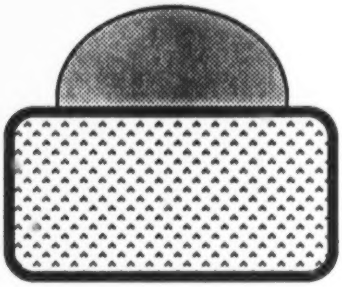
Paraplegic to Walk

Transparency 7

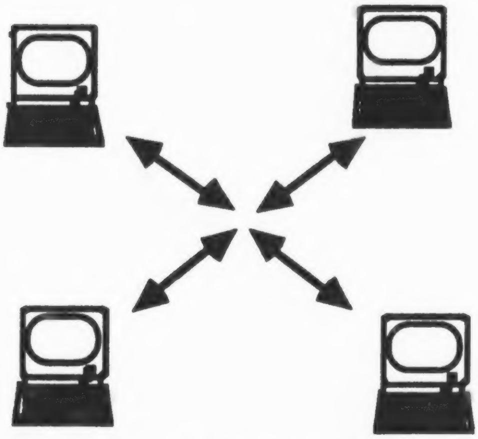
FUTURE



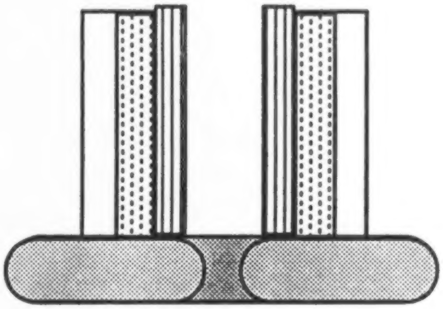
CD ROM



Bubble Memory



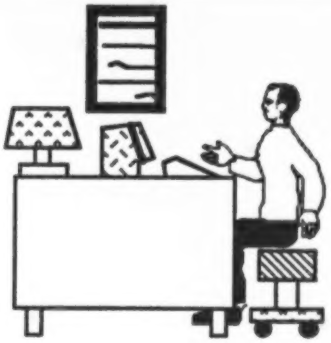
Idea Networks



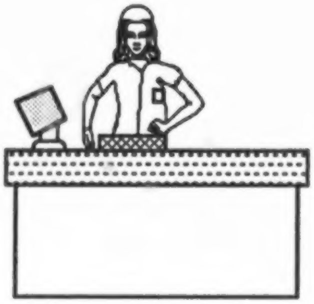
Supercomputer (CRAY)

Transparency 8

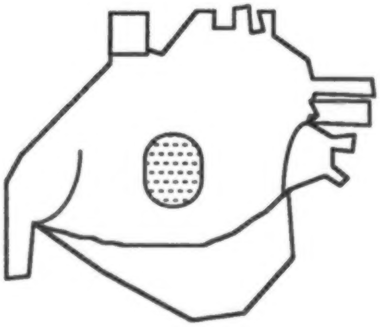
MEDICAL
Diagnosis/Psychiatric



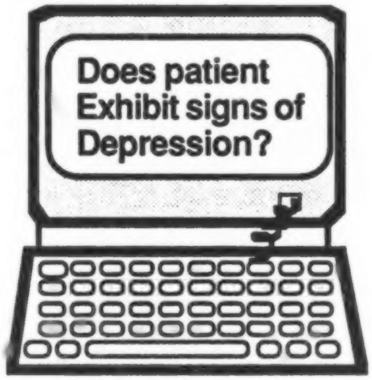
In the Office



In Hospitals

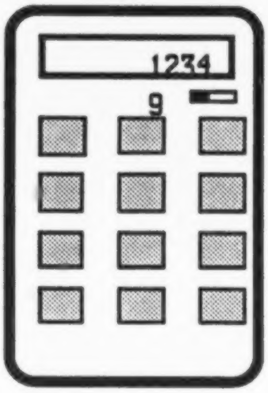


Artificial Heart

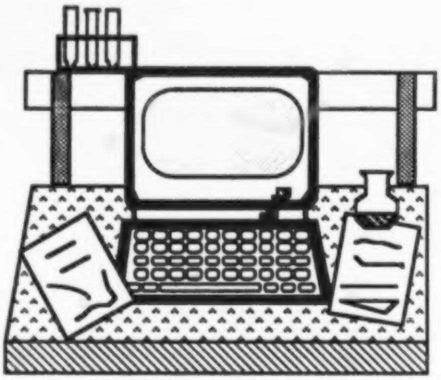

Does patient
Exhibit signs of
Depression?

Psychology

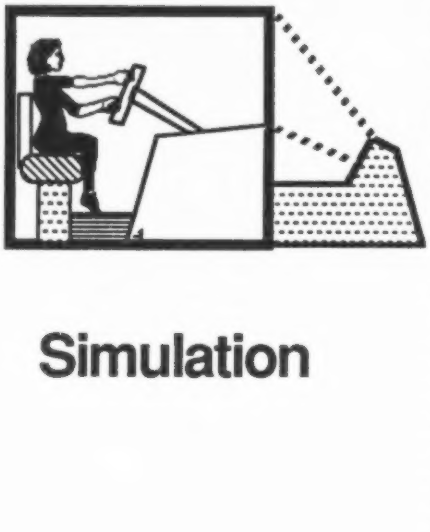
EDUCATION



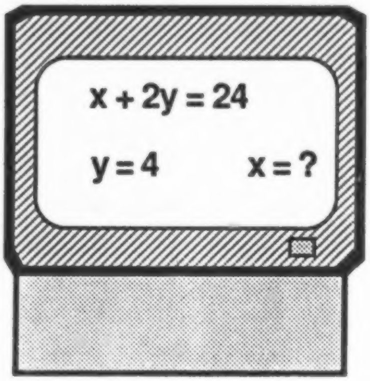
Calculator



Lab Setting

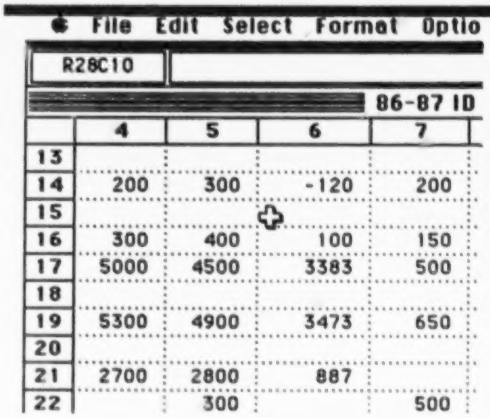


Simulation



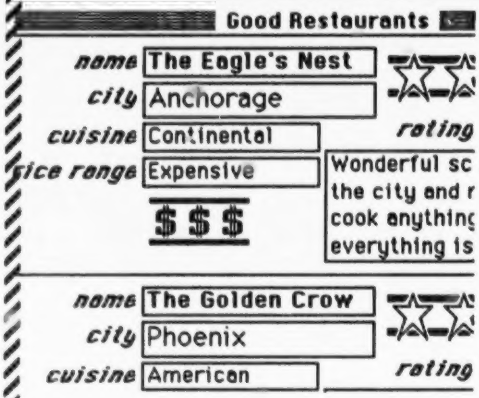
Testing

APPLICATIONS



	4	5	6	7
13				
14	200	300	-120	200
15				
16	300	400	100	150
17	5000	4500	3383	500
18				
19	5300	4900	3473	650
20				
21	2700	2800	887	
22		300		500

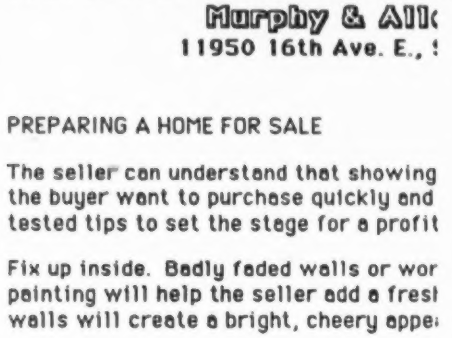
Spreadsheets



name The Eagle's Nest
city Anchorage
cuisine Continental
rice range Expensive
$ $ $
rating
Wonderful sc
the city and r
cook anythin
everything is

name The Golden Crow
city Phoenix
cuisine American
rating

Data Bases



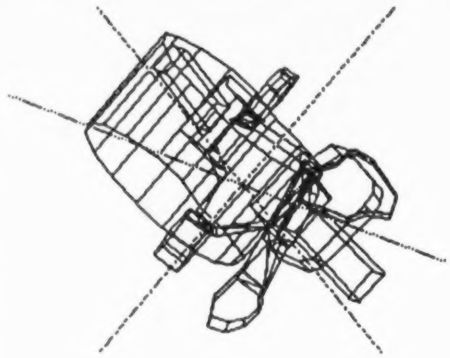
Murphy & All
11950 16th Ave. E.,

PREPARING A HOME FOR SALE

The seller can understand that showing
the buyer want to purchase quickly and
tested tips to set the stage for a profit

Fix up inside. Badly faded walls or wor
painting will help the seller add a fres
walls will create a bright, cheery appe

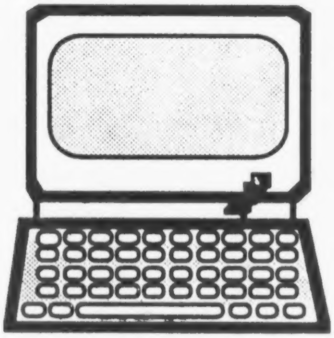
Word Processing

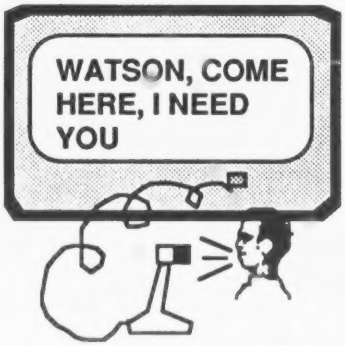


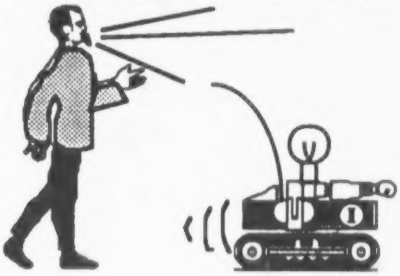
Graphics

Transparency 11

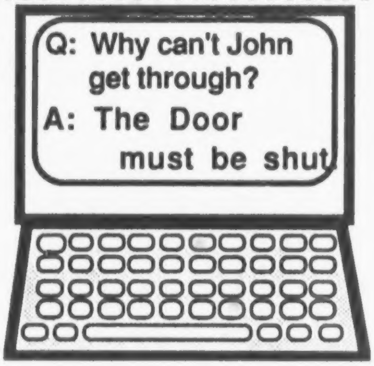
FUTURE




WATSON, COME HERE, I NEED YOU



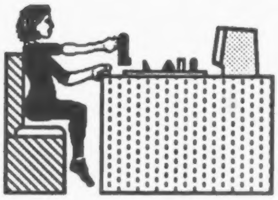
Voice Control


Q: Why can't John get through?
A: The Door must be shut

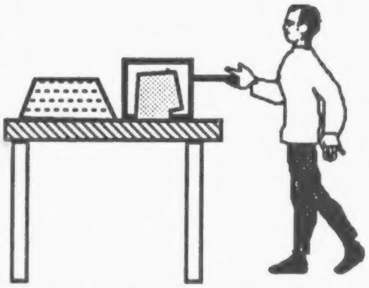
Machine Intuition

Transparency 12

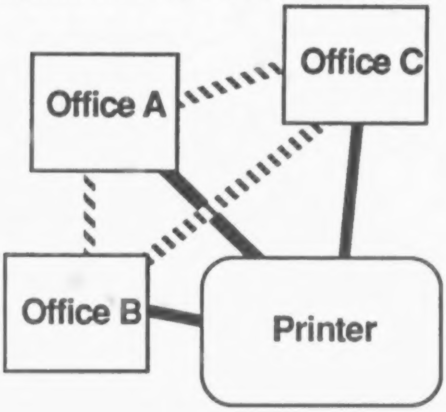
Artificial Intelligence



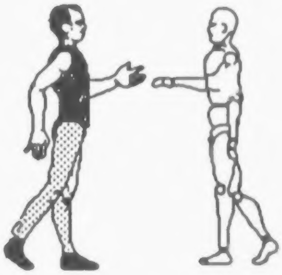
Chess



Collating Medical Data


Office A
Office C
Office B
Printer

Intelligent Branching



The Future?

Transparency 13

Session 31
Using Evaluation during In-service

Please check one:
Elementary ______
Middle __________
Secondary _______

Computers in Mathematics Classrooms

National Council of Teachers of Mathematics

Workshop Evaluation - Final

Please evaluate each of the following by circling the appropriate number: 1 means poor; 5 means excellent. Please add any additional comments.

	Poor				Excellent
Objectives clear?	1	2	3	4	5
met?	1	2	3	4	5
Presenters prepared?	1	2	3	4	5
effective?	1	2	3	4	5
Participants involved?	1	2	3	4	5
interactive?	1	2	3	4	5
Organization?	1	2	3	4	5
Flow?	1	2	3	4	5
Materials useful here?	1	2	3	4	5
later?	1	2	3	4	5
OVERALL RATING	1	2	3	4	5

Changes?

Other comments: use reverse side.

Evaluation Design

1. Preassessment
2. In-process data collection
3. Postassessment
4. Follow-up -- survey, newsletter, network

Writing a Report

1. Program objectives
2. Program description
3. Evaluation procedures
4. Conclusions

Keep the Readers in Mind: What would they like to know?

How will they use it?

566